Supplementary Document

This document contains supplementary information for the paper, such as proofs for lemmas and propositions in the base model and in the extensions. It is not intended for publication and will be posted on authors' website for readers' references. The structure of the document is as follows.

SA. Supplementary Information for the Base Model

SA.1. Summary Tables of Main Results

Table S.1 Impact of sales information on consumer search and purchase (compared to no information)

Information Type	Ranking Information	Volume Information
Search	Directional search (search bestseller first)	Directional search (search bestseller first)
Purchase	Total expected sales always increases as the belief that the bestseller is of high value increases	Total expected sales can decrease, when the two products have close sales and products are more likely to be of low value ex ante

Table S.2 Impact of (finer) sales information on consumers' purchased product value, search cost incurred before purchase, and overall surplus

	From No Info to Ranking Info			From Ranking Info to Volume Info		
	Search Cost			Search Cost		
	Low	Medium	High	Low	Medium	High
Purchased Product Value	Same	Decrease	Increase	Decrease	Ambiguous	Increase
Search Cost Incurred	First search increases Second search decreases			Decrease	First search decreases Second search increases	Increase
Overall Surplus	Increase			Increase		

Table S.3 Preferred type of bestseller information from the perspective of platform and consumers, respectively.

From the Perspective of	Preferred Type of Bestseller Information					
	Search-Cost Distribution		Number of Early Consumers		Product Values	
	Convex	Concave	High	Low	High	Low
Platform (sales-maximizing)	Volume	Ranking	Ranking	Volume	Ranking	Volume
Consumers (welfare-maximizing)	Volume					

SA.2. Discussion on Consumer Behavior in China and in the U.S.

Our results lead to a conjecture that the major shopping platforms' different practices in bestseller information provision could possibly reflect differences in consumers' search-cost distribution and quality distribution of the products that consumers consider purchasing in China and the U.S., which are evidenced by distinctive behavior of Chinese and American consumers in product search and shopping:

- For example, Chinese consumers are much more strongly reliant on social influences than their American counterparts. A consumer survey by KPMG[11] reveals that 60.8% of Chinese shoppers searched online for reviews and recommendations when gathering information for their potential purchases – a much higher percentage than that in the United States (39.4%). This is aligned with the report in a Forbes article[12] that about 75% of online users in China post after-purchase feedback at least once a month, while the figure is below 20% in the United States. It is also consistent with the consumer study in Doran (2000), which demonstrates that, compared to North American shoppers, Chinese consumers utilize much more interpersonal information and "exhibit more uniformity" in terms of their sources of information during product search.[13]

- Another consumer study[14], by Fitch, a global retail consultancy, discovers that Chinese shoppers are also "more engaged" than those from the United States, regarding shopping and learning about products. In particular, while shopping for fashion, electronics and grocery products, Chinese consumers are more active than American consumers in all of the three important states, "having fun" (70% vs 41%), "learning something" (79% vs 57%), and "being inspired" (55% vs 50%). Only 3% of consumers in China classify themselves as reluctant shoppers, significantly lower than the percentage in the States (15%).

- This is in line with the observations that Chinese consumers tend to seek more information before purchase than American consumers. They often use real-time messaging services within e-commerce platforms to ask sellers detailed questions about products, which is less common in the US market. It is noted that Chinese consumers require an average of twelve touch points before committing to a purchase, a sharp contrast to their US counterparts, who typically need just four points.[15]

These differences in consumers' product search and shopping behavior may be attributed to various cultural and market factors:

- For example, Chinese consumers' stronger dependence on social influences may correspond to collectivist tendencies in Eastern cultures, while American consumers tend to make decision more independently as Western cultures emphasize individualism (Doran, 2000). In line with it, Kwan (2025) find that "Mainland Chinese rely on social monitoring and trust cues in e-commerce transactions", while "Americans tend to rely on individual integrity trust cues."[16]

[11] assets.kpmg.com/content/dam/kpmg/cn/pdf/en/2016/11/china-s-connected-consumer-2016.pdf

[12] www.forbes.com/sites/onmarketing/2014/08/07/what-us-marketers-can-learn-from-social-commerce-in-china/

[13] escholarship.mcgill.ca/concern/theses/12579t935

[14] `www.wpp.com/en/-/media/Project/WPP/Files/Imported-News/chinese-dream_feb14.pdf`

[15] blog.adobe.com/en/publish/2020/01/11/5-takeaways-from-nrf-2020-vision-retail-s-big-show

[16] www.emerald.com/mip/article-abstract/doi/10.1108/MIP-10-2024-0702/1317347/How-do-cultural-lay-beliefs-affect-trust-decisions?

• In the meanwhile, Chinese consumers' more active involvement and need for more information before purchase are consistent with the argument in Zhao et al. (2021) that, because institutional trust environment may not have been fully developed in China, Chinese consumers may be alert and vigilant in purchasing.[17] The shopping behavior is also aligned with Chinese consumers' lower sense of safety when shopping online: a recent study[18] by the Interactive Advertising Bureau (IAB) reveals that only 13% of Chinese shoppers feel completely safe shopping online, much lower than the percentage in the U.S. (30%). In particular, according to the study, the lower confidence of Chinese consumers about online shopping can be associated with their concerns about vendor integrity and product quality. These concerns can be related to an observation[19] that "based on the unique business environment, a primary method to conducting e-commerce" in China is C2C (Consumer to Consumer), while in America, "it is more prevalent to have B2C (Business to Consumer)". This observation is evidenced by a finding in the IAB study that 69% of Chinese customers have purchased from C2C sellers, while 44% of U.S. customers have never done so. Compared with C2C, B2C is generally believed to provide "better pre-sales and after-sales services, and product quality can be guaranteed" (Shen 2019) and involve lower transaction risks (Xu et al. 2010, Lu et al. 2024, Sun et al 2025). Because of these market-related factors, Chinese consumers tend to require more information and social proof to build trust before purchase.

SA.3. Proofs for Results in the Base Model

Proof of Lemma 1 For (i), first notice that as the value of a product is always higher than the reservation utility and a consumer must search a product before purchasing it, a consumer purchases a product if and only if she performs the first search. Recall that a consumer who does not search has zero utility, i.e., the reservation utility. Now we prove that consumers with search cost $s \le p_h u_h + p_l u_l$ perform the first search. First by (1) the expected utility of the first search is $p_h u_h + p_l \max(u_l, u_h p_h + u_l p_l - s) - s$, where $\max(u_l, u_h p_h + u_l p_l - s)$ is the expected utility when the first search is a low type as a consumer may perform the second search depending on her search cost. As $p_h u_h + p_l \max(u_l, u_h p_h + u_l p_l - s) - s \ge p_h u_h + p_l u_l - s$, a consumer with search cost lower than $p_h u_h + p_l u_l$ will search as the expected utility of doing so is greater than the reservation utility. To see that a consumer with search cost higher than $p_h u_h + p_l u_l$ does not search, notice that when $s > p_h u_h + p_l u_l$, $u_h p_h + u_l p_l - s < 0 < u_l$, implying that the consumer does not perform the second search and thus her expected utility of first search is $p_h u_h + p_l u_l - s$. Therefore, the consumer does not perform the first search as the expected utility is lower than the reservation utility.

For (ii), as consumers have symmetric belief for both products, each product is equally likely to be searched first. If the first search reveals a high-value product, then the consumer buys it as this is already the best he can get; if the first search reveals a low value product, then the consumer needs to evaluate whether or not to search the second product. The expected utility of second search is $p_h(u_h - u_l) - s = p_h \Delta - s$. Thus, only consumers with search cost lower than $p_h \Delta$ perform the second search and they purchase the high type product if there is one and randomly purchase a product otherwise; consumers with higher search cost purchase the product that they explore in their first and only search. □

[17] pmc.ncbi.nlm.nih.gov/articles/PMC8155702/

[18] www.iab.com/wp-content/uploads/2016/11/IAB-US-China-Digital-Commerce-Study_FINAL.pdf

[19] www.atlantis-press.com/proceedings/isemss-19/125918571

Proof of Proposition 1 If the two products have the same value, by symmetry every consumer will buy either product with equal probability. As the total number of consumers making purchases in the first period is n_1, the number of consumers purchasing either product follows the distribution G_s. When the two products differ in value, by Lemma 1 all consumers with search cost smaller than $p_h(u_h - u_l)$ purchase the high type and the remaining $n_1 - m$ consumers (who only perform one search) purchase either product with equal probability. Thus, the sales number of the high type product, denoted by Y, equals to $m + Z$ where $Z \in [0, n_1 - m]$ represents the number of consumers who search only once and purchase the high type product and follows $Binomial(x, n_1 - m, 1/2)$. Since $\Pr[Y \le x] = \Pr[Z \le x - m]$, $Y \in [m, n_1]$ follows $Binomial(x - m, n_1 - m, 1/2)$. For notational convenience, we extend the domain of Y to $[0, n_1]$ and define the probability of $Y \in [0, m]$ to be zero. That is, Y follows the distribution G_a. The distribution of the low type is as claimed because the two products have total sales n_1. As the high-value product's sale follows $G_a(x)$ when the two products' values are different, the probability that the high-value product has higher sales is $1 - G_a(n_1/2)$ when n_1 is odd and is $1 - G_a(n_1/2) + g_a(n_1/2)/2$ when n_1 is even. Both probabilities are strictly less than one when $n_1 \ge 2m$. □

Proof of Proposition 2 We first show that product i^*, the product with higher sales, is more likely of high value than product $-i^*$, the product with lower sales. We focus on the case that n_1 is odd. The case that n_1 is even is similar and thus omitted. Under ranking information, by the Bayes formula, the updated belief that product i^* is of high type is

$$\Pr[u_{i^*} = u_h | X_{i^*} \ge \frac{n_1}{2}] = \frac{p_h(p_h \bar{G}_s(\frac{n_1}{2}) + p_l(1 - G_a(\frac{n_1}{2})))}{p_h(p_h \bar{G}_s(\frac{n_1}{2}) + p_l(1 - G_a(\frac{n_1}{2}))) + p_l(p_l \bar{G}_s(\frac{n_1}{2}) + p_h G_a(\frac{n_1}{2}))}$$

where $p_h p_h \bar{G}_s(\frac{n_1}{2})$ =Pr(product i is high type, product $3-i$ is high type)Pr(product i has higher sales|product i is high type, product $3-i$ is high type) for given $i \in \{1, 2\}$, and the other terms follow similarly. Meanwhile, the belief that product $-i^*$ is high type is

$$\Pr[u_{-i^*} = u_h | X_{i^*} \ge \frac{n_1}{2}] = \frac{p_h(p_h \bar{G}_s(\frac{n_1}{2}) + p_l G_a(\frac{n_1}{2}))}{p_h(p_h \bar{G}_s(\frac{n_1}{2}) + p_l G_a(\frac{n_1}{2})) + p_l(p_l \bar{G}_s(\frac{n_1}{2}) + p_h(1 - G_a(\frac{n_1}{2})))}$$

As $G_a(\frac{n_1}{2}) = \sum_{x=0}^{\lfloor \frac{n_1}{2} \rfloor} g_a(x) = \sum_{x=0}^{\lfloor \frac{n_1}{2} \rfloor}$ Binomial$(x - m, n_1 - m, 1/2)$, $G_a(\frac{n_1}{2})$ is the probability that among $n_1 - m$ trials, at most $\lfloor \frac{n_1}{2} \rfloor - m$ succeed. As $\frac{\lfloor \frac{n_1}{2} \rfloor - m}{n_1 - m} < 1/2$, $G_a(\frac{n_1}{2}) < 1/2$. It follows that product i^* is more likely of high value than product $-i^*$.

Under volume information,

$$\Pr[u_{i^*} = u_h | X_{i^*} = x, X_{i^*} \ge \frac{n_1}{2}] = \frac{p_h(p_h g_s(x) + p_l g_a(x))}{p_h(p_h g_s(x) + p_l g_a(x)) + p_l(p_h g_a(n_1 - x) + p_l g_s(n_1 - x))}$$

and the posterior for product $-i^*$ is

$$\Pr[u_{-i^*} = u_h | X_{i^*} = x, X_{i^*} \ge \frac{n_1}{2}] = \frac{p_h(p_h g_s(n_1 - x) + p_l g_a(n_1 - x))}{p_h(p_h g_s(n_1 - x) + p_l g_a(n_1 - x)) + p_l(p_h g_a(x) + p_l g_s(x))}$$

By the property of binomial distribution we have $g_s(x) = g_s(n_1 - x)$. To prove $\Pr[u_{i^*} = u_h | X_{i^*} = x, X_{i^*} \ge \frac{n_1}{2}] \ge \Pr[u_{-i^*} = u_h | X_{i^*} = x, X_{i^*} \ge \frac{n_1}{2}]$, it thus suffices to show $g_a(x) \ge g_a(n_1 - x)$. Notice that as $x \ge n_1/2$, we have $x \ge n_1 - x$. Therefore, if $x < m$, it must be that $g_a(x) = g_a(n_1 - x) = 0$. Thus, we will focus on the case $x \ge m$. Next we show $g_a(x) = \binom{n_1 - m}{x - m}(\frac{1}{2})^{n_1 - m} > \binom{n_1 - m}{n_1 - x - m}(\frac{1}{2})^{n_1 - m} = g_a(n_1 - x)$, where $m = n_0 F(p_h \Delta)$.

To see the inequality, notice that $x \ge \frac{n_1}{2}$ and for $t_1, t_2 \in [0, n_1 - m]$, $\binom{n_1 - m}{t_1} > \binom{n_1 - m}{t_2}$ if and only if $|t_1 - (n_1 - m)/2| < |t_2 - (n_1 - m)/2|$. Notice that $(n_1 - m)/2 - (x - m) \le m/2$ and $(n_1 - m)/2 - (n_1 - x - m) \ge m/2$.

There are two cases: (i) $(n_1-m)/2-(x-m)>0$, then $|x-m-(n_1-m)/2|<|n_1-x-m-(n_1-m)/2|$. (ii) $(n_1-m)/2-(x-m)<0$, then $|x-m-(n_1-m)/2|=x-m-(n_1-m)/2$ and $|n_1-x-m-(n_1-m)/2|=(n_1-m)/2-(n_1-x-m)$. As $(n_1-m)/2-(n_1-x-m)-(x-m-(n_1-m)/2)=m$, $|x-m-(n_1-m)/2|<|n_1-x-m-(n_1-m)/2|$. It follows that the expected value of product i^* is greater. By Lemma S.1 below, it is optimal to search product i^* first. □

Lemma S.1. *Assume that product value follows a two-point distribution. When the probability of high value differs by product, it is optimal to first search the product with higher probability of high value if a consumer searches.*

Proof of Lemma S.1 We will prove that it is always optimal to first search the product with higher expected value when either product's value follows a two point distribution with possible values v_1, v_2 $(v_1>v_2)$.

Let u_1, u_2 be the value of the two products. Define $q_{jk}:=\Pr(u_1=v_j, u_2=v_k), j=1,2; k=1,2$. Assume that product 1 has higher expected value. That is, $(q_{11}+q_{12})v_1+(1-(q_{11}+q_{12}))v_2>(q_{11}+q_{21})v_1+(1-(q_{11}+q_{21}))v_2$, which is equivalent to $q_{12}>q_{21}$.

Consider a consumer's expected utility if she searches product 1 first. If product 1 is high type, then the consumer gets v_1 and this happens with probability $q_{11}+q_{12}$. If it is low type, then she compares $\frac{q_{21}}{q_{22}+q_{21}}(v_1-v_2)$ with s to decide whether she should search product 2. In that case her utility is $\max[v_2, v_2+\frac{q_{21}}{q_{22}+q_{21}}(v_1-v_2)-s]$, which happens with probability $q_{21}+q_{22}$. Let the expected utility be r_1, then

$$r_1=v_1(q_{11}+q_{12})+(q_{21}+q_{22})\max[v_2, v_2+\frac{q_{21}}{q_{22}+q_{21}}(v_1-v_2)-s]$$

Consider now the case that the consumer searches product 2 first. If product 2 is high type, she gets v_1, which happens with probability $q_{11}+q_{21}$. If it is low type, she compares $\frac{q_{12}}{q_{12}+q_{22}}(v_1-v_2)$ with s, and the value is $\max[v_2, v_2+\frac{q_{12}}{q_{12}+q_{22}}(v_1-v_2)-s]$, which happens with probability $q_{12}+q_{22}$. Let the expected utility be r_2, then

$$r_2=v_1(q_{11}+q_{21})+(q_{12}+q_{22})\max[v_2, v_2+\frac{q_{12}}{q_{12}+q_{22}}(v_1-v_2)-s]$$

Notice we have $q_{12}>q_{21}$ so $\frac{q_{12}}{q_{12}+q_{22}}>\frac{q_{21}}{q_{22}+q_{21}}$.

Now we compare r_1 and r_2 in three cases. The first case is $\frac{q_{12}}{q_{12}+q_{22}}(v_1-v_2)-s<0$, which implies $\frac{q_{21}}{q_{22}+q_{21}}(v_1-v_2)-s<0$ and thus $r_1>r_2$ (as $q_{12}>q_{21}$).

The second case is $\frac{q_{12}}{q_{12}+q_{22}}(v_1-v_2)>s>\frac{q_{21}}{q_{22}+q_{21}}(v_1-v_2)$. Thus,

$$\begin{aligned}
r_1-r_2&=v_1(q_{11}+q_{12})+(q_{21}+q_{22})v_2-v_1(q_{11}+q_{21})-(q_{12}+q_{22})(v_2+\frac{q_{12}}{q_{22}+q_{12}}(v_1-v_2)-s)\\
&=v_1(q_{12}-q_{21})-v_2(q_{12}-q_{21})-q_{12}(v_1-v_2)+(q_{12}+q_{22})s\\
&=(v_1-v_2)(q_{12}-q_{21})+(q_{12}+q_{22})s-q_{12}(v_1-v_2)\\
&=(q_{12}+q_{22})s-q_{21}(v_1-v_2)\\
&=(q_{12}+q_{22})(s-\frac{q_{21}}{q_{22}+q_{12}}(v_1-v_2))\\
&\geq(q_{12}+q_{22})(s-\frac{q_{21}}{q_{22}+q_{21}}(v_1-v_2))\\
&\geq 0
\end{aligned}$$

The last inequality follows from the condition of this case.

The last case is $s < \frac{q_{21}}{q_{22}+q_{21}}(v_1 - v_2)$. In this case, we have

$$\begin{aligned}
r_1 - r_2 &= v_1(q_{11}+q_{12}) + (q_{21}+q_{22})(v_2 + \frac{q_{21}}{q_{22}+q_{21}}(v_1-v_2) - s) - v_1(q_{11}+q_{21}) \\
&\quad - (q_{12}+q_{22})(v_2 + \frac{q_{12}}{q_{22}+q_{12}}(v_1-v_2) - s) \\
&= (q_{12}-q_{21})v_1 - (q_{12}-q_{21})v_2 + q_{21}(v_1-v_2) - q_{12}(v_1-v_2) + (q_{12}-q_{21})s \\
&= (q_{12}-q_{21})s \\
&\geq 0
\end{aligned}$$

As in all the cases we have $r_1 \geq r_2$, it is always better to first search product 1 than product 2. □

Proof of Lemma 2 Before present the proof, we first derive the expression of π_1^r, π_2^r, $\pi_1^v(x)$, and $\pi_2^v(x)$. Note that $i^* = 1$ is equivalent to $X_1 \geq \frac{n_1}{2}$. By the Bayes' rule, if n_1 is odd,

$$\begin{aligned}
\pi_1^r &= \Pr[u_1 = u_h | X_1 \geq \tfrac{n_1}{2}] \\
&= \frac{\Pr[u_1=u_2=u_h, X_1 \geq \frac{n_1}{2}] + \Pr[u_1=u_h, u_2=u_l, X_1 \geq \frac{n_1}{2}]}{\begin{pmatrix} \Pr[u_1=u_2=u_h, X_1 \geq \frac{n_1}{2}] + \Pr[u_1=u_h, u_2=u_l, X_1 \geq \frac{n_1}{2}] \\ +\Pr[u_1=u_l, u_2=u_h, X_1 \geq \frac{n_1}{2}] + \Pr[u_1=u_2=u_l, X_1 \geq \frac{n_1}{2}] \end{pmatrix}} \\
&= \frac{p_h^2 \bar{G}_s(\frac{n_1}{2}) + p_h p_l \bar{G}_a(\frac{n_1}{2})}{p_h^2 \bar{G}_s(\frac{n_1}{2}) + p_h p_l \bar{G}_a(\frac{n_1}{2}) + p_h p_l G_a(\frac{n_1}{2}) + p_l^2 \bar{G}_s(\frac{n_1}{2})} \\
&= p_h^2 + 2p_h p_l (1 - G_a(\tfrac{n_1}{2}))
\end{aligned}$$

The second equality takes account of four possible scenarios: (i) both products are of high value, (ii) the higher-ranked product is of high value, while the lower-ranked one is of low value, (iii) the higher-ranked product is of low value, while the lower-ranked one is of high value, and (iv) both products are of low value, and the last equality is by the facts $G_s(\frac{n_1}{2}) = \frac{1}{2}$ and $\frac{1}{2}p_h^2 + p_h p_l + \frac{1}{2}p_l^2 = \frac{1}{2}$. The expression of π_1^r when n_1 is even can be derived similarly.

Similarly, the posterior under the sales-volume information is as follows: for $x \geq n_1/2$,

$$\begin{aligned}
\pi_1^v(x) &= \frac{\Pr[u_1=u_2=u_h, X_1=x] + \Pr[u_1=u_h, u_2=u_l, X_1=x]}{\begin{pmatrix} \Pr[u_1=u_2=u_h, X_1=x] + \Pr[u_1=u_h, u_2=u_l, X_1=x] \\ +\Pr[u_1=u_l, u_2=u_h, X_1=x] + \Pr[u_1=u_2=u_l, X_1=x] \end{pmatrix}} \\
&= \frac{p_h^2 g_s(x) + p_h p_l g_a(x)}{p_h^2 g_s(x) + p_h p_l g_a(x) + p_h p_l g_a(n_1 - x) + p_l^2 g_s(x)}
\end{aligned}$$

For π_2^r, by the Bayes' rule, when n_1 is odd,

$$\pi_2^r = \Pr[u_2 = u_h | X_1 \geq \frac{n_1}{2}, u_1 = u_l] \tag{S.1}$$

$$= \frac{\Pr[u_1=u_l, u_2=u_h, X_1 \geq \frac{n_1}{2}]}{\Pr[u_1=u_l, u_2=u_h, X_1 \geq \frac{n_1}{2}] + \Pr[u_1=u_2=u_l, X_1 \geq \frac{n_1}{2}]}$$

$$= \frac{p_h G_a(\frac{n_1}{2})}{p_h G_a(\frac{n_1}{2}) + p_l \bar{G}_s(\frac{n_1}{2})} \tag{S.2}$$

The expression of π_2^r under an even n_1 can be derived similarly. Furthermore, for $x \geq n_1/2$,

$$\pi_2^v(x) = \frac{\Pr[u_1=u_l, u_2=u_h, X_1=x]}{\Pr[u_1=u_l, u_2=u_h, X_1=x] + \Pr[u_1=u_2=u_l, X_1=x]}$$

$$= \frac{p_h g_a(n_1 - x)}{p_h g_a(n_1 - x) + p_l g_s(x)} \tag{S.3}$$

ν_1^r and $\nu_1^v(x)$ can be derived similarly.

For ranking information, when n_1 is odd, we have

$$\pi_1^r = p_h^2 + 2p_h p_l(1 - G_a(\frac{n_1}{2}))$$
$$\nu_1^r = p_h^2 + 2p_h p_l G_a(\frac{n_1}{2})$$

Thus, $\nu_1^r \leq \pi_1^r$ as $G_a(\frac{n_1}{2}) \leq 1/2$. To see that $\pi_2^r \leq \nu_1^r$, notice that

$$\pi_2^r = \frac{p_h G_a(\frac{n_1}{2})}{p_h G_a(\frac{n_1}{2}) + p_l/2} = \frac{2p_h p_l G_a(\frac{n_1}{2})}{2p_h p_l G_a(\frac{n_1}{2}) + p_l^2}$$

Let $a = p_h, b = 2p_h p_l G_a(\frac{n_1}{2})$, then

$$\begin{aligned}
\nu_1^r - \pi_2^r &= a^2 + b - \frac{b}{b + (1-a)^2} \\
&= \frac{(a^2+b)(b+(1-a)^2) - b}{b+(1-a)^2} \\
&= \frac{b^2 + (2a^2 - 2a)b + a^2(1-a)^2}{b+(1-a)^2} \\
&= \frac{(b - a(1-a))^2}{b+(1-a)^2} \geq 0
\end{aligned}$$

The case for n_1 is even is similar and omitted here.

For volume information, by definition we have:

$$\begin{aligned}
\nu_1^v(x) =& \Pr[u_{-i^*} = u_h | X_{i^*} = x] \\
=& \frac{\Pr[u_{i^*} = u_{-i^*} = u_h, X_{i^*} = x] + \Pr[u_{i^*} = u_l, u_{-i^*} = u_h, X_{i^*} = x]}{\left(\begin{array}{c} \Pr[u_{i^*} = u_{-i^*} = u_h, X_{i^*} = x] + \Pr[u_{i^*} = u_h, u_{-i^*} = u_l, X_{i^*} = x] \\ + \Pr[u_{i^*} = u_l, u_{-i^*} = u_h, X_{i^*} = x] + \Pr[u_{i^*} = u_{-i^*} = u_l, X_{i^*} = x] \end{array} \right)} \\
=& \frac{p_h^2 g_s(x) + p_h p_l g_a(n_1 - x)}{p_h^2 g_s(x) + p_h p_l g_a(x) + p_h p_l g_a(n_1 - x) + p_l^2 g_s(x)}
\end{aligned}$$

From the proof of Proposition 2, we have $\pi_1^v(x) \geq \nu_1^v(x)$. To prove $\nu_1^v(x) \geq \pi_2^v(x)$, note:

$$\begin{aligned}
\nu_1^v(x) =& \frac{p_h(p_h g_s(x) + p_l g_a(n_1 - x))}{p_h(p_h g_s(x) + p_l g_a(n_1 - x)) + p_l(p_l g_s(x) + p_h g_a(x))} \\
=& \frac{p_h}{p_h + p_l \frac{p_l g_s(x) + p_h g_a(x)}{p_h g_s(x) + p_l g_a(n_1 - x)}} \\
\pi_2^v(x) =& \frac{p_h g_a(n_1 - x)}{p_h g_a(n_1 - x) + p_l g_s(n_1 - x)} = \frac{p_h}{p_h + p_l \frac{g_s(n_1 - x)}{g_a(n_1 - x)}}
\end{aligned}$$

Noting that $g_s(x) = g_s(n_1 - x)$, denote $A(x) := \frac{p_l g_s(x) + p_h g_a(x)}{p_h g_s(x) + p_l g_a(n_1 - x)}$ and $B(x) := \frac{g_s(n_1 - x)}{g_a(n_1 - x)}$ and it is equivalent to show $A(x) \leq B(x), \forall x \geq \frac{n_1}{2}$. Note that

$$\begin{aligned}
B(x) - A(x) &= \frac{g_s(n_1 - x)}{g_a(n_1 - x)} - \frac{p_l g_s(x) + p_h g_a(x)}{p_h g_s(x) + p_l g_a(n_1 - x)} \\
&= \frac{p_h(g_s(x) g_s(n_1 - x) - g_a(x) g_a(n_1 - x))}{g_a(n_1 - x)(p_h g_s(x) + p_l g_a(n_1 - x))}
\end{aligned}$$

Recall that for $x < m$, $g_a(x) = 0$ and thus $B(x) \geq A(x)$. Now, consider $x \geq m$ and it is equivalent to show $\frac{g_s(x) g_s(n_1 - x)}{g_a(x) g_a(n_1 - x)} \geq 1$. Recall that m is a nonnegative integer. If $m = 0$, $g_a(x) \equiv g_s(x)$ and hence $\frac{g_s(x) g_s(n_1 - x)}{g_a(x) g_a(n_1 - x)} = 1$. For $m \geq 1$, we have

$$\begin{aligned}
\frac{g_s(x) g_s(n_1 - x)}{g_a(x) g_a(n_1 - x)} &= \frac{\binom{n_1}{x}(1/2)^{n_1}}{\binom{n_1 - m}{x - m}(1/2)^{n_1 - m}} \frac{\binom{n_1}{x}(1/2)^{n_1}}{\binom{n_1 - m}{n_1 - x - m}(1/2)^{n_1 - m}} \\
&= 4^{-m} \frac{n_1 \cdot (n_1 - 1) \cdot \ldots \cdot (n_1 - m + 1)}{(x - m + 1) \cdot (x - m + 2) \cdot \ldots \cdot x} \frac{n_1 \cdot (n_1 - 1) \cdot \ldots \cdot (n_1 - m + 1)}{(n_1 - x - m + 1) \cdot (n_1 - x - m + 2) \cdot \ldots \cdot (n_1 - x)} \\
&\geq 1
\end{aligned}$$

To see the inequality, notice that we have $\frac{c^2}{e\cdot f}\geq 4$ for $e+f\leq c$, where e,f,c are non-negative real numbers. Thus, $n_1^2\geq 4x(n_1-x)$, $(n_1-1)^2\geq 4(x-1)(n_1-x-1)$, ..., and $(n_1-m+1)^2\geq 4(x-m+1)(n_1-x-m+1)$. Combining these m inequalities, we have the desired inequality. □

Proof of Lemma 3 Recall that a second period consumer makes a first search (and purchases) if and only if $\pi_1^t u_h+(1-\pi_1^t)\max(u_l,\pi_2^t u_h+(1-\pi_2^t)u_l-s)-s\geq 0$. For (i), we will show that the search threshold is $u_h\pi_1^t+u_l(1-\pi_1^t)$. Similar to Lemma 1, a consumer with search cost less than $u_h\pi_1^t+u_l(1-\pi_1^t)$ will search. For a consumer whose search cost is greater than $u_h\pi_1^t+u_l(1-\pi_1^t)$, as $\pi_1^t\geq\pi_2^t$, she does not perform the second search and thus her expected utility of first search is $u_h\pi_1^t+u_l(1-\pi_1^t)-s$, which is negative and thus she does not search at all. The proof of (ii) is similar to that of Lemma 1. □

Proof of Proposition 3 When n_1 is odd, we have

$$\pi_1^r=p_h^2+2p_hp_l(1-G_a(\frac{n_1}{2}))=p_h(p_h+2p_l(1-G_a(\frac{n_1}{2})))\geq p_h(p_h+p_l)=p_h$$
$$\nu_1^r=p_h^2+2p_hp_lG_a(\frac{n_1}{2})=p_h(p_h+2p_lG_a(\frac{n_1}{2}))\leq p_h(p_h+p_l)=p_h$$
$$\pi_2^r=\frac{p_hG_a(\frac{n_1}{2})}{p_hG_a(\frac{n_1}{2})+p_l/2}=\frac{p_h}{p_h+p_l/(2G_a(\frac{n_1}{2}))}\leq\frac{p_h}{p_h+p_l}=p_h$$

where all inequalities follow from the fact $G_a(\frac{n_1}{2})\leq 1/2$.

When n_1 is even,

$$\pi_1^r=p_h^2+2p_hp_l(1-G_a(\frac{n_1}{2})+g_a(\frac{n_1}{2})/2)=p_h(p_h+2p_l(1-G_a(\frac{n_1}{2})+g_a(\frac{n_1}{2})/2)\geq p_h(p_h+p_l)=p_h$$
$$\nu_1^r=p_h^2+2p_hp_l(G_a(\frac{n_1}{2})-g_a(\frac{n_1}{2})/2)=p_h(p_h+2p_l(G_a(\frac{n_1}{2})-g_a(\frac{n_1}{2})/2)\leq p_h(p_h+p_l)=p_h$$
$$\pi_2^r=\frac{p_h(G_a(\frac{n_1}{2})-g_a(\frac{n_1}{2})/2)}{p_h(G_a(\frac{n_1}{2})-g_a(\frac{n_1}{2})/2)+p_l/2}=\frac{p_h}{p_h+p_l/(2(G_a(\frac{n_1}{2})-g_a(\frac{n_1}{2})/2))}\leq\frac{p_h}{p_h+p_l}=p_h$$

where all inequalities follow from $G_a(\frac{n_1}{2})-g_a(\frac{n_1}{2})/2\leq 1/2$. □

Proof of Lemma 4 We focus on the case where n_1 is odd. The case of even n_1 is similar and omitted here. Throughout this proof, $i=1,2$. For (i), we have

$$\Pr\left[u_i=u_h|u_1\neq u_2,X_i\geq\frac{n_1}{2}\right]=\frac{\Pr\left[u_i=u_h,u_1\neq u_2,X_i\geq\frac{n_1}{2}\right]}{\Pr\left[u_i=u_h,u_1\neq u_2,X_i\geq\frac{n_1}{2}\right]+\Pr\left[u_i=u_l,u_1\neq u_2,X_i\geq\frac{n_1}{2}\right]}$$
$$=\frac{p_hp_l(1-G_a(\frac{n_1}{2}))}{p_hp_l(1-G_a(\frac{n_1}{2}))+p_hp_lG_a(\frac{n_1}{2})}=1-G_a(\frac{n_1}{2})\geq\frac{1}{2}$$

where the inequality follows from $G_a(\frac{n_1}{2})\leq\frac{1}{2}$. In the meanwhile,

$$\Pr\left[u_i=u_h|u_1\neq u_2\right]=\frac{\Pr\left[u_i=u_h,u_1\neq u_2\right]}{\Pr\left[u_i=u_h,u_1\neq u_2\right]+\Pr\left[u_i=u_l,u_1\neq u_2\right]}=\frac{p_hp_l}{p_hp_l+p_lp_h}=\frac{1}{2}$$

Thus, $\Pr\left[u_i=u_h|u_1\neq u_2,X_i\geq\frac{n_1}{2}\right]\geq\Pr\left[u_i=u_h|u_1\neq u_2\right]$.

Furthermore,

$$\Pr\left[u_i=u_h|u_1=u_2,X_i\geq\frac{n_1}{2}\right]=\frac{\Pr\left[u_i=u_h,u_1=u_2,X_i\geq\frac{n_1}{2}\right]}{\Pr\left[u_i=u_h,u_1=u_2,X_i\geq\frac{n_1}{2}\right]+\Pr\left[u_i=u_l,u_1=u_2,X_i\geq\frac{n_1}{2}\right]}$$
$$=\frac{p_h^2G_s(\frac{n_1}{2})}{p_h^2G_s(\frac{n_1}{2})+p_l^2G_s(\frac{n_1}{2})}=\frac{p_h^2}{p_h^2+p_l^2}$$

and

$$\begin{aligned}\Pr[u_i=u_h|u_1=u_2] &= \Pr[i=1]\Pr[u_1=u_h|u_1=u_2]+\Pr[i=2]\Pr[u_2=u_h|u_1=u_2]\\ &= \frac{1}{2}\frac{p_h^2}{p_h^2+p_l^2}+\frac{1}{2}\frac{p_h^2}{p_h^2+p_l^2}\\ &= \frac{p_h^2}{p_h^2+p_l^2}\end{aligned}$$

Hence, $\Pr\left[u_i=u_h|u_1=u_2,X_i\geq\frac{n_1}{2}\right]=\Pr[u_i=u_h|u_1=u_2]$. For (ii),

$$\begin{aligned}\Pr\left[u_1\neq u_2|X_i\geq\frac{n_1}{2}\right] &= \frac{\Pr[u_i=u_h,u_{3-i}=u_l,X_i\geq\frac{n_1}{2}]+\Pr[u_i=u_l,u_{3-i}=u_h,X_i\geq\frac{n_1}{2}]}{\begin{pmatrix}\Pr[u_i=u_{3-i}=u_h,X_i\geq\frac{n_1}{2}]+\Pr[u_i=u_h,u_{3-i}=u_l,X_i\geq\frac{n_1}{2}]\\+\Pr[u_i=u_l,u_{3-i}=u_h,X_i\geq\frac{n_1}{2}]+\Pr[u_i=u_{3-i}=u_l,X_i\geq\frac{n_1}{2}]\end{pmatrix}}\\ &= \frac{p_hp_l\bar{G}_a(\frac{n_1}{2})+p_hp_lG_a(\frac{n_1}{2})}{p_h^2\bar{G}_s(\frac{n_1}{2})+p_hp_l\bar{G}_a(\frac{n_1}{2})+p_hp_lG_a(\frac{n_1}{2})+p_l^2\bar{G}_s(\frac{n_1}{2})}\\ &= 2p_hp_l\\ &= \Pr[u_1\neq u_2]\end{aligned}$$

Furthermore, $\Pr\left[u_1=u_2|X_i\geq\frac{n_1}{2}\right]=1-\Pr\left[u_1\neq u_2|X_i\geq\frac{n_1}{2}\right]=1-\Pr[u_1\neq u_2]=\Pr[u_1=u_2]$. □

Proof of Proposition 4 For (i),

$$\pi_2^v(x)=\frac{p_hg_a(n_1-x)}{p_hg_a(n_1-x)+p_lg_s(n_1-x)}=\frac{p_h}{p_h+p_l\frac{g_s(n_1-x)}{g_a(n_1-x)}}$$

and it suffices to show $\frac{g_s(n_1-x)}{g_a(n_1-x)}\geq 1$ for $x\geq\frac{n_1}{2}$. To this end, we first show that $g_s(n_1-x)/g_a(n_1-x)$ increases in x. Recall that, when $0\leq x<m$, $g_a(x)=0$ while $g_s(x)>0$, implying $g_a(x)/g_s(x)=0$. When $x\geq m$, we have

$$\frac{g_a(x)}{g_s(x)}=\frac{\binom{n_1-m}{x-m}(1/2)^{n_1-m}}{\binom{n_1}{x}(1/2)^{n_1}}=2^m\frac{(x-m+1)\cdot(x-m+2)\cdot\ldots\cdot x}{n_1\cdot(n_1-1)\cdot\ldots\cdot(n_1-m+1)}$$

which is increasing in x. Thus, $g_a(x)/g_s(x)$ increases in x and $g_s(n_1-x)/g_a(n_1-x)$ increases in x. Moreover, if $m=0$, $\frac{g_s(\frac{n_1}{2})}{g_a(\frac{n_1}{2})}=1$ and if $m\geq 1$, $\frac{g_s(\frac{n_1}{2})}{g_a(\frac{n_1}{2})}=2^{-m}\frac{n_1\cdot(n_1-1)\cdot\ldots\cdot(n_1-m+1)}{(\frac{n_1}{2}-m+1)\cdot(\frac{n_1}{2}-m+2)\cdot\ldots\cdot\frac{n_1}{2}}>1$ since $\frac{n_1-k}{\frac{n_1}{2}-k}\geq 2$ for $k\geq 0$. Thus, (i) is proved.

For (ii), we first show that $\pi_1^v(x)$ increases in x.

$$\begin{aligned}\pi_1^v(x) &= \frac{p_h(p_hg_s(x)+p_lg_a(x))}{p_h(p_hg_s(x)+p_lg_a(x))+p_l(p_hg_a(n_1-x)+p_lg_s(x))}\\ &= \frac{\rho^2g_s(x)+\rho g_a(x)}{\rho^2g_s(x)+\rho g_a(x)+g_s(x)+\rho g_a(n_1-x)}\end{aligned}$$

where $\rho=p_h/p_l$ and it is equivalent to show

$$\frac{\rho^2g_s(x)+\rho g_a(x)}{g_s(x)+\rho g_a(n_1-x)}=\rho\frac{\rho+g_a(x)/g_s(x)}{1+\rho g_a(n_1-x)/g_s(n_1-x)}$$

increases in x, where the equality follows from $g_s(x)=g_s(n_1-x)$. Recall that in part (i) we have shown that $g_a(x)/g_s(x)$ increases in x and $g_a(n_1-x)/g_s(n_1-x)$ decreases in x. Thus, $\pi_1^v(x)$ increases in x.

Next, we prove $\pi_1^v(n_1)\geq p_h$. Note that if $m=0$, then $g_a(x)\equiv g_s(x)$ and $\pi_1^v(n_1)=p_h$. If $m\geq 1$, we have

$$\begin{aligned}\pi_1^v(n_1) &= \frac{p_h(p_hg_s(n_1)+p_lg_a(n_1))}{p_h(p_hg_s(n_1)+p_lg_a(n_1))+p_l(p_hg_a(0)+p_lg_s(n_1))}\\ &= \frac{p_h(p_hg_s(n_1)+p_lg_a(n_1))}{p_h(p_hg_s(n_1)+p_lg_a(n_1))+p_l^2g_s(n_1)}\\ &= \frac{p_h}{p_h+p_l\frac{p_lg_s(n_1)}{p_hg_s(n_1)+p_lg_a(n_1)}}\end{aligned}$$

where $g_s(n_1) = (\frac{1}{2})^{n_1}$ and $g_a(n_1) = (\frac{1}{2})^{n_1-m}$, implying that $g_s(n_1) < g_a(n_1)$ and $\frac{p_l g_s(n_1)}{p_h g_s(n_1)+p_l g_a(n_1)} < 1$. Hence, $\pi_1^v(n_1) > p_h$ in this case. Summarizing both cases, we have $\pi_1^v(n_1) \geq p_h$.

Now we prove the properties of $\pi_1^v(\frac{n_1}{2})$. Note that

$$\pi_1^v(\frac{n_1}{2}) = \frac{p_h}{p_h + p_l \frac{p_h g_a(\frac{n_1}{2})+p_l g_s(\frac{n_1}{2})}{p_h g_s(\frac{n_1}{2})+p_l g_a(\frac{n_1}{2})}},$$

where

$$\frac{p_h g_a(\frac{n_1}{2}) + p_l g_s(\frac{n_1}{2})}{p_h g_s(\frac{n_1}{2}) + p_l g_a(\frac{n_1}{2})} - 1 = \frac{\rho \frac{g_a(\frac{n_1}{2})}{g_s(\frac{n_1}{2})} + 1}{\rho + \frac{g_a(\frac{n_1}{2})}{g_s(\frac{n_1}{2})}} - 1 = \frac{(1-\rho)(1 - \frac{g_a(\frac{n_1}{2})}{g_s(\frac{n_1}{2})})}{\rho + \frac{g_a(\frac{n_1}{2})}{g_s(\frac{n_1}{2})}}$$

Recall that $\frac{g_a(\frac{n_1}{2})}{g_s(\frac{n_1}{2})} \leq 1$. Thus, $\frac{p_h g_a(\frac{n_1}{2})+p_l g_s(\frac{n_1}{2})}{p_h g_s(\frac{n_1}{2})+p_l g_a(\frac{n_1}{2})} \leq 1$ if and only if $\rho \geq 1$, i.e., $p_h \geq \frac{1}{2}$. Thus, if $p_h \geq \frac{1}{2}$, $\pi_1^v(\frac{n_1}{2}) \geq p_h$, implying that $\pi_1^v(x) \geq p_h$ for all $x \geq \frac{n_1}{2}$; if, however, $p_h < \frac{1}{2}$, $\pi_1^v(\frac{n_1}{2}) < p_h$, implying that $\pi_1^v(x) \geq p_h$ if and only if x is sufficiently high (e.g., when $x = n_1$). □

Proof of Lemma 5 See proof for Proposition 4 (ii). □

Proof of Lemma 6 (i) Notice that we have

$$\Pr[u_{i^*} = u_h | u_1 \neq u_2, X_{i^*} = x] = \frac{\Pr[u_{i^*} = u_h, u_1 \neq u_2, X_{i^*} = x]}{\Pr[u_{i^*} = u_h, u_1 \neq u_2, X_{i^*} = x] + \Pr[u_{i^*} = u_l, u_1 \neq u_2, X_{i^*} = x]} = \frac{g_a(x)}{g_a(x) + g_a(n_1 - x)}$$

$$\Pr[u_1 \neq u_2 | X_{i^*} = x] = \frac{\Pr[u_1 \neq u_2, X_{i^*} = x]}{\Pr[u_1 \neq u_2, X_{i^*} = x] + \Pr[u_1 = u_2, X_{i^*} = x]} = \frac{p_h p_l (g_a(x) + g_a(n_1 - x))}{p_h p_l (g_a(x) + g_a(n_1 - x)) + (p_h^2 + p_l^2) g_s(x)}$$

We have shown that $\frac{g_a(x)}{g_s(x)}$ increases in x. As $\frac{g_a(x)}{g_a(n_1-x)} = \frac{g_a(x)}{g_s(x)} \cdot \frac{g_s(n_1-x)}{g_a(n_1-x)}$ is a product of two non-negative increasing functions, it follows that $\frac{g_a(x)}{g_a(x)+g_a(n_1-x)}$ is increasing in x. For $\Pr[u_1 \neq u_2 | X_{i^*} = x]$, it suffices to show

$$\frac{g_a(x) + g_a(n_1 - x)}{g_s(x)}$$

increases in x. When $n_1 - x < m$, $g_a(n_1 - x) = 0$ and $(g_a(x) + g_a(n_1 - x))/g_s(x)$ increases in x. When $x < m$, as $x \geq n_1/2$, $x \geq n_1 - x$ and $g_a(x) = g_a(n_1 - x) = 0$. When $n_1 - x \geq m$ and $x \geq m$, we have

$$\begin{aligned}\frac{g_a(x) + g_a(n_1 - x)}{g_s(x)} &= \frac{\binom{n_1-m}{x-m}(1/2)^{n_1-m} + \binom{n_1-m}{n_1-x-m}(1/2)^{n_1-m}}{\binom{n_1}{x}(1/2)^{n_1}} \\ &= 2^m \frac{(x-m+1)\cdot(x-m+2)\cdot \ldots \cdot x + (n_1-x-m+1)\cdot(n_1-x-m+2)\cdot \ldots \cdot (n_1-x)}{n_1 \cdot (n_1-1) \cdot \ldots \cdot (n_1-m+1)}\end{aligned}$$

Now consider the numerator and let $f(x) := (x-m+1)\cdot(x-m+2)\cdot \ldots \cdot x + (n_1-x-m+1)\cdot(n_1-x-m+2)\cdot \ldots \cdot (n_1-x)$, then

$$\begin{aligned}& f(x) - f(x-1) \\ =& (x-(x-m))(x-m+1)\cdot \ldots \cdot (x-1) - (n_1-x+1-(n_1-x-m+1))(n_1-x-m+2)\cdot(n_1-x-m+3)\cdot \ldots \cdot (n_1-x) \\ =& m[(x-m+1)\cdot \ldots \cdot (x-1) - (n_1-x-m+2)\cdot(n_1-x-m+3)\cdot \ldots \cdot (n_1-x)]\end{aligned}$$

It follows that $f(x) - f(x-1) \geq 0$ if and only if $x - 1 \geq n_1 - x$, which is $x \geq \frac{n_1+1}{2}$. Now consider two cases: if n_1 is odd, then f increases in $x > \frac{n_1}{2}$; if n_1 is even, then as $\frac{n_1}{2} + 1 > \frac{n_1+1}{2}$, $f(\frac{n_1}{2}+1) > f(\frac{n_1}{2})$ and f increases in $x \geq \frac{n_1}{2}$. It follows that $\frac{g_a(x)+g_a(n_1-x)}{g_s(x)}$ increases for $x \geq \frac{n_1}{2}$ and $x \leq n_1 - m$.

It remains to show that, for $m \geq 1$, $(g_a(x) + g_a(n_1 - x))/g_s(x)$ increases when x increases from $n_1 - m$ to $n_1 - m + 1$. Notice that when $m > n_1/2$, as $x \geq n_1/2$, it must be $n_1 - x < m$ and, thus, $(g_a(x) + g_a(n_1 - x))/g_s(x) =$

$g_a(x)/g_s(x)$ increases in x for $x \geq n_1/2$. Therefore, it suffices to show that $(g_a(x)+g_a(n_1-x))/g_s(x)$ increases when x increases from n_1-m to n_1-m+1, given $1 \leq m \leq n_1/2$. That is,

$$\frac{g_a(n_1-m)+g_a(m)}{g_s(n_1-m)} \leq \frac{g_a(n_1-m+1)}{g_s(n_1-m+1)},$$

which is equivalent to

$$2^m \frac{(n_1-2m+1)\cdot(n_1-2m+2)\cdot\ldots\cdot(n_1-m)+1\cdot 2\cdot\ldots\cdot m}{n_1\cdot(n_1-1)\cdot\ldots\cdot(n_1-m+1)}$$
$$\leq 2^m \frac{(n_1-2m+2)\cdot(n_1-2m+3)\cdot\ldots\cdot(n_1-m)\cdot(n_1-m+1)}{n_1\cdot(n_1-1)\cdot\ldots\cdot(n_1-m+2)\cdot(n_1-m+1)}.$$

It is straightforward to verify that the inequality above holds when $m=1$. For $m>1$, we have

$$\begin{aligned}
&(n_1-2m+2)\cdot(n_1-2m+3)\cdot\ldots\cdot(n_1-m+1)-(n_1-2m+1)\cdot(n_1-2m+2)\cdot\ldots\cdot(n_1-m)-1\cdot 2\cdot\ldots\cdot m\\
=&(n_1-2m+2)\cdot(n_1-2m+3)\cdot\ldots\cdot(n_1-m)\cdot(n_1-m+1-n_1+2m-1)-1\cdot 2\cdot\ldots\cdot m\\
=&(n_1-2m+2)\cdot(n_1-2m+3)\cdot\ldots\cdot(n_1-m)\cdot m-1\cdot 2\cdot\ldots\cdot m\\
>&0
\end{aligned}$$

where the last inequality follows from the fact $n_1-2m+2>1$, $n_1-2m+3>2$, ..., and $n_1-m>m-1$. Hence, $\Pr[u_1 \neq u_2 | X_{i^*}=x]$ increases in x and the proof is completed.

(ii) For $\pi_1^v(x)$, see the proof for Proposition 4 (ii). For $\pi_2^v(x)$, we have

$$\pi_2^v(x) = \frac{p_h g_a(n_1-x)}{p_h g_a(n_1-x)+p_l g_s(n_1-x)} = \frac{p_h}{p_h+p_l\frac{g_s(n_1-x)}{g_a(n_1-x)}}$$

The claim follows as we have shown that $\frac{g_s(n_1-x)}{g_a(n_1-x)}$ increases in x (see proof of Proposition 4 (i)). □

Proof of Lemma 7 Notice that π_1^r is a number, which can be considered as a degenerate random variable and it suffices to prove $\pi_1^r=\mathbb{E}[\pi_1^v(X_{i^*})]$, where $X_{i^*}\geq\frac{n_1}{2}$ stands for the sales of the higher sales product and is a random variable. Specifically, if either n_1 is odd or $x\neq\frac{n_1}{2}$, $Pr(X_{i^*}=x)=2(p_h^2+p_l^2)g_s(x)+2p_hp_lg_a(x)+2p_hp_lg_a(n_1-x)$. On the other hand, if n_1 is even, then $Pr(X_{i^*}=\frac{n_1}{2})=(p_h^2+p_l^2)g_s(\frac{n_1}{2})+2p_hp_lg_a(\frac{n_1}{2})$. Therefore, if n_1 is odd,

$$\begin{aligned}
\mathbb{E}[\pi_1^v(X_{i^*})] &= \mathbb{E}[\frac{p_h(p_hg_s(X_{i^*})+p_lg_a(X_{i^*}))}{p_h(p_hg_s(X_{i^*})+p_lg_a(X_{i^*}))+p_l(p_lg_s(X_{i^*})+p_hg_a(n_1-X_{i^*}))}]\\
&=\sum_{x=(n_1+1)/2}^{n_1}\frac{p_h(p_hg_s(x)+p_lg_a(x))}{p_h(p_hg_s(x)+p_lg_a(x))+p_l(p_lg_s(x)+p_hg_a(n_1-x))}(2(p_h^2+p_l^2)g_s(x)+2p_hp_lg_a(x)+2p_hp_lg_a(n_1-x))\\
&=2\sum_{x=(n_1+1)/2}^{n_1}p_h(p_hg_s(x)+p_lg_a(x))\\
&=p_h^2+2p_hp_l(1-G_a(\frac{n_1}{2}))\\
&=\pi_1^r
\end{aligned}$$

Now consider n_1 is even,

$$\begin{aligned}
\mathbb{E}[\pi_1^v(X_{i^*})] &= \sum_{x=\frac{n_1}{2}+1}^{n_1}\frac{p_h(p_hg_s(x)+p_lg_a(x))}{p_h(p_hg_s(x)+p_lg_a(x))+p_l(p_lg_s(x)+p_hg_a(n_1-x))}(2(p_h^2+p_l^2)g_s(x)+2p_hp_lg_a(x)+2p_hp_lg_a(n_1-x))\\
&+\frac{p_h(p_hg_s(\frac{n_1}{2})+p_lg_a(\frac{n_1}{2}))}{p_h(p_hg_s(\frac{n_1}{2})+p_lg_a(\frac{n_1}{2}))+p_l(p_lg_s(\frac{n_1}{2})+p_hg_a(\frac{n_1}{2}))}((p_h^2+p_l^2)g_s(\frac{n_1}{2})+2p_hp_lg_a(\frac{n_1}{2}))\\
&=2\sum_{x=\frac{n_1}{2}+1}^{n_1}p_h(p_hg_s(x)+p_lg_a(x))+p_h(p_hg_s(\frac{n_1}{2})+p_lg_a(\frac{n_1}{2}))\\
&=p_h^2+2p_hp_l(1-G_a(\frac{n_1}{2})+g_a(\frac{n_1}{2})/2)\\
&=\pi_1^r
\end{aligned}$$

For π_2^r, when n_1 is odd, as it is known that product i^* has low value and it has higher sales, the conditional probability that product i^* has sales $x \geq \frac{n_1}{2}$ is $\frac{Pr(X_{i^*}=x, u_{i^*}=u_l)}{Pr(X_{i^*}\geq \frac{n_1}{2}, u_{i^*}=u_l)} = \frac{p_h g_a(n_1-x)+p_l g_s(x)}{p_h G_a(\frac{n_1}{2})+p_l \bar{G}_s(\frac{n_1}{2})}$, then

$$\begin{aligned}\mathbb{E}[\pi_2^v(X_{i^*})] &= \mathbb{E}[\frac{p_h g_a(n_1-x)}{p_h g_a(n_1-x)+p_l g_s(x)}]\\ &= \sum_{x=(n_1+1)/2}^{n_1} \frac{p_h g_a(n_1-x)}{p_h g_a(n_1-x)+p_l g_s(x)} \frac{p_h g_a(n_1-x)+p_l g_s(x)}{p_h G_a(\frac{n_1}{2})+p_l \bar{G}_s(\frac{n_1}{2})}\\ &= \frac{p_h G_a(\frac{n_1}{2})}{p_h G_a(\frac{n_1}{2})+p_l \bar{G}_s(\frac{n_1}{2})}\\ &= \pi_2^r\end{aligned}$$

When n_1 is even, the conditional probability that product i^* has sales $x > \frac{n_1}{2}$ is $\frac{p_h g_a(n_1-x)+p_l g_s(x)}{p_h(G_a(\frac{n_1}{2})-g_a(\frac{n_1}{2})/2)+p_l/2}$ and has sales $\frac{n_1}{2}$ is $\frac{p_h g_a(\frac{n_1}{2})/2+p_l g_s(\frac{n_1}{2})/2}{p_h(G_a(\frac{n_1}{2})-g_a(\frac{n_1}{2})/2)+p_l/2}$. Thus,

$$\begin{aligned}\mathbb{E}[\pi_2^v(X_{i^*})] &= \mathbb{E}[\frac{p_h g_a(n_1-x)}{p_h g_a(n_1-x)+p_l g_s(x)}]\\ &= \sum_{x=\frac{n_1}{2}+1}^{n_1} \frac{p_h g_a(n_1-x)}{p_h g_a(n_1-x)+p_l g_s(x)} \frac{p_h g_a(n_1-x)+p_l g_s(x)}{p_h(G_a(\frac{n_1}{2})-g_a(\frac{n_1}{2}/2))+p_l/2}\\ &\quad + \frac{p_h g_a(\frac{n_1}{2})}{p_h g_a(\frac{n_1}{2})+p_l g_s(\frac{n_1}{2})} \frac{p_h g_a(\frac{n_1}{2})/2+p_l g_s(\frac{n_1}{2})/2}{p_h(G_a(\frac{n_1}{2})-g_a(\frac{n_1}{2})/2)+p_l/2}\\ &= \frac{p_h G_a(\frac{n_1}{2})-p_h g_a(\frac{n_1}{2})/2}{p_h G_a(\frac{n_1}{2})-p_h g_a(\frac{n_1}{2})/2+p_l/2}\\ &= \pi_2^r\end{aligned}$$

□

Proof of Proposition 5 The corollary follows from the fact that $\pi_1^v(x)$ is a mean preserving spread of π_1^r and $\pi_1^v(x)$ increases in $x \geq \frac{n_1}{2}$. □

Proof of Proposition 6 For the first part of (i), as we have shown that sales in the second period equals to $n_2 F(\pi_1^t u_h + (1-\pi_1^t)u_l)$, $S_r \geq S_\phi$ follows directly from $\pi_1^r \geq p_h$ and monotonicity of F(as it is a cdf). For the second part of (i), we construct the following example. Let $p_h = 0.6, u_h = 6, u_l = 2$. Thus, $p_h u_h + p_l u_l = 4.4$ and $p_h(u_h - u_l) = 2.4$. Consider a distribution function F such that $F(2.4) = \epsilon/6$, $F(4.4) = \epsilon/2$, and $F(4.88) = 3/4$. Let $n_0 = \lceil 6/\epsilon \rceil$, then $n_1 = 3, m = 1$, implying $G_a(\frac{n_1}{2}) = (\frac{1}{2})^2$, $\pi_1^r = p_h^2 + 2p_h p_l(1-G_a(\frac{n_1}{2})) = 0.72$, and $\pi_1^r u_h + (1-\pi_1^r)u_l = 4.88$. It follows that

$$\frac{S_\phi}{S_r} = \frac{n_2 F(p_h u_h + p_l u_l)}{n_2 F(\pi_1^r u_h + (1-\pi_1^r)u_l)} = \frac{F(p_h u_h + p_l u_l)}{F(\pi_1^r u_h + (1-\pi_1^r)u_l)} = \frac{2\epsilon}{3} < \epsilon$$

Similarly, the first part of (ii) follows from the facts that $\pi_1^v(\frac{n_1}{2}) < p_h$ when $p_h < \frac{1}{2}$ (Lemma 5) and $\pi_1^v(x)$ increases in x (Lemma 6).

We exemplify that S_v can be strictly less than S_ϕ by constructing a specific instance. Consider an example with $u_h = 2, u_l = 1$, then $u_h \pi_1^t + u_l(1-\pi_1^t) = \pi_1^t + 1$ and $\pi_1^t \Delta = \pi_1^t$. Let $n_0 = 20, n_2 = 20$, $p_h = 0.1, p_l = 0.9$ and consider a F with probability mass $f(0.1) = 0.1, f(1.092) = 0.35, f(1.1) = 0.05, f(2) = 0.5$.

Then, $n_0 F(p_h + 1) = 10$, $n_0 F(p_h) = 2$ as $F(1.1) = 0.5, F(0.1) = 0.1$. So, $n_1 = 10, m = 2$. Moreover, as $\pi_1^v(5) = \frac{p_h^2 g_s(5)+p_h p_l g_a(5)}{p_h^2 g_s(5)+p_h p_l g_a(5)+p_h p_l g_a(5)+p_l^2 g_s(5)} = 0.092, \pi_1^v(10) = \frac{p_h^2 g_s(10)+p_h p_l g_a(10)}{p_h^2 g_s(10)+p_h p_l g_a(10)+p_h p_l g_a(0)+p_l^2 g_s(10)} = 0.3136$ (where $g_s(x) = (1/2)^{10}\binom{10}{x}, g_a(x) = (1/2)^8 \binom{8}{x-2}$ if $x \geq 2$ and $g_a(x) = 0$ if $x < 2$), $n_2 F(\pi_1^v(10)+1) = 10$ and $n_2 F(\pi_1^v(5)+1) = 9$.

So the total sales in the second period under volume information is 9 when the sales in the first period for the two products are 5 and 5. As we have shown that $\pi_1^v(x)$ increases in x and $n_2F(\pi_1^v(10)+1)=10$, the sales in the second period under volume information is at most 10 for all other possible sales realization in the first period. Thus, $S_\phi=10$ and $S_v<10$ and we have $S_v<S_\phi$. $\square$

Proof of Proposition 7 Part (i) of the proposition follows as $\pi_1^v(x)<\pi_1^r$ when x is low and $\pi_1^v(x)>\pi_1^r$ when x is high (Proposition 5).

For the first part of part (ii), notice that we have

$$\begin{aligned} S_v &= \mathbb{E}_X[n_2F(u_h\pi_1^v(X)+(1-\pi_1^v(X))u_l)] \\ &= \mathbb{E}_Y[H(Y)] \end{aligned}$$

where $H(t):=n_2F(u_ht+u_l(1-t))$ and $Y=\pi_1^v(X)$. Also $S_r=n_2F(u_h\pi_1^r+(1-\pi_1^r)u_l)=H(\mathbb{E}_Y[Y])$. By Jensen's inequality we have the desired result. For the second part of part (ii), we construct a specific example such that $\frac{S_r}{S_v}\le\epsilon$. Let $p_h=0.6, u_h=6, u_l=2$. Thus, $p_hu_h+p_lu_l=4.4$ and $p_h(u_h-u_l)=2.4$. Consider a distribution function F such that $F(2.4)=\epsilon/36$, $F(4.4)=\epsilon/12$, $F(4.88)=\epsilon/6$, and $F(5.36)=50/111$. Let $n_0=\lceil 36/\epsilon\rceil$, then $n_1=3, m=1$, implying $\pi_1^r=0.72$, $\pi_1^ru_h+(1-\pi_1^r)u_l=4.88$, $\pi_1^v(3)=\frac{p_h}{p_h+p_l\frac{p_l(\frac{1}{2})^3}{p_h(\frac{1}{2})^3+p_l(\frac{1}{2})^2}}=0.84$, and $\pi_1^v(3)u_h+(1-\pi_1^v(3))u_l=5.36$. The probability that the bestseller's first-period sales is 3 is $p(3|3,1)=2\cdot(0.6^2\cdot 1/8+0.4^2\cdot 1/8+2\cdot 0.4\cdot 0.6\cdot 1/4)=0.37$, so

$$\begin{aligned} S_v &= n_2\sum_{x=(n_1+1)/2}^{n_1} p(x|n_1,m)F(\pi_1^v(x)u_h+(1-\pi_1^v(x))u_l) \\ &\ge n_2\cdot p(n_1|n_1,m)F(\pi_1^v(x)u_h+(1-\pi_1^v(x))u_l) \\ &= n_2\cdot 0.37\cdot\frac{50}{111} \\ &= \frac{n_2}{6} \end{aligned}$$

As $S_r=n_2F(\pi_1^ru_h+(1-\pi_1^r)u_l)=\frac{\epsilon n_2}{6}$, it follows that

$$\frac{S_r}{S_v}\le\frac{\epsilon n_2/6}{n_2/6}=\epsilon.$$

$\square$

Proof of Proposition 8 In preparation, we first note that, as a product's value is either low or high, the expected purchased value increases (resp. decreases) if the probability of a consumer purchasing a high value product increases (resp. decreases), conditional on purchase. In the meanwhile, we remark that regardless of the information available, a consumer who performs two searches purchases a high value product with probability $1-p_l^2$. The logic behind is that a consumer who searches twice will purchase a high value product whenever there is one and the probability that there is at least one high value product is $1-p_l^2$.

(i) When $\pi_2^r(u_h-u_l)<s\le p_h(u_h-u_l)$, the consumer is willing to search twice under no information but only once under ranking information. As it occurs with a positive probability that the higher-ranked product has low value and the lower-ranked product has high value, it follows that the probability of the consumer purchasing a high value product is lower than $1-p_l^2$ and thus her expected purchased value decreases.

(ii) When $\pi_2^v(n_1)(u_h-u_l)<s\le\pi_2^r(u_h-u_l)$, the consumer is willing to search twice under ranking information; under volume informative, however, she is willing to search only once when the sales of the higher sales product

is high and is willing to search twice otherwise. As it occurs with a positive probability that the higher-ranked product has low value and the lower-ranked product has high value, the expected purchased value decreases. □

Proof of Proposition 9 For (i), the conclusion follows from the fact $\pi_1^r \geq p_h \geq \pi_2^r$ (notice that $\pi_1^\phi = \pi_2^\phi = p_h$).

For (ii), we prove for each case of search cost:

- When $\pi_2^v(n_1)(u_h - u_l) < s \leq \pi_2^r(u_h - u_l)$, under ranking information, the consumer always performs the first search and also conducts the second search if the first search reveals a low value product. Under volume information, on the other hand, the consumer may choose not to perform the first search when sales of the higher-ranked product is low, or choose not to do the second search (if the first search reveals a low value product) when sales of the higher-ranked product is high. Thus, both search probabilities weakly decrease.
- When $\pi_1^r u_h + (1-\pi_1^r)u_l < s \leq \pi_1^v(n_1)u_h + (1-\pi_1^v(n_1))u_l$, the consumer never searches under ranking information. Under volume information, the consumer may perform the first search when sales of the higher-ranked product is high and thus the first search probability is weakly higher than that under ranking information. Similarly, the consumer never performs the second search under ranking information and thus the second-search probability is also weakly higher under volume information. □

Proof of Proposition 10 Consider a consumer whose search cost is s. For this particular consumer, his expected surplus when there is no information is

$$w_n = \max[0, p_h u_h + p_l \max[u_l, p_h u_h + p_l u_l - s] - s]$$

where the second maximum operator reflects his decision of whether or not to perform the second search and the first maximum operator reflects his decision of whether or not to perform the first search.

Similarly, the expected surplus under ranking information is

$$w_r = \max[0, \pi_1^r u_h + (1-\pi_1^r)\max[u_l, \pi_2^r u_h + (1-\pi_2^r)u_l - s] - s]$$

The case for volume information is more complicated as the sales x is random. For a fixed value of x, the expected surplus under sales volume information is

$$\max[0, \pi_1^v(x)u_h + (1-\pi_1^v(x))\max[u_l, \pi_2^v(x)u_h + (1-\pi_2^v(x))u_l - s] - s]$$

So, the overall expected surplus is

$$w_s = \mathbb{E}[\max[0, \pi_1^v(x)u_h + (1-\pi_1^v(x))\max[u_l, \pi_2^v(x)u_h + (1-\pi_2^v(x))u_l - s] - s]$$

where the expectation is taken over x.

We first show that the surplus increases from no information to ranking information. There are four cases:

1) $s > p_h u_h + p_l u_l$: In this case the consumer's surplus under no information is 0, so his surplus can never be lower under ranking information.

2) $p_h(u_h - u_l) < s \leq p_h u_h + p_l u_l$: Recall that $\pi_2^r \leq p_h$, and thus $\pi_2^r(u_h - u_l) < s$. In this case the surplus under no information and ranking information are $p_h u_h + (1-p_h)u_l - s$ and $\pi_1^r u_h + (1-\pi_1^r)u_l - s$, respectively. As $\pi_1^r \geq p_h$, ranking information leads to higher surplus.

3) $s \leq \pi_2^r(u_h - u_l)$: In this case the surplus are $p_h u_h + (1-p_h)(p_h u_h + (1-p_h)u_l - s) - s$ and $\pi_1^r u_h + (1-\pi_1^r)(\pi_2^r u_h + (1-\pi_2^r)u_l - s) - s$.

Notice that we have $p_hu_h+(1-p_h)p_hu_h=\pi_1^ru_h+(1-\pi_1^r)\pi_2^ru_h$, which is $p_h+(1-p_h)p_h=\pi_1^r+(1-\pi_1^r)\pi_2^r$. To see this identity, notice that the two sides are the overall probability that the consumer can get a high surplus product under no information and ranking information, respectively. As the consumer is willing to search twice under both no information and ranking information, this is also the probability that at least one product has high surplus and the identity follows (this can also be checked by direct calculation). Similarly, the probability that the consumer gets a low surplus product is also the same. Thus the only difference between the two surplus is search cost: as $(1+1-p_h)s\geq(1+1-\pi_1^r)s$, the consumer pays less search cost under ranking information and it follows that the consumer's surplus is higher under ranking information.

4) $\pi_2^r(u_h-u_l)<s\leq p_h(u_h-u_l)$: In this case the surplus are $p_hu_h+(1-p_h)(p_hu_h+(1-p_h)u_l-s)-s$ and $\pi_1^ru_h+(1-\pi_1^r)u_l-s$. Note that

$$\begin{aligned}
&\pi_1^ru_h+(1-\pi_1^r)u_l-s\\
=&\max[0,\pi_1^ru_h+(1-\pi_1^r)\max[u_l,\pi_2^ru_h+(1-\pi_2^r)u_l-s]-s]\\
>&\max[0,\pi_1^ru_h+(1-\pi_1^r)(\pi_2^ru_h+(1-\pi_2^r)u_l-s)-s]\\
\geq&\max[0,p_hu_h+(1-p_h)(p_hu_h+(1-p_h)u_l-s)-s]\\
=&p_hu_h+(1-p_h)(p_hu_h+(1-p_h)u_l-s)-s
\end{aligned}\tag{S.4}$$

where the last inequality follows from the identity $p_h+(1-p_h)p_h=\pi_1^r+(1-\pi_1^r)\pi_2^r$.

Now we compare the surplus under ranking and volume information. There are three cases depending on the value of s: (i) $s>\pi_1^ru_h+(1-\pi_1^r)u_l$, (ii) $\pi_2^r(u_h-u_l)<s\leq\pi_1^ru_h+(1-\pi_1^r)u_l$, (iii) $s\leq\pi_2^r(u_h-u_l)$. For case (ii), the consumer's surplus under ranking information is $\pi_1^ru_h+(1-\pi_1^r)u_l-s$. We have

$$\begin{aligned}
&\mathbb{E}[\max[0,\pi_1^v(x)u_h+(1-\pi_1^v(x))\max[u_l,\pi_2^v(x)u_h+(1-\pi_2^v(x))u_l-s]-s]]\\
\geq&\mathbb{E}[\pi_1^v(x)u_h+(1-\pi_1^v(x))\max[u_l,\pi_2^v(x)u_h+(1-\pi_2^v(x))u_l-s]-s]\\
\geq&\mathbb{E}[\pi_1^v(x)u_h+(1-\pi_1^v(x))u_l-s]\\
=&\pi_1^ru_h+(1-\pi_1^r)u_l-s
\end{aligned}\tag{S.5}$$

Other cases follow similarly. Thus, the consumer's surplus is higher under volume information than that under ranking information. □

Justification for "sales information becomes more informative as n_0 increases" We first use normal distribution to approximate the binomial distribution G_a. Recall that from the base model we have $m=\lfloor n_0F(p_h(u_h-u_l))\rfloor$ and $n_1=\lfloor n_0F(p_hu_h+p_lu_l)\rfloor$. Here as we are using normal approximation, we drop the floor operator and assume $m=n_0F(p_h(u_h-u_l))$ and $n_1=n_0F(p_hu_h+p_lu_l)$. Notice that $G_a(\cdot)$ is the cumulative distribution function for the sales of the high-value product ranging from m to n_1 when the two products are of different values, i.e.,

$$G_a(x)=\sum_{i=0}^{x}g_a(i)=\sum_{i=m}^{x}\frac{(i-m)!(n_1-i)!}{(n_1-m)!}\cdot\frac{1}{2^{n_1-m}}$$

where $g_a(x)=\text{Binomial}(x-m,n_1-m,1/2),x\geq m$ $(g_a(x)=0,x<m)$ is the corresponding probability density function. Therefore, $G_a(\cdot)$ has mean

$$\frac{n_1+m}{2}=n_0\frac{F(p_h(u_h-u_l))+F(p_hu_h+p_lu_l)}{2}$$

and variance

$$\frac{n_1 - m}{4} = n_0 \frac{F(p_h u_h + p_l u_l) - F(p_h(u_h - u_l))}{4}.$$

Furthermore, note that

$$\frac{\frac{n_1}{2} - \frac{n_1+m}{2}}{\sqrt{\frac{n_1-m}{4}}} = -\frac{m}{\sqrt{n_1 - m}} = -\sqrt{n_0}\frac{F(p_h(u_h - u_l))}{\sqrt{F(p_h u_h + p_l u_l) - F(p_h(u_h - u_l))}}$$

Hence, by normal approximation, we have

$$G_a(\frac{n_1}{2}) \sim \Phi(-\sqrt{n_0}\frac{F(p_h(u_h - u_l))}{\sqrt{F(p_h u_h + p_l u_l) - F(p_h(u_h - u_l))}}) \tag{S.6}$$

where $\Phi(\cdot)$ is the cumulative distribution function for the standard normal distribution.

As in (S.6), $G_a(\frac{n_1}{2})$ decreases in n_0. By definition of π_1^r, we have: π_1^r increases in n_0. Furthermore, as the expectation of $\pi_1^v(X_{i^*})$ equals π_1^r, we have: the expectation of $\pi_1^v(X_{i^*})$ also increases in n_0. □

SB. Three Products

The base model examines social learning of two products. We now extend the base model to consider learning of three products and show that our main results remain valid. Similar to the setting in the base model, we assume that the products are ex-ante homogeneous: each product's value is independently distributed - the value is high (i.e., u_h) with probability p_h and low (i.e., u_l) with probability p_l. Consumers may sequentially search the products before making their purchasing decisions. All the other assumptions remain the same as in the base model. We start our analysis from the generation of the bestseller information in the first period.

SB.1. First Period

Recall that the first-period consumers do not have access to any bestseller information. The following lemma characterizes the search and purchasing behavior of the first-period consumers. It confirms that the behavior is of very similar structure to that in the base model. Specifically, the early consumers with search cost $s \leq p_h(u_h - u_l)$ make "informed" purchases in that they always purchase a high-value product if one exists, while those with search cost $p_h(u_h - u_l) < s \leq p_h u_h + p_l u_l$ randomly pick and purchase a product.

Lemma S.2. *In the first period, consumers with search cost $s \leq p_h u_h + p_l u_l$ purchase a product and consumers with higher search cost leave without buying. Among the consumers who make a purchase,*

- *consumers with search cost $s \leq p_h(u_h - u_l)$ can search up to three products: they purchase a high-value product if there is any and randomly purchase a product if all the products are of low value;*
- *consumers with search cost $p_h(u_h - u_l) < s \leq p_h u_h + p_l u_l$ randomly search a product and purchase the searched product regardless of its value.*

While the first-period consumers' search and purchasing behavior is akin to its counterpart in the two-product model, the presence of three products leads to additional variability in the sales distribution. For example, in the case that there are more than one high-value products, the high-value product purchased by an "informed" consumer is the first high-value product that she discovers in her sequential search and thus is completely random. In addition, the purchases made by "un-informed" consumers are spread out among three products, instead of two products. As in the base model, let n_0 be the number of consumes in the first period. The proposition below characterizes the sales distribution in the first period.

Proposition S.1. *The sales distribution in the first period is characterized as follows:*

(i) Conditional on all products being of high value or all products being of low value: the sales of the three products follow a multinomial distribution with probability $(1/3, 1/3, 1/3)$ *and trial number* n_1*;*

(ii) Conditional on one product being of high value and the other two products being of low value: let x *be the sales for the high-type product and* y, z *be the sales for the two low-type products, respectively. Then,* $(x-m, y, z)$ *follows a multinomial distribution with probability* $(1/3, 1/3, 1/3)$ *and trial number* $n_1 - m$*;*

(iii) Conditional on two products being of high value and one product being of low value: let x *and* y *be the sales for the two high-value products and* z *be the sales for the low-value product, respectively. Then* $x = x_1 + x_2$ *and* $y = m - x_1 + y_2$*, where* x_1 *follows a binomial distribution with success probability* $1/2$ *and trial number* m *and* (x_2, y_2, z) *follows a multinomial distribution with probability* $(1/3, 1/3, 1/3)$ *and trial number* $n_1 - m$*,*
where $n_1 = \lfloor n_0 F(p_h u_h + p_l u_l) \rfloor$ *denotes the number of early consumers willing to perform the first search and* $m = \lfloor n_0 F(p_h(u_h - u_l)) \rfloor$ *denotes the number of early consumers willing to perform both the second and the third search.*

SB.2. Second Period

Now we analyze the late-arriving consumers' search and purchasing behavior in the second period. As in the base model, we first examine these consumers' posterior beliefs about product values after observing the bestseller information generated by the first-period consumer purchases.

Our first finding is that consumers' posterior beliefs about the products' values are nicely ordered in that a product with a higher sales ranking is always believed as more likely to be of high value. This ordering applies both before a late-arriving consumer performs a product search and after her first search reveals a low-value product. Proposition S.2 follows.

Proposition S.2. *(i) After observing the first-period sales information (either ranking or volume) and before conducting any product search, a second-period consumer believes that a product with higher first-period sales is more likely to be of high value;*
(ii) After her first search reveals a low-value product, a second-period consumer believes that, between the two remaining products, the one with higher first-period sales is more likely to be of high value.

Based on Proposition S.2, we prove in Proposition S.3 that it is rational for the second-period consumers to first search the product with the highest (or higher) sales ranking.

Proposition S.3. *(i) In the second period, if a consumer finds it worthwhile to search, it is optimal for her to first search the product with the highest first-period sales.*
(ii) If a second-period consumer's first search reveals a low type and she finds it worthwhile to continue her search, between the two remaining products it is optimal for her to search the one with higher first-period sales.

Propositions S.2 and S.3 echo with their counterparts under the two-product model and confirm that, in the three-product setting, it is optimal for the second-period consumers to search the products in a decreasing order of the products' first-period sales.

The results so far answer the question of *how* to search in the second period. What remains unclear is *when* to search in the second period. That is, given a consumer's search cost s and the first-period sales information, when it is worthwhile for her to conduct the first, second, or third search. Same as in the main model, we examine the consumer's expected utilities of each search. Let $t=\phi, r, v$ represent no sales information, sales ranking information, and sales volume information, respectively and let $\pi_1^t, \pi_2^t, \pi_3^t$ denote the beliefs under sales information t that the highest sales product is high type, the second highest sales product is high type given that the highest sales product is low type, and the lowest sales product is high type given the other two products are low types, respectively. Without loss of generality, consider the situation that product 1 has the highest first-period sales, product 2 has the second-highest, and product 3 has the lowest. Denote their (random) first-period sales by η_1, η_2, and η_3, respectively, and the corresponding realization by x_1, x_2, and x_3, respectively. Specifically, the posterior beliefs are defined as follows:

$$\begin{aligned}
\pi_1^r &= \Pr[u_1=u_h|\eta_1\geq\eta_2\geq\eta_3]\\
\pi_2^r &= \Pr[u_2=u_h|\eta_1\geq\eta_2\geq\eta_3, u_1=u_l]\\
\pi_3^r &= \Pr[u_3=u_h|\eta_1\geq\eta_2\geq\eta_3, u_1=u_2=u_l]\\
\pi_1^v(x_1,x_2,x_3) &= \Pr[u_1=u_h|\eta_1\geq\eta_2\geq\eta_3, \eta_1=x_1,\eta_2=x_2,\eta_3=x_3]\\
\pi_2^v(x_1,x_2,x_3) &= \Pr[u_2=u_h|\eta_1\geq\eta_2\geq\eta_3, \eta_1=x_1,\eta_2=x_2,\eta_3=x_3, u_1=u_l]\\
\pi_3^v(x_1,x_2,x_3) &= \Pr[u_3=u_h|\eta_1\geq\eta_2\geq\eta_3, \eta_1=x_1,\eta_2=x_2,\eta_3=x_3, u_1=u_2=u_l]
\end{aligned}$$

Under sales information t, a second-period consumer's expected utility for the third search is $\pi_3^t u_h+(1-\pi_3^t)u_l-s$, the expected utility for the second search is $\pi_2^t u_h+(1-\pi_2^t)\max[u_l, \pi_3^t u_h+(1-\pi_3^t)u_l-s]-s$, and the expected utility for the first search is $\pi_1^t u_h+(1-\pi_1^t)\max[u_l,\pi_2^t u_h+(1-\pi_2^t)\max[u_l,\pi_3^t u_h+(1-\pi_3^t)u_l-s]-s]-s$.

Notice that the utility for a consumer to leave without purchasing is zero and a consumer considers the second/third search when the previous searches reveal low-value products only. Therefore a second-period consumer performs the third search when

$$\pi_3^t u_h+(1-\pi_3^t)u_l-s\geq u_l,$$

the second search when

$$\pi_2^t u_h+(1-\pi_2^t)\max[u_l,\pi_3^t u_h+(1-\pi_3^t)u_l-s]-s\geq u_l,$$

and the first search when

$$\pi_1^t u_h+(1-\pi_1^t)\max[u_l,\pi_2^t u_h+(1-\pi_2^t)\max[u_l,\pi_3^t u_h+(1-\pi_3^t)u_l-s]-s]-s\geq 0.$$

In what follows, we examine the impact of bestseller information on posterior beliefs and expected sales. Let S_ϕ, S_r, S_v be the expected sales in the second period under no information, ranking information, and volume information, respectively.

Ranking Information

First consider ranking information. Proposition S.4 is consistent with its counterpart in the base model.

Proposition S.4. *When sales ranking information is released, a second-period consumer's belief that the highest sales product is of high value is higher than the prior belief, i.e.,* $\pi_1^r\geq p_h$.

Recall that when there is no sales information, consumers perform the first search and make purchase if and only if $s \le p_h u_h + p_l u_l$. With ranking information, the expected utility of performing the first search is $\pi_1^r u_h + (1-\pi_1^r)\max[u_l, \pi_2^r u_h + \max[u_l, \pi_3^r u_h + (1-\pi_3^r)u_l] - s] - s \ge \pi_1^r u_h + (1-\pi_1^r)u_l - s$. Therefore, among others, consumers with search cost $s \le \pi_1^r u_h + (1-\pi_1^r)u_l$ perform the first search and make purchase when sales ranking information is released. As $\pi_1^r \ge p_h$, sales ranking information increases the expected sales in the second period. Corollary S.1 follows.

Corollary S.1. *The expected sales in the second period under ranking information is greater than or equal to that under no information, i.e., $S_r \ge S_\phi$.*

This result is well aligned with that in the base model and confirms that ranking information is beneficial for the platform under three products.

Volume Information

Now we proceed to volume information. We show in Lemma S.3 that the reinforcement by homogeneity effect still exists when there are three products. In particular, upon observing the sales volumes, consumers' belief that the product with highest sales is of high value may become lower than the prior.

Lemma S.3. *There exist problem instances such that that $\pi_1^v(x_1, x_2, x_3) < p_h$.*

Lemma S.3 is proved by an example. As illustrated by the example, the main driving force for the posterior belief lower than prior is again the reinforcement by homogeneity effect. In particular, in the example, when the three products have the same sales, a second-period consumer believes that all the three products are of the same value, which in turn strengthens the initial belief, i.e., the posterior belief that either product is of high type is lower than the initial belief when p_h is low.

Building on the result in Lemma S.3, we show in Proposition S.5 that sales volume information can lead to lower expected sales in the second period. This proposition confirms, for the case of three products, that volume information can backfire and hurt the platform.

Proposition S.5. *There exist instances such that $S_v < S_\phi$.*

We conclude this subsection by showing that an important result in the base model about the belief under volume information being a mean preserving spread of the belief under ranking information continues to hold under three products.

Proposition S.6. *Under the three product extension, the belief that the bestseller is high type under volume information, $\pi_1^v(\cdot,\cdot,\cdot)$, is a mean preserving spread of the belief that the best seller is high type under ranking information, π_1^r.*

SB.3. Three Products versus Two Products

So far we have shown that the key results under two products remain valid for three products. To explore the implications of a greater number of products on consumer learning and purchasing, we numerically compare the posterior beliefs and expected sales under three products and under two products. We focus on the effects of ranking information in the numerical exploration.

To evaluate the expected sales under three products, we first numerically confirm an important ordering of the posterior beliefs, π_1^r, π_2^r and π_3^r. The parameter values in our numerical study are as follows. We set $n_0 = 20$ and $F(x) = (\alpha + (1-\alpha)\Phi((x-\mu)/\sigma))\mathbb{I}(x \geq 0)$, where $\Phi(\cdot)$ is the cumulative distribution function for the standard normal distribution and $\mathbb{I}(\cdot)$ is the indicator function. We set $\alpha = 0.08$, $\mu = 4.5$, $\sigma = 1.5$, $u_h = 4 + \Delta$, and $u_l = \Delta$. We vary p_h from 0.1 to 0.9 with interval 0.001 and Δ from 0 to 10 with interval 0.01. That is, we test over a total of 800 values of p_h and 1,000 values of Δ, resulting in 800,000 problem instances. In all of the 800,000 instances, we observe $\pi_1^r \geq \pi_2^r \geq \pi_3^r$. The following table exemplifies the posteriors π_1^r, π_2^r, and π_3^r.

Table S.4 Value of $\pi_1^r, \pi_2^r, \pi_3^r$**:** $\mu = 4.5, \sigma = 1.5, n_0 = 20, u_h = 6 + \Delta, u_l = 2 + \Delta, \alpha = 0.08$

	$p_h = 0.4$			$p_h = 0.6$		
Δ	π_1^r	π_2^r	π_3^r	π_1^r	π_2^r	π_3^r
0	0.6489	0.3544	0.0471	0.8404	0.5702	0.0672
0.5	0.6119	0.3901	0.0876	0.8195	0.6011	0.1110
1	0.5798	0.4029	0.1391	0.8031	0.6186	0.1479
1.5	0.5662	0.4031	0.1659	0.7913	0.6217	0.1894
2	0.5537	0.4078	0.1827	0.7800	0.6284	0.2171

The ordering $\pi_1^r \geq \pi_2^r \geq \pi_3^r$ is important as it allows us to characterize the search strategy of the second-period consumers. Recall that the expected utility of performing the first search for a second-period consumer with ranking information is

$$\pi_1^r u_h + (1-\pi_1^r)\max[u_l, \pi_2^r u_h + (1-\pi_2^r)\max[u_l, \pi_3^r u_h + (1-\pi_3^r)u_l - s] - s] - s$$

Therefore, given $\pi_1^r \geq \pi_2^r \geq \pi_3^r$, a second-period consumer performs the first search if $s \leq \pi_1^r u_h + (1-\pi_1^r)u_l$ and the expected second-period sales with ranking information is $n_2 F(\pi_1^r u_h + (1-\pi_1^r)u_l)$. Note that this expression is identical to its counterpart under two products. When comparing the three-product and two-product models, the difference in the expected second-period sales is then driven by the difference in π_1^r.

In what follows, we compare the expected second-period sales with ranking information under three products and under two products. Under the same search-cost distribution as stated earlier in this subsection, we further set $n_2 = 80$, $p_h = 0.1 + \delta_p$, $u_h = 4 + \delta_q$, and $u_l = \delta_q$ for δ_p varying from 0.04 to 0.8 with interval 0.04 (20 values of δ_p) and δ_q varying from 0.05 to 2.5 with interval 0.05 (50 values of δ_q), resulting in a total of $1,000$ problem instances. We observe that, in all of these 1,000 instances, the expected second-period sales under ranking information is higher when there are three products than when there are two products. For illustration, the following table presents some instances for second-period expected sales with ranking information.

Table S.5 Expected sales in the second period with ranking information:

$\mu = 4.5, \sigma = 1.5, n_0 = 20, u_h = 6 + \delta_q, u_l = 2 + \delta_q, \alpha = 0.08, p_h = 0.4 + \delta_p$

Three Products				Two Products			
δ_q	$\delta_p = 0.1$	$\delta_p = 0.2$	$\delta_p = 0.3$	δ_q	$\delta_p = 0.1$	$\delta_p = 0.2$	$\delta_p = 0.3$
0	49.43	59.18	61.45	0	44.12	54.32	59.01
0.5	57.12	65.49	67.72	0.5	52.66	61.88	65.98
1	63.61	70.66	72.55	1	60.64	68.18	71.38
1.5	69.43	74.52	75.92	1.5	67.33	72.90	75.30
2	73.73	77.03	77.97	2	72.38	76.09	77.57

The results so far lead to a very interesting implication: *ranking information's sales-enhancing effect seems to be more pronounced under a greater number of products.* This implication is drawn based on (i) when no sales information is available in the second period, the second-period consumers behave exactly like the first-period ones and, thus, Lemma S.2 implies that the expected second-period sales without sales information is $n_0 F(p_h u_h + p_l u_l)$, regardless of whether there are two or three products; and (ii) the observation that, in all of the 1,000 numerical instances tested, the expected second-period sales under ranking information is higher when there are three products than when there are two products.

We conjecture that the aforementioned implication is related to two facts: first, the number of first-period consumers who have made "informed" purchases (i.e., those who are willing to conduct the third search and thus would always purchase a high-value product if at least one of the products is of high value) is independent of the number of products, as implied by Lemma S.2; and second, under three products the probability that at least one of the products is of high value is higher than that under two products (the former is $1-p_l^3$ and the latter is $1-p_l^2$). These two facts jointly imply that the expected sales of high-value products to the "informed" consumers are higher under three products. This may have contributed to the observation that consumers are more confident about the bestseller being of high value and thus are more willing to perform a first search under three products than under two products.

SB.4. Appendix

Proof of Lemma S.2 We start from the third search. A consumer considers whether or not to conduct the third search if and only if both of her first two searches reveal low-value products. Her expected utility of not conducting the third search is u_l and that of conducting it is $p_h u_h + p_l u_l - s$. Therefore, a consumer performs the third search if and only if $p_h u_h + p_l u_l - s \geq u_l$, i.e., $s \leq p_h(u_h - u_l)$.

Now, back to the second search. A consumer considers the second search if and only if the first search reveals a low type. Her expected utility of not performing the second search is u_l and that of performing it is

$$p_h u_h + p_l \max[u_l, p_h u_h + p_l u_l - s] - s$$

where the maximum operator corresponds to the option of the third search. Thus, a consumer performs the second search if and only if

$$p_h u_h + p_l \max[u_l, p_h u_h + p_l u_l - s] - s \geq u_l$$

which is equivalent to $s \leq p_h(u_h - u_l)$. To see the equivalence, note that if $s \leq p_h(u_h - u_l)$, $p_h u_h + p_l u_l - s \geq u_l$, implying that $p_h u_h + p_l \max[u_l, p_h u_h + p_l u_l - s] - s \geq p_h u_h + p_l u_l - s \geq u_l$. If, however, $s > p_h(u_h - u_l)$, $p_h u_h + p_l u_l - s < u_l$, implying that $p_h u_h + p_l \max[u_l, p_h u_h + p_l u_l - s] - s = p_h u_h + p_l u_l - s < u_l$.

Based on the analysis of the second and third searches, now we consider the first search. The expected utility of first search is

$$p_h u_h + p_l \max[u_l, p_h u_h + p_l \max[u_l, p_h u_h + p_l u_l - s] - s] - s$$

where the two maximum operators correspond to the options of the second search and of the third search, respectively. As the utility of not searching is zero, a consumer performs the first search if and only if

$$p_h u_h + p_l \max[u_l, p_h u_h + p_l \max[u_l, p_h u_h + p_l u_l - s] - s] - s \geq 0$$

which is equivalent to $s \le p_h u_h + p_l u_l$. To see the equivalence, note that, if $s \le p_h u_h + p_l u_l$, $p_h u_h + p_l \max[u_l, p_h u_h + p_l \max[u_l, p_h u_h + p_l u_l - s] - s] - s \ge p_h u_h + p_l u_l - s \ge 0$. If, however, $s > p_h u_h + p_l u_l$, $p_h u_h + p_l u_l - s < 0$, implying that $p_h u_h + p_l \max[u_l, p_h u_h + p_l u_l - s] - s = p_h u_h + p_l u_l - s < 0$. Thus, $p_h u_h + p_l \max[u_l, p_h u_h + p_l \max[u_l, p_h u_h + p_l u_l - s] - s] - s = p_h u_h + p_l u_l - s < 0$. □

Proof of Proposition S.1 The proof is similar to its counterpart in the main body of the paper. Note that for the $n_1 - m$ consumers with relatively high search cost, they only search once and purchase each of the three products with equal probability. For the m consumer with low search cost, they keep searching until either a high type product is revealed or all of the three products are found to be low type. When there is no high type product, these consumers purchase each of the three products with equal probability. When there is only one product, all of these m consumers purchase the high type product. When there are two high type products, these consumers purchase one of the two high-type products with equal probability. When there are three high-type products, these consumers purchase each of the three high-type products with equal probability. The proposition follows by summarizing all of these cases. □

Proof of Proposition S.2 (i) The proof applies a technical lemma, Lemma S.4, which is claimed and proved after the proof of this proposition.

Without loss of generality, consider the situation that product 1 has the highest first-period sales, product 2 has the second-highest, and product 3 has the lowest. This is without loss of generality since the products are ex-ante homogenous and we can always re-label the products to attain such a ranking. Let u_1, u_2, and u_3 denote the value of product 1, product 2, and product 3, respectively, and let η_1, η_2, and η_3 denote the first-period sales of product 1, product 2, and product 3, respectively. Let t_1, t_2, t_3, t_4, t_5, and t_6 denote the probabilities that the sales ranking is HHL, HLH, HLL, LHH, LHL, and LLH, respectively, where, for instance, HHL stands for the case that the product with highest first-period sales is high type, the one with the second-highest first-period sales is high type, and the one with the lowest first-period sales is low type. That is, $t_1 = \Pr(HHL) = \Pr(\eta_1 \ge \eta_2 \ge \eta_3, u_1 = u_h, u_2 = u_h, u_3 = u_l)$.

The posterior belief about product 1 being high type is:

$$\begin{aligned}
\Pr(u_1 = u_h | \eta_1 \ge \eta_2 \ge \eta_3) =& \frac{\Pr(u_1 = u_h, \eta_1 \ge \eta_2 \ge \eta_3)}{\Pr(\eta_1 \ge \eta_2 \ge \eta_3)} \\
=& \frac{\Pr(u_1 = u_h, u_2 = u_h, u_3 = u_h, \eta_1 \ge \eta_2 \ge \eta_3)}{1/6} + \frac{\Pr(u_1 = u_h, u_2 = u_h, u_3 = u_l, \eta_1 \ge \eta_2 \ge \eta_3)}{1/6} \\
&+ \frac{\Pr(u_1 = u_h, u_2 = u_l, u_3 = u_h, \eta_1 \ge \eta_2 \ge \eta_3)}{1/6} + \frac{\Pr(u_1 = u_h, u_2 = u_l, u_3 = u_l, \eta_1 \ge \eta_2 \ge \eta_3)}{1/6} \\
=& 6[p_h^3/6 + t_1 + t_2 + t_3]
\end{aligned}$$

where $\Pr(\eta_1 \ge \eta_2 \ge \eta_3) = 1/6$ as there are six permutations of the three products in their sales rankings and, since the three products are symmetric ex ante, each of the six possible permutations occurs with equal probability. Similarly,

$$\begin{aligned}
\Pr(u_2 = u_h | \eta_1 \ge \eta_2 \ge \eta_3) =& \frac{\Pr(u_2 = u_h, \eta_1 \ge \eta_2 \ge \eta_3)}{\Pr(\eta_1 \ge \eta_2 \ge \eta_3)} \\
=& \frac{\Pr(u_1 = u_h, u_2 = u_h, u_3 = u_h, \eta_1 \ge \eta_2 \ge \eta_3)}{1/6} + \frac{\Pr(u_1 = u_h, u_2 = u_h, u_3 = u_l, \eta_1 \ge \eta_2 \ge \eta_3)}{1/6} \\
&+ \frac{\Pr(u_1 = u_l, u_2 = u_h, u_3 = u_h, \eta_1 \ge \eta_2 \ge \eta_3)}{1/6} + \frac{\Pr(u_1 = u_l, u_2 = u_h, u_3 = u_l, \eta_1 \ge \eta_2 \ge \eta_3)}{1/6} \\
=& 6[p_h^3/6 + t_1 + t_4 + t_5]
\end{aligned}$$

and

$$\begin{aligned}\Pr(u_3=u_h|\eta_1\geq\eta_2\geq\eta_3)=&\frac{\Pr(u_3=u_h,\eta_1\geq\eta_2\geq\eta_3)}{\Pr(\eta_1\geq\eta_2\geq\eta_3)}\\=&\frac{\Pr(u_1=u_h,u_2=u_h,u_3=u_h,\eta_1\geq\eta_2\geq\eta_3)}{1/6}+\frac{\Pr(u_1=u_l,u_2=u_h,u_3=u_h,\eta_1\geq\eta_2\geq\eta_3)}{1/6}\\+&\frac{\Pr(u_1=u_h,u_2=u_l,u_3=u_h,\eta_1\geq\eta_2\geq\eta_3)}{1/6}+\frac{\Pr(u_1=u_l,u_2=u_l,u_3=u_h,\eta_1\geq\eta_2\geq\eta_3)}{1/6}\\=&6[p_h^3/6+t_4+t_2+t_6]\end{aligned}$$

To prove the proposition, it suffices to show $t_2\geq t_4$, $t_3\geq t_5$, $t_1\geq t_2$ and $t_5\geq t_6$. These properties follow from Lemma S.4.

(ii) Same as in the proof of part (i), without loss of generality, consider the situation where product 1 has the highest first-period sales, product 2 has the second-highest, and product 3 has the lowest. Consider the following three sub-cases:

(ii-a) consider first the subcase that a consumer first searches product 1 and discovers that it is low type. Now, we compare her posterior belief about the remaining two products:

$$\begin{aligned}&\Pr(u_2=u_h|\eta_1\geq\eta_2\geq\eta_3,u_1=u_l)\\=&\frac{\Pr(u_1=u_l,u_2=u_h,u_3=u_h,\eta_1\geq\eta_2\geq\eta_3)}{\Pr(\eta_1\geq\eta_2\geq\eta_3,u_1=u_l)}+\frac{\Pr(u_1=u_l,u_2=u_h,u_3=u_l,\eta_1\geq\eta_2\geq\eta_3)}{\Pr(\eta_1\geq\eta_2\geq\eta_3,u_1=u_l)}\end{aligned}$$

and

$$\begin{aligned}&\Pr(u_3=u_h|\eta_1\geq\eta_2\geq\eta_3,u_1=u_l)\\=&\frac{\Pr(u_1=u_l,u_2=u_h,u_3=u_h,\eta_1\geq\eta_2\geq\eta_3)}{\Pr(\eta_1\geq\eta_2\geq\eta_3,u_1=u_l)}+\frac{\Pr(u_1=u_l,u_2=u_l,u_3=u_h,\eta_1\geq\eta_2\geq\eta_3)}{\Pr(\eta_1\geq\eta_2\geq\eta_3,u_1=u_l)}\end{aligned}$$

By Lemma S.4 (i), $\Pr(u_1=u_l,u_2=u_h,u_3=u_l,\eta_1\geq\eta_2\geq\eta_3)\geq\Pr(u_1=u_l,u_2=u_l,u_3=u_h,\eta_1\geq\eta_2\geq\eta_3)$. Hence, $\Pr(u_2=u_h|\eta_1\geq\eta_2\geq\eta_3,u_1=u_l)\geq\Pr(u_3=u_h|\eta_1\geq\eta_2\geq\eta_3,u_1=u_l)$.

(ii-b) consider next the subcase that a consumer first searches product 2 and discovers that it is low type. Now, we compare her posterior belief about the remaining two products:

$$\begin{aligned}&\Pr(u_1=u_h|\eta_1\geq\eta_2\geq\eta_3,u_2=u_l)\\=&\frac{\Pr(u_1=u_h,u_2=u_l,u_3=u_h,\eta_1\geq\eta_2\geq\eta_3)}{\Pr(\eta_1\geq\eta_2\geq\eta_3,u_2=u_l)}+\frac{\Pr(u_1=u_h,u_2=u_l,u_3=u_l,\eta_1\geq\eta_2\geq\eta_3)}{\Pr(\eta_1\geq\eta_2\geq\eta_3,u_2=u_l)}\end{aligned}$$

and

$$\begin{aligned}&\Pr(u_3=u_h|\eta_1\geq\eta_2\geq\eta_3,u_2=u_l)\\=&\frac{\Pr(u_1=u_h,u_2=u_l,u_3=u_h,\eta_1\geq\eta_2\geq\eta_3)}{\Pr(\eta_1\geq\eta_2\geq\eta_3,u_2=u_l)}+\frac{\Pr(u_1=u_l,u_2=u_l,u_3=u_h,\eta_1\geq\eta_2\geq\eta_3)}{\Pr(\eta_1\geq\eta_2\geq\eta_3,u_2=u_l)}\end{aligned}$$

By Lemma S.4 (ii), $\Pr(u_1=u_h,u_2=u_l,u_3=u_l,\eta_1\geq\eta_2\geq\eta_3)\geq\Pr(u_1=u_l,u_2=u_l,u_3=u_h,\eta_1\geq\eta_2\geq\eta_3)$. Hence, $\Pr(u_1=u_h|\eta_1\geq\eta_2\geq\eta_3,u_2=u_l)\geq\Pr(u_3=u_h|\eta_1\geq\eta_2\geq\eta_3,u_2=u_l)$.

(ii-c) consider the last subcase that a consumer first searches product 3 and discovers that it is low type. Now, we compare her posterior belief about the remaining two products:

$$\begin{aligned}&\Pr(u_1=u_h|\eta_1\geq\eta_2\geq\eta_3,u_3=u_l)\\=&\frac{\Pr(u_1=u_h,u_2=u_h,u_3=u_l,\eta_1\geq\eta_2\geq\eta_3)}{\Pr(\eta_1\geq\eta_2\geq\eta_3,u_3=u_l)}+\frac{\Pr(u_1=u_h,u_2=u_l,u_3=u_l,\eta_1\geq\eta_2\geq\eta_3)}{\Pr(\eta_1\geq\eta_2\geq\eta_3,u_3=u_l)}\end{aligned}$$

and

$$\begin{aligned}&\Pr(u_2=u_h|\eta_1\geq\eta_2\geq\eta_3,u_3=u_l)\\=&\frac{\Pr(u_1=u_h,u_2=u_h,u_3=u_l,\eta_1\geq\eta_2\geq\eta_3)}{\Pr(\eta_1\geq\eta_2\geq\eta_3,u_3=u_l)}+\frac{\Pr(u_1=u_l,u_2=u_h,u_3=u_l,\eta_1\geq\eta_2\geq\eta_3)}{\Pr(\eta_1\geq\eta_2\geq\eta_3,u_3=u_l)}\end{aligned}$$

By Lemma S.4 (iii), $\Pr(u_1=u_h,u_2=u_l,u_3=u_l,\eta_1\geq\eta_2\geq\eta_3)\geq\Pr(u_1=u_l,u_2=u_h,u_3=u_l,\eta_1\geq\eta_2\geq\eta_3)$. Hence, $\Pr(u_2=u_h|\eta_1\geq\eta_2\geq\eta_3,u_1=u_l)\geq\Pr(u_3=u_h|\eta_1\geq\eta_2\geq\eta_3,u_1=u_l)$. □

Lemma S.4.

(i) For $v_1\in\{u_h,u_l\}$, $Pr(\eta_1\geq\eta_2\geq\eta_3,u_1=v_1,u_2=u_h,u_3=u_l)\geq Pr(\eta_1\geq\eta_2\geq\eta_3,u_1=v_1,u_2=u_l,u_3=u_h)$;

(ii) For $v_2\in\{u_h,u_l\}$, $Pr(\eta_1\geq\eta_2\geq\eta_3,u_1=u_h,u_2=v_2,u_3=u_l)\geq Pr(\eta_1\geq\eta_2\geq\eta_3,u_1=u_l,u_2=v_2,u_3=u_h)$;

(iii) For $v_3\in\{u_h,u_l\}$, $Pr(\eta_1\geq\eta_2\geq\eta_3,u_1=u_h,u_2=u_l,u_3=v_3)\geq Pr(\eta_1\geq\eta_2\geq\eta_3,u_1=u_l,u_2=u_h,u_3=v_3)$.

Proof of Lemma S.4 Below we prove part (i) and $v_1=u_h$. Other parts can be proved similarly. Note that

$$\begin{aligned}\Pr(HHL)&=\Pr(\eta_1\geq\eta_2\geq\eta_3,u_1=u_h,u_2=u_h,u_3=u_l)\\&=\Pr(\eta_1\geq\eta_2\geq\eta_3|u_1=u_h,u_2=u_h,u_3=u_l)\cdot\Pr(u_1=u_h,u_2=u_h,u_3=u_l)\\&=\Pr(\eta_1\geq\eta_2\geq\eta_3|u_1=u_h,u_2=u_h,u_3=u_l)\cdot p_h^2p_l\\&=p_h^2p_l\sum_{x\geq t\geq n_1-x-t}\Pr(\eta_1=x,\eta_2=t,\eta_3=n_1-x-t|u_1=u_h,u_2=u_h,u_3=u_l)\end{aligned}$$

and

$$\begin{aligned}\Pr(HLH)&=\Pr(\eta_1\geq\eta_2\geq\eta_3,u_1=u_h,u_2=u_l,u_3=u_h)\\&=\Pr(\eta_1\geq\eta_2\geq\eta_3|u_1=u_h,u_2=u_l,u_3=u_h)\cdot\Pr(u_1=u_h,u_2=u_l,u_3=u_h)\\&=\Pr(\eta_1\geq\eta_2\geq\eta_3|u_1=u_h,u_2=u_l,u_3=u_h)\cdot p_h^2p_l\\&=p_h^2p_l\sum_{x\geq t\geq n_1-x-t}\Pr(\eta_1=x,\eta_2=t,\eta_3=n_1-x-t|u_1=u_h,u_2=u_l,u_3=u_h)\end{aligned}$$

Therefore it suffices to show

$$\Pr(\eta_1=x,\eta_2=t,\eta_3=n_1-x-t|u_1=u_h,u_2=u_h,u_3=u_l)\geq\Pr(\eta_1=x,\eta_2=t,\eta_3=n_1-x-t|u_1=u_h,u_2=u_l,u_3=u_h)$$

for all x,t such that $x\geq t\geq n_1-x-t$. By Proposition S.1,

$$\begin{aligned}&\Pr(\eta_1=x,\eta_2=t,\eta_3=n_1-x-t|u_1=u_h,u_2=u_h,u_3=u_l)\\=&\sum_{s=0}^{m}\frac{m!}{(m-s)!s!}\frac{1}{2^m}\frac{(n_1-m)!}{(x-s)!(t-m+s)!(n_1-x-t)!}\frac{1}{2^{n_1-m}}\end{aligned}$$

and

$$\begin{aligned}&\Pr(\eta_1=x,\eta_2=t,\eta_3=n_1-x-t|u_1=u_h,u_2=u_l,u_3=u_h)\\=&\sum_{s=0}^{m}\frac{m!}{(m-s)!s!}\frac{1}{2^m}\frac{(n_1-m)!}{(x-s)!t!(n_1-x-t-m+s)!}\frac{1}{2^{n_1-m}}.\end{aligned}$$

It suffices to show

$$\begin{aligned}&\frac{m!}{(m-s)!s!}\frac{1}{2^m}\frac{(n_1-m)!}{(x-s)!(t-m+s)!(n_1-x-t)!}\frac{1}{2^{n_1-m}}\\\geq&\frac{m!}{(m-s)!s!}\frac{1}{2^m}\frac{(n_1-m)!}{(x-s)!t!(n_1-x-t-m+s)!}\frac{1}{2^{n_1-m}}\end{aligned}$$

for all s, which is equivalent to

$$(t-m+s)!(n_1-x-t)! \leq t!(n_1-x-t-m+s)!$$

Notice that $y!(n_1-x-m+s-y)!$ increases in y for $y \geq (n_1-x+s-m)/2$. As $(t-m+s)+(n_1-x-t)=n_1-x-m+s$, $\max[t-m+s, n_1-x-t] \geq (n_1-x+s-m)/2$. Since $s \leq m$ and $t \geq n_1-x-t$, $t \geq \max[t-m+s, n_1-x-t]$, which implies $(t-m+s)!(n_1-x-t)! \leq t!(n_1-x-t-m+s)!$ and completes the proof. □

Proof of Proposition S.3 We first note that part (ii) is a direct corollary of Proposition S.2 (ii) and Lemma S.1. This is because Proposition S.2 (ii) shows that when a product is revealed to be a low type, between the two remaining products the one with higher sales has a higher probability of being a high type and Lemma S.1 proves that it is optimal to search the product with a higher probability of being a high type when there are two products.

Now we prove part (i). The proof is similar to that of Lemma S.1. A useful technical lemma, Lemma S.5, is claimed and proved after the proof of the proposition.

Without loss of generality, consider the situation that product 1 has the highest first-period sales, product 2 has the second-highest, and product 3 has the lowest. For notational convenience, define $v_1 = u_h, v_2 = u_l$ and let $q_{ijk} = \Pr(u_1 = v_i, u_2 = v_j, u_3 = v_k), i,j,k = 1,2$. Below we first compare a second-period consumer's expected utility of first searching product 1 with that of first searching product 2. We will then examine the case of her first searching product three.

By part (ii), if a consumer first searches product 1 and finds it to be a low type, it is optimal for her to search product 2 next if she decides to continue her search. Thus, the expected utility of first searching product 1 is

$$\begin{aligned} w_1 &:= \Pr(u_1=v_1)v_1 + \Pr(u_1=v_2)\max[v_2,t_1] - s \\ &= (q_{111}+q_{112}+q_{121}+q_{122})v_1 + (1-q_{111}-q_{112}-q_{121}-q_{122})\max[v_2,t_1] - s \end{aligned} \tag{S.7}$$

with

$$\begin{aligned} t_1 &:= \Pr(u_2=v_1|u_1=v_2)v_1 + \Pr(u_2=v_2|u_1=v_2)\max[v_2, \Pr(u_3=v_1|u_1=u_2=v_2)v_1 + \Pr(u_3=v_2|u_1=u_2=v_2)v_2 - s] - s \\ &= \frac{q_{211}+q_{212}}{q_{211}+q_{212}+q_{221}+q_{222}}v_1 + \frac{q_{221}+q_{222}}{q_{211}+q_{212}+q_{221}+q_{222}}\max[v_2, \frac{q_{221}}{q_{221}+q_{222}}v_1 + \frac{q_{222}}{q_{221}+q_{222}}v_2 - s] - s \end{aligned} \tag{S.8}$$

where the maximum operator in (S.7) represents the possibility of a second search and the maximum operator in (S.8) represents the possibility of a third search.

Similarly, if a consumer searches product 2 first, her optimal choice for the second search is product 1 and the expected utility of first searching product 2 is

$$\begin{aligned} w_2 &:= \Pr(u_2=v_1)v_1 + \Pr(u_2=v_2)\max[v_2,t_2] - s \\ &= (q_{111}+q_{112}+q_{211}+q_{212})v_1 + (1-q_{111}-q_{112}-q_{211}-q_{212})\max[v_2,t_2] - s \end{aligned}$$

where

$$\begin{aligned} t_2 &:= \Pr(u_1=v_1|u_2=v_2)v_1 + \Pr(u_1=v_2|u_2=v_2)\max[v_2, \Pr(u_3=v_1|u_1=u_2=v_2)v_1 + \Pr(u_3=v_2|u_1=u_2=v_2)v_2 - s] - s \\ &= \frac{q_{121}+q_{122}}{q_{121}+q_{122}+q_{221}+q_{222}}v_1 + \frac{q_{221}+q_{222}}{q_{121}+q_{122}+q_{221}+q_{222}}\max[v_2, \frac{q_{221}}{q_{221}+q_{222}}v_1 + \frac{q_{222}}{q_{221}+q_{222}}v_2 - s] - s \end{aligned}$$

Note that w_1 and w_2 share the same utility for the third search, which we denote by $k_1 := \max[v_2, \frac{q_{221}}{q_{221}+q_{222}}v_1 + \frac{q_{222}}{q_{221}+q_{222}}v_2 - s]$. Note that $v_2 \le k_1 < v_1$. For notational convenience, define $q_{ij} := q_{ij1} + q_{ij2}, i, j = 1, 2$. We have:

$$w_1 = (q_{11}+q_{12})v_1 + (1-q_{11}-q_{12})\max[v_2, \frac{q_{21}}{q_{21}+q_{22}}v_1 + \frac{q_{22}}{q_{21}+q_{22}}k_1 - s] - s$$
$$w_2 = (q_{11}+q_{21})v_1 + (1-q_{11}-q_{21})\max[v_2, \frac{q_{12}}{q_{12}+q_{22}}v_1 + \frac{q_{22}}{q_{12}+q_{22}}k_1 - s] - s$$

As product 1 has higher sales than product 2, by Proposition S.2(i), product 1 is more likely to be a high type than product 2. That is, $q_{11}+q_{12} > q_{11}+q_{21}$, i.e., $q_{12} > q_{21}$. It follows that $w_1 > w_2$ by Lemma S.5. That is, searching product 1 first brings a higher expected utility than searching product 2 first.

Now consider searching product 3 first. If a consumer searches product 3 first, by part (ii), her optimal choice for a second search is product 1 and, thus, the expected utility of first searching product 3 is

$$\begin{aligned} w_3 &:= \Pr(u_3 = v_1)v_1 + \Pr(u_3 = v_2)\max[v_2, t_3] - s \\ &= (q_{111}+q_{121}+q_{211}+q_{221})v_1 + (1-q_{111}-q_{121}-q_{211}-q_{221})\max[v_2, t_3] - s \end{aligned}$$

where

$$\begin{aligned} t_3 &:= \Pr(u_1 = v_1|u_3 = v_2)v_1 + \Pr(u_1 = v_2|u_3 = v_2)\max[v_2, \Pr(u_2 = v_1|u_1 = u_3 = v_2)v_1 + \Pr(u_2 = v_2|u_1 = u_3 = v_2)v_2 - s] - s \\ &= \frac{q_{112}+q_{122}}{q_{112}+q_{122}+q_{212}+q_{222}}v_1 + \frac{q_{212}+q_{222}}{q_{112}+q_{122}+q_{212}+q_{222}}\max[v_2, \frac{q_{212}}{q_{212}+q_{222}}v_1 + \frac{q_{222}}{q_{212}+q_{222}}v_2 - s] - s \end{aligned}$$

To compare w_1 and w_3, we consider another scenario where the consumer first searches product 1, and then search product 3 (if product one is low type), and then product 2 (if both product 1 and product 3 are low type). The expected utility of following such a strategy is

$$\begin{aligned} w_4 &:= \Pr(u_1 = v_1)v_1 + \Pr(u_1 = v_2)\max[v_2, t_4] - s \\ &= (q_{111}+q_{112}+q_{121}+q_{122})v_1 + (1-q_{111}-q_{112}-q_{121}-q_{122})\max[v_2, t_4] - s \end{aligned}$$

where

$$\begin{aligned} t_4 &:= \Pr(u_3 = v_1|u_1 = v_2)v_1 + \Pr(u_3 = v_2|u_1 = v_2)\max[v_2, \Pr(u_2 = v_1|u_1 = u_3 = v_2)v_1 + \Pr(u_2 = v_2|u_1 = u_3 = v_2)v_2 - s] - s \\ &= \frac{q_{211}+q_{221}}{q_{211}+q_{212}+q_{221}+q_{222}}v_1 + \frac{q_{212}+q_{222}}{q_{211}+q_{212}+q_{221}+q_{222}}\max[v_2, \frac{q_{212}}{q_{212}+q_{222}}v_1 + \frac{q_{222}}{q_{212}+q_{222}}v_2 - s] - s \end{aligned}$$

Notice that t_3 and t_4 share the same utility for the third search, which we denote by $k_3 := \max[v_2, \frac{q_{212}}{q_{212}+q_{222}}v_1 + \frac{q_{222}}{q_{212}+q_{222}}v_2 - s]$. Note that $v_2 \le k_3 < v_1$. Define $\tau_{ij} := q_{i1j} + q_{i2j}, i, j = 1, 2$. Then we have

$$\begin{aligned} w_3 &= (q_{111}+q_{121}+q_{211}+q_{221})v_1 + (1-q_{111}-q_{121}-q_{211}-q_{221}) \\ &\quad \cdot \max[v_2, \frac{q_{112}+q_{122}}{q_{112}+q_{122}+q_{212}+q_{222}}v_1 + \frac{q_{212}+q_{222}}{q_{112}+q_{122}+q_{212}+q_{222}}k_3 - s] - s \\ &= (\tau_{11}+\tau_{21})v_1 + (1-\tau_{11}-\tau_{21})\max[v_2, \frac{\tau_{12}}{\tau_{12}+\tau_{22}}v_1 + \frac{\tau_{22}}{\tau_{12}+\tau_{22}}k_3 - s] - s \\ w_4 &= \Pr(u_1 = v_1)v_1 + \Pr(u_1 = v_2)\max[v_2, \Pr(u_3 = v_1|u_1 = v_2)v_1 + \Pr(u_3 = v_2|u_1 = v_2)k_3 - s] - s \\ &= (\tau_{11}+\tau_{12})v_1 + (1-\tau_{11}-\tau_{12})\max[v_2, \frac{\tau_{21}}{\tau_{21}+\tau_{22}}v_1 + \frac{\tau_{22}}{\tau_{21}+\tau_{22}}k_3 - s] - s \end{aligned}$$

As product 1 has higher sales than product 2, by Proposition S.2, product 1 is more likely to be a high type than product 2. That is, $(\tau_{11}+\tau_{12}) > (\tau_{11}+\tau_{21})$, i.e, $\tau_{12} > \tau_{21}$. By Lemma S.5, $w_4 > w_3$.

Now, since $w_1 \ge w_4$ (by part (ii)), $w_4 > w_3$ implies $w_1 > w_3$. $\square$

Lemma S.5. *Given* $q_{ij} \geq 0, i, j = 1, 2$, $q_{11} + q_{12} + q_{21} + q_{22} = 1$, *and* $q_{12} > q_{21}$, *for* $v_2 \leq k_1 < v_1$ *and* $s > 0$,

$$\begin{aligned}&(q_{11}+q_{12})v_1 + (1-q_{11}-q_{12})\max[v_2, \frac{q_{21}}{q_{21}+q_{22}}v_1 + \frac{q_{22}}{q_{21}+q_{22}}k_1 - s]\\ >&(q_{11}+q_{21})v_1 + (1-q_{11}-q_{21})\max[v_2, \frac{q_{12}}{q_{12}+q_{22}}v_1 + \frac{q_{22}}{q_{12}+q_{22}}k_1 - s]\end{aligned}$$

Proof of Lemma S.5 Let

$$\begin{aligned}r_1 &= (q_{11}+q_{12})v_1 + (1-q_{11}-q_{12})\max[v_2, \frac{q_{21}}{q_{21}+q_{22}}v_1 + \frac{q_{22}}{q_{21}+q_{22}}k_1 - s]\\ r_2 &= (q_{11}+q_{21})v_1 + (1-q_{11}-q_{21})\max[v_2, \frac{q_{12}}{q_{12}+q_{22}}v_1 + \frac{q_{22}}{q_{12}+q_{22}}k_1 - s]\end{aligned}$$

We compare r_1 and r_2 in the following three cases. Notice that $\frac{q_{12}}{q_{12}+q_{22}}v_1 + \frac{q_{22}}{q_{12}+q_{22}}k_1 - s > \frac{q_{21}}{q_{21}+q_{22}}v_1 + \frac{q_{22}}{q_{21}+q_{22}}k_1 - s$ as $q_{12} > q_{21}$ and $v_1 > k_1$.

(i) If $\frac{q_{12}}{q_{12}+q_{22}}v_1 + \frac{q_{22}}{q_{12}+q_{22}}k_1 - s \leq v_2$,

$$\begin{aligned}r_1 - r_2 &= (q_{11}+q_{12})v_1 + (1-q_{11}-q_{12})v_2 - ((q_{11}+q_{21})v_1 + (1-q_{11}-q_{21})v_2)\\ &= (q_{12}-q_{21})(v_1-v_2) > 0\end{aligned}$$

since $v_1 > v_2$ and $q_{12} > q_{21}$.

(ii) If $\frac{q_{12}}{q_{12}+q_{22}}v_1 + \frac{q_{22}}{q_{12}+q_{22}}k_1 - s > v_2 > \frac{q_{21}}{q_{21}+q_{22}}v_1 + \frac{q_{22}}{q_{21}+q_{22}}k_1 - s$,

$$\begin{aligned}r_1 - r_2 &= (q_{11}+q_{12})v_1 + (1-q_{11}-q_{12})v_2 - (q_{11}+q_{21})v_1 - (q_{12}+q_{22})(\frac{q_{12}}{q_{12}+q_{22}}v_1 + \frac{q_{22}}{q_{12}+q_{22}}k_1 - s)\\ &= (q_{12}-q_{21})v_1 + (q_{21}+q_{22})v_2 - q_{12}v_1 - q_{22}k_1 + (q_{12}+q_{22})s\\ &= (q_{21}+q_{22})v_2 + (q_{12}+q_{22})s - q_{22}k_1 - q_{21}v_1\\ &> (q_{21}+q_{22})v_2 + (q_{21}+q_{22})s - q_{22}k_1 - q_{21}v_1\\ &> (q_{21}+q_{22})v_2 + q_{21}v_1 + q_{22}k_1 - (q_{21}+q_{22})v_2 - q_{22}k_1 - q_{21}v_1\\ &= 0\end{aligned}$$

where the last inequality follows from the condition in this case.

(iii) If $\frac{q_{21}}{q_{21}+q_{22}}v_1 + \frac{q_{22}}{q_{21}+q_{22}}k_1 - s \geq v_2$, we have

$$\begin{aligned}r_1 - r_2 &= (q_{11}+q_{12})v_1 + (1-q_{11}-q_{12})(\frac{q_{21}}{q_{21}+q_{22}}v_1 + \frac{q_{22}}{q_{21}+q_{22}}k_1 - s)\\ &\quad - [(q_{11}+q_{21})v_1 + (1-q_{11}-q_{21})(\frac{q_{12}}{q_{12}+q_{22}}v_1 + \frac{q_{22}}{q_{12}+q_{22}}k_1 - s)]\\ &= (q_{11}+q_{12})v_1 + q_{21}v_1 + q_{22}k_1 - (q_{21}+q_{22})s - (q_{11}+q_{21})v_1 - q_{12}v_1 - q_{22}k_1 + (q_{12}+q_{22})s\\ &= (q_{12}-q_{21})s\\ &> 0\end{aligned}$$

Summarizing cases (i) through (iii), we have $r_1 > r_2$. □

Proof of Proposition S.4 We have

$$\begin{aligned}\pi_1^r &= \Pr(u_1 = u_h | \eta_1 \geq \eta_2 \geq \eta_3)\\ &= \frac{\Pr(u_1 = u_h, \eta_1 \geq \eta_2 \geq \eta_3)}{\Pr(\eta_1 \geq \eta_2 \geq \eta_3)}\end{aligned}$$

$$
\begin{aligned}
&= \frac{\Pr(u_1=u_h,u_2=u_h,u_3=u_h,\eta_1\ge\eta_2\ge\eta_3)}{1/6} + \frac{\Pr(u_1=u_h,u_2=u_h,u_3=u_l,\eta_1\ge\eta_2\ge\eta_3)}{1/6} \\
&\quad + \frac{\Pr(u_1=u_h,u_2=u_l,u_3=u_h,\eta_1\ge\eta_2\ge\eta_3)}{1/6} + \frac{\Pr(u_1=u_h,u_2=u_l,u_3=u_l,\eta_1\ge\eta_2\ge\eta_3)}{1/6} \\
&= p_h^3 + 6p_h^2p_l\Pr(\eta_1\ge\eta_2\ge\eta_3|u_1=u_h,u_2=u_h,u_3=u_l) \\
&\quad + 6p_h^2p_l\Pr(\eta_1\ge\eta_2\ge\eta_3|u_1=u_h,u_2=u_l,u_3=u_h) + 6p_hp_l^2\Pr(\eta_1\ge\eta_2\ge\eta_3|u_1=u_h,u_2=u_l,u_3=u_l) \\
&= p_h[p_h^2 + 6p_hp_l(\Pr(\eta_1\ge\eta_2\ge\eta_3|u_1=u_h,u_2=u_h,u_3=u_l) + \Pr(\eta_1\ge\eta_2\ge\eta_3|u_1=u_h,u_2=u_l,u_3=u_h)) \\
&\quad + 6p_l^2\Pr(\eta_1\ge\eta_2\ge\eta_3|u_1=u_h,u_2=u_l,u_3=u_l)]
\end{aligned}
$$

By Lemma S.4, we have $\Pr(\eta_1\ge\eta_2\ge\eta_3,u_1=u_h,u_2=u_h,u_3=u_l)\ge\Pr(\eta_1\ge\eta_2\ge\eta_3,u_1=u_h,u_2=u_l,u_3=u_h)\ge\Pr(\eta_1\ge\eta_2\ge\eta_3,u_1=u_l,u_2=u_h,u_3=u_h)$. Notice that

$$
\begin{aligned}
&\Pr(\eta_1\ge\eta_2\ge\eta_3,u_1=u_h,u_2=u_h,u_3=u_l) \\
=&\Pr(\eta_1\ge\eta_2\ge\eta_3|u_1=u_h,u_2=u_h,u_3=u_l)\cdot\Pr(u_1=u_h,u_2=u_h,u_3=u_l) \\
=&p_h^2p_l\Pr(\eta_1\ge\eta_2\ge\eta_3|u_1=u_h,u_2=u_h,u_3=u_l)
\end{aligned}
$$

and similarly

$$
\begin{aligned}
\Pr(\eta_1\ge\eta_2\ge\eta_3,u_1=u_h,u_2=u_l,u_3=u_h) &= p_h^2p_l\Pr(\eta_1\ge\eta_2\ge\eta_3|u_1=u_h,u_2=u_l,u_3=u_h) \\
\Pr(\eta_1\ge\eta_2\ge\eta_3.u_1=u_l,u_2=u_h,u_3=u_h) &= p_h^2p_l\Pr(\eta_1\ge\eta_2\ge\eta_3|u_1=u_l,u_2=u_h,u_3=u_h)
\end{aligned}
$$

Therefore Lemma S.4 implies $\Pr(\eta_1\ge\eta_2\ge\eta_3|u_1=u_h,u_2=u_h,u_3=u_l)\ge\Pr(\eta_1\ge\eta_2\ge\eta_3|u_1=u_h,u_2=u_l,u_3=u_h)\ge\Pr(\eta_1\ge\eta_2\ge\eta_3|u_1=u_l,u_2=u_h,u_3=u_h)$. By relabeling products, this is equivalent to $\Pr(\eta_1\ge\eta_2\ge\eta_3|u_1=u_h,u_2=u_h,u_3=u_l)\ge\Pr(\eta_1\ge\eta_3\ge\eta_2|u_1=u_h,u_2=u_h,u_3=u_l)\ge\Pr(\eta_3\ge\eta_2\ge\eta_1|u_1=u_h,u_2=u_h,u_3=u_l)$.

Similarly, Lemma S.4 states $\Pr(\eta_1\ge\eta_2\ge\eta_3,u_1=u_h,u_2=u_l,u_3=u_l)\ge\Pr(\eta_1\ge\eta_2\ge\eta_3,u_1=u_l,u_2=u_h,u_3=u_l)\ge\Pr(\eta_1\ge\eta_2\ge\eta_3,u_1=u_l,u_2=u_l,u_3=u_h)$, which is equivalent to $\Pr(\eta_1\ge\eta_2\ge\eta_3,u_1=u_h,u_2=u_l,u_3=u_l)\ge\Pr(\eta_2\ge\eta_1\ge\eta_3,u_1=u_h,u_2=u_l,u_3=u_l)\ge\Pr(\eta_3\ge\eta_2\ge\eta_1,u_1=u_h,u_2=u_l,u_3=u_l)$ by relabeling products. This further implies $\Pr(\eta_1\ge\eta_2\ge\eta_3|u_1=u_h,u_2=u_l,u_3=u_l)\ge\Pr(\eta_2\ge\eta_1\ge\eta_3|u_1=u_h,u_2=u_l,u_3=u_l)\ge\Pr(\eta_3\ge\eta_2\ge\eta_1|u_1=u_h,u_2=u_l,u_3=u_l)$.

As

$$
\begin{aligned}
&\Pr(\eta_1\ge\eta_2\ge\eta_3|u_1=u_h,u_2=u_h,u_3=u_l)+\Pr(\eta_2\ge\eta_1\ge\eta_3|u_1=u_h,u_2=u_h,u_3=u_l) \\
+&\Pr(\eta_1\ge\eta_3\ge\eta_2|u_1=u_h,u_2=u_h,u_3=u_l)+\Pr(\eta_2\ge\eta_3\ge\eta_1|u_1=u_h,u_2=u_h,u_3=u_l) \\
+&\Pr(\eta_3\ge\eta_2\ge\eta_1|u_1=u_h,u_2=u_h,u_3=u_l)+\Pr(\eta_3\ge\eta_1\ge\eta_2|u_1=u_h,u_2=u_h,u_3=u_l) \\
=&1
\end{aligned}
$$

It follows that

$$
\begin{aligned}
&\Pr(\eta_1\ge\eta_2\ge\eta_3|u_1=u_h,u_2=u_h,u_3=u_l)+\Pr(\eta_1\ge\eta_2\ge\eta_3|u_1=u_h,u_2=u_l,u_3=u_h) \\
=&\Pr(\eta_1\ge\eta_2\ge\eta_3|u_1=u_h,u_2=u_h,u_3=u_l)+\Pr(\eta_1\ge\eta_3\ge\eta_2|u_1=u_h,u_2=u_h,u_3=u_l) \\
\ge&\frac{2}{3}[\Pr(\eta_1\ge\eta_2\ge\eta_3|u_1=u_h,u_2=u_h,u_3=u_l)+\Pr(\eta_1\ge\eta_3\ge\eta_2|u_1=u_h,u_2=u_h,u_3=u_l) \\
&+\Pr(\eta_3\ge\eta_2\ge\eta_1|u_1=u_h,u_2=u_h,u_3=u_l)]
\end{aligned}
$$

$$
\begin{aligned}
=&\frac{2}{3}\cdot\frac{1}{2}[\Pr(\eta_1\geq\eta_2\geq\eta_3|u_1=u_h,u_2=u_h,u_3=u_l)+\Pr(\eta_1\geq\eta_3\geq\eta_2|u_1=u_h,u_2=u_h,u_3=u_l)\\
&+\Pr(\eta_2\geq\eta_1\geq\eta_3|u_1=u_h,u_2=u_h,u_3=u_l)+\Pr(\eta_2\geq\eta_3\geq\eta_1|u_1=u_h,u_2=u_h,u_3=u_l)\\
&+\Pr(\eta_3\geq\eta_2\geq\eta_1|u_1=u_h,u_2=u_h,u_3=u_l)+\Pr(\eta_3\geq\eta_1\geq\eta_2|u_1=u_h,u_2=u_h,u_3=u_l)]\\
=&\frac{1}{3}
\end{aligned}
$$

The first equality follows from $\Pr(\eta_1\geq\eta_2\geq\eta_3|u_1=u_h,u_2=u_l,u_3=u_h)=\Pr(\eta_1\geq\eta_3\geq\eta_2|u_1=u_h,u_2=u_h,u_3=u_l)$ (by relabeling product 2 and 3). The first inequality follows from $\Pr(\eta_1\geq\eta_2\geq\eta_3|u_1=u_h,u_2=u_h,u_3=u_l)\geq\Pr(\eta_3\geq\eta_2\geq\eta_1|u_1=u_h,u_2=u_h,u_3=u_l)$ and $\Pr(\eta_1\geq\eta_3\geq\eta_2|u_1=u_h,u_2=u_h,u_3=u_l)\geq\Pr(\eta_3\geq\eta_2\geq\eta_1|u_1=u_h,u_2=u_h,u_3=u_l)$. The second equality follows from $\Pr(\eta_1\geq\eta_2\geq\eta_3|u_1=u_h,u_2=u_h,u_3=u_l)=\Pr(\eta_2\geq\eta_1\geq\eta_3|u_1=u_h,u_2=u_h,u_3=u_l)$, $\Pr(\eta_1\geq\eta_3\geq\eta_2|u_1=u_h,u_2=u_h,u_3=u_l)=\Pr(\eta_2\geq\eta_3\geq\eta_1|u_1=u_h,u_2=u_h,u_3=u_l)$, and $\Pr(\eta_3\geq\eta_2\geq\eta_1|u_1=u_h,u_2=u_h,u_3=u_l)=\Pr(\eta_3\geq\eta_1\geq\eta_2|u_1=u_h,u_2=u_h,u_3=u_l)$.

Similarly, we have

$$
\begin{aligned}
&\Pr(\eta_1\geq\eta_2\geq\eta_3|u_1=u_h,u_2=u_l,u_3=u_l)+\Pr(\eta_1\geq\eta_3\geq\eta_2|u_1=u_h,u_2=u_l,u_3=u_l)\\
+&\Pr(\eta_2\geq\eta_1\geq\eta_3|u_1=u_h,u_2=u_l,u_3=u_l)+\Pr(\eta_3\geq\eta_1\geq\eta_2|u_1=u_h,u_2=u_l,u_3=u_l)\\
+&\Pr(\eta_3\geq\eta_2\geq\eta_1|u_1=u_h,u_2=u_l,u_3=u_l)+\Pr(\eta_2\geq\eta_3\geq\eta_1|u_1=u_h,u_2=u_l,u_3=u_l)\\
=&1
\end{aligned}
$$

it follows that

$$
\begin{aligned}
&\Pr(\eta_1\geq\eta_2\geq\eta_3|u_1=u_h,u_2=u_l,u_3=u_l)\\
\geq&\frac{1}{3}[\Pr(\eta_1\geq\eta_2\geq\eta_3|u_1=u_h,u_2=u_l,u_3=u_l)+\Pr(\eta_2\geq\eta_1\geq\eta_3|u_1=u_h,u_2=u_l,u_3=u_l)\\
&+\Pr(\eta_3\geq\eta_2\geq\eta_1|u_1=u_h,u_2=u_l,u_3=u_l)]\\
=&\frac{1}{3}\cdot\frac{1}{2}[\Pr(\eta_1\geq\eta_2\geq\eta_3|u_1=u_h,u_2=u_l,u_3=u_l)+\Pr(\eta_1\geq\eta_3\geq\eta_2|u_1=u_h,u_2=u_l,u_3=u_l)\\
+&\Pr(\eta_2\geq\eta_1\geq\eta_3|u_1=u_h,u_2=u_l,u_3=u_l)+\Pr(\eta_3\geq\eta_1\geq\eta_2|u_1=u_h,u_2=u_l,u_3=u_l)\\
+&\Pr(\eta_3\geq\eta_2\geq\eta_1|u_1=u_h,u_2=u_l,u_3=u_l)+\Pr(\eta_2\geq\eta_3\geq\eta_1|u_1=u_h,u_2=u_l,u_3=u_l)]\\
=&\frac{1}{6}
\end{aligned}
$$

Hence

$$
\begin{aligned}
&\Pr(u_1=u_h|\eta_1\geq\eta_2\geq\eta_3)\\
=&p_h[p_h^2+6p_hp_l(\Pr(\eta_1\geq\eta_2\geq\eta_3|u_1=u_h,u_2=u_h,u_3=u_l)+\Pr(\eta_1\geq\eta_2\geq\eta_3|u_1=u_h,u_2=u_l,u_3=u_h))\\
&+6p_l^2\Pr(\eta_1\geq\eta_2\geq\eta_3|u_1=u_h,u_2=u_l,u_3=u_l)]\\
\geq&p_h[p_h^2+2p_hp_l+p_l^2]\\
=&p_h
\end{aligned}
$$

□

Proof of Lemma S.3 This lemma is proved by the following example.

Example S.1. Consider a case where $n_0 = 5$, $u_h = 6, u_l = 2$, and $p_h = 0.1$. Let F be such that $F(x) = 0, x < 0.4, F(x) = 0.6, 0.4 \leq x < 2.5, F(x) = 1, x \geq 2.5$. As $p_h u_h + p_l u_l = 2.4, p_h(u_h - u_l) = 0.4$, the numbers of consumers who are willing to perform the first search and the second/third search when prior searches reveals low-value products are both 3. Consider the sales realization $\eta_1 = \eta_2 = \eta_3 = 1$. Notice that as all the three consumers in the first period are willing to perform the second or the third search when the products revealed are of low value, a low-value product has no sales in the first period when there is at least one high-value product. Consequently, the realization of $(1,1,1)$ sales indicates that either all products are of high value, or all products are of low value. As the three products have equal sales, the belief that either product is of high value is

$$
\begin{aligned}
&\Pr(u_1 = u_h | \eta_1 = 1, \eta_2 = 1, \eta_3 = 1) \\
=&\frac{\Pr(u_1 = u_h, \eta_1 = 1, \eta_2 = 1, \eta_3 = 1)}{\Pr(\eta_1 = 1, \eta_2 = 1, \eta_3 = 1)}
\end{aligned}
$$

We have

$$
\begin{aligned}
&\Pr(u_1 = u_h, \eta_1 = 1, \eta_2 = 1, \eta_3 = 1) \\
=&\Pr(u_1 = u_h, u_2 = u_h, u_3 = u_h, \eta_1 = 1, \eta_2 = 1, \eta_3 = 1) + \Pr(u_1 = u_h, u_2 = u_h, u_3 = u_l, \eta_1 = 1, \eta_2 = 1, \eta_3 = 1) \\
&+ \Pr(u_1 = u_h, u_2 = u_l, u_3 = u_h, \eta_1 = 1, \eta_2 = 1, \eta_3 = 1) + \Pr(u_1 = u_h, u_2 = u_l, u_3 = u_l, \eta_1 = 1, \eta_2 = 1, \eta_3 = 1) \\
=&\Pr(u_1 = u_h, u_2 = u_h, u_3 = u_h) \cdot \Pr(\eta_1 = 1, \eta_2 = 1, \eta_3 = 1 | u_1 = u_h, u_2 = u_h, u_3 = u_h) \\
=&0.1^3 \cdot 2/9 \\
=&0.00022
\end{aligned}
$$

and

$$
\begin{aligned}
&\Pr(u_1 = u_l, \eta_1 = 1, \eta_2 = 1, \eta_3 = 1) \\
=&\Pr(u_1 = u_l, u_2 = u_h, u_3 = u_h, \eta_1 = 1, \eta_2 = 1, \eta_3 = 1) + \Pr(u_1 = u_l, u_2 = u_h, u_3 = u_l, \eta_1 = 1, \eta_2 = 1, \eta_3 = 1) \\
&+ \Pr(u_1 = u_l, u_2 = u_l, u_3 = u_h, \eta_1 = 1, \eta_2 = 1, \eta_3 = 1) + \Pr(u_1 = u_l, u_2 = u_l, u_3 = u_l, \eta_1 = 1, \eta_2 = 1, \eta_3 = 1) \\
=&\Pr(u_1 = u_l, u_2 = u_l, u_3 = u_l) \cdot \Pr(\eta_1 = 1, \eta_2 = 1, \eta_3 = 1 | u_1 = u_l, u_2 = u_l, u_3 = u_l) \\
=&0.9^3 \cdot 2/9 \\
=&0.162
\end{aligned}
$$

where $\Pr(\eta_1 = 1, \eta_2 = 1, \eta_3 = 1 | u_1 = u_h, u_2 = u_h, u_3 = u_h) = \Pr(\eta_1 = 1, \eta_2 = 1, \eta_3 = 1 | u_1 = u_l, u_2 = u_l, u_3 = u_l) = 2/9$ as this is the probability that the three consumers purchase different products. This is the probability that the second consumer purchases a product that is not purchased by the first consumer and the third consumer purchases the product that is purchased by neither of the first two consumers, which is $1/3 \cdot 2/3 = 2/9$.

It follows that

$$
\begin{aligned}
\pi_1^v(1,1,1) =&\Pr(u_1 = u_h | \eta_1 = 1, \eta_2 = 1, \eta_3 = 1) \\
=&\frac{\Pr(u_1 = u_h, \eta_1 = 1, \eta_2 = 1, \eta_3 = 1)}{\Pr(\eta_1 = 1, \eta_2 = 1, \eta_3 = 1)} \\
=&\frac{0.00022}{0.00022 + 0.162} \\
=&0.0014
\end{aligned}
$$

Hence $\pi_1^v(1,1,1) < p_h$. $\square$

Proof of Proposition S.5 The following example shows that the expected sales can be lower due to learning through volume information. Let $u_h = 10$, $u_l = 0$, and $p_h = 0.1$. Let the search cost be uniformly distributed on $[0.5, 1]$ and suppose that there are 3 consumers in the first period and 7 consumers in the second period. Then as $p_h u_h + p_l u_l = p_h(u_h - u_l) = 1$, all consumers in the first period are willing to perform the first search or the second/third search when the prior search reveals a low-value product. When no sales information is released, all second-period consumers search and purchase and the expected sales in the second period is 7.

When volume information is released, there is a positive probability that the sales in the first period is $1, 1, 1$. By the same calculation as in Example S.1 (in proof of Lemma S.3), we have $\pi_1^v(1,1,1) = 0.0014$. Moreover, a same proof as in Example S.1 shows that all the three products must be of the same value as all products have positive sales. Thus, if the first search reveals a low-value product, then all the products are of low value and no second-period consumer would perform the second or the third search. As $\pi_1^v(1,1,1)u_h + (1-\pi_1^v(1,1,1))u_l = 0.0014 \cdot 10 < 0.5$, no second-period consumer performs the first search. It follows that the expected sales in the second period is lower than 7 when sales volume information is released. Therefore the expected sales under volume information is lower than that of no sales information, i.e., this finding in the base model is robust under three products. □

Proof of Proposition S.6 Let i, j be such that $i \geq j \geq n_1 - i - j$. The belief under volume information when the sales are $i, j, n_1 - i - j$ is

$$\Pr(u_1 = u_h | \eta_1 = i, \eta_2 = j, \eta_3 = n_1 - i - j) = \frac{\Pr(u_1 = u_h, \eta_1 = i, \eta_2 = j, \eta_3 = n_1 - i - j)}{\Pr(\eta_1 = i, \eta_2 = j, \eta_3 = n_1 - i - j)}$$

and

$$\begin{aligned}
&\mathbb{E}[\Pr(u_1 = u_h | \eta_1 = i, \eta_2 = j, \eta_3 = n_1 - i - j)] \\
=&\frac{\sum_{i \geq j \geq n_1 - i - j} \Pr(\eta_1 = i, \eta_2 = j, \eta_3 = n_1 - i - j) \cdot \frac{\Pr(u_1 = u_h, \eta_1 = i, \eta_2 = j, \eta_3 = n_1 - i - j)}{\Pr(\eta_1 = i, \eta_2 = j, \eta_3 = n_1 - i - j)}}{\Pr(\eta_1 \geq \eta_2 \geq \eta_3)} \\
=&\frac{\sum_{i \geq j \geq n_1 - i - j} \Pr(u_1 = u_h, \eta_1 = i, \eta_2 = j, \eta_3 = n_1 - i - j)}{\Pr(\eta_1 \geq \eta_2 \geq \eta_3)} \\
=&\frac{\Pr(u_1 = u_h, \eta_1 \geq \eta_2 \geq \eta_3)}{\Pr(\eta_1 \geq \eta_2 \geq \eta_3)} \\
=&\Pr(u_1 = u_h | \eta_1 \geq \eta_2 \geq \eta_3)
\end{aligned}$$

□

SC. Asymmetric Products

In the base model we assume that the two products are ex ante symmetric. This is a baseline situation for two products that are considered close substitute to each other. In this extension we consider a generalized setting where the two products are ex ante asymmetric with different prior probabilities of being high value.

Without loss of generality we assume that product 1 (resp. product 2) is believed ex ante to be of high value with a higher (resp. lower) probability. For expositional convenience, hereafter we may refer to product 1 (resp. product 2) as the more (resp. less) promising product. Let p_1, p_2 ($p_1 > p_2 > 0$) be the prior probabilities that product 1 and product 2 are of high value, respectively. The probabilities p_1, p_2 are public knowledge and common prior beliefs shared by the consumers and the platform. All other assumptions remain the same as in the base model. We first analyze the generation of bestseller information in the first period.

SC.1. First Period

We start by analyzing consumers' optimal search and purchase strategy in the first period. The following lemma characterizes the optimal search and purchase strategy for a first-period consumer.

LEMMA S.6. *In the first period, if a consumer decides to search a product, it is optimal to search product* 1 *first. Moreover, a consumer performs the first search if and only if* $s \le p_1 u_h + (1-p_1)u_l$ *and the second search if and only if product* 1 *is revealed to be of low value and* $s \le p_2(u_h - u_l)$*.*

Unlike in the base model where an early consumer's first-search choice is completely random, in the extended model it is always optimal for a first-period consumer to search product 1 first as it is more likely to be of high value than product 2. Therefore, the threshold for the first search is determined by p_1, the prior that product 1 is of high value, while the threshold for the second search is determined by p_2, product 2's prior of being high value. When the two products are symmetric, i.e., $p_1 = p_2 = p_h$, a first-period consumer's optimal search and decision strategy is the same as in the base model.

As in the base model, let n_0 be the number of first-period consumers, $n_1 := \lfloor n_0 F(p_1 u_h + (1-p_1)u_l) \rfloor$, and $m := \lfloor n_0 F(p_2(u_h - u_l)) \rfloor$. That is, n_1 and m are the number of consumers willing to perform the first and the second search in the first period, respectively. We refer to the m consumers with search cost $s \le p_2(u_h - u_l)$ as the *low-cost consumers* and the $n_1 - m$ consumers with search cost $p_2(u_h - u_l) \le s \le p_1 u_h + (1-p_1)u_l$ as the *high-cost consumers*. As in the base model, the m low-cost consumers make "informed" purchases as they are willing to perform the second search if the first search reveals a low-value product, while the purchases made by the $n_1 - m$ high-cost consumers are "uninformed" as these consumers only search once. The following proposition characterizes the sales distribution in the first period.

PROPOSITION S.7. *The sales distribution in the first period is characterized as follows:*

(i) conditional on product 1 *being of high value: product* 1 *has sales* n_1 *and product* 2 *has sales zero;*

(ii) conditional on product 1 *being of low value and product* 2 *being of high value: the* $n_1 - m$ *high-cost consumers purchase product* 1 *and the* m *low-cost consumers purchase product* 2*;*

(iii) conditional on both products being of low value: the $n_1 - m$ *high-cost consumer purchase product* 1 *and the* m *low-cost consumers purchase either product with equal probabilities.*

Proposition S.7 implies that the first-period sales of product 1 is at least $n_1 - m$. This is true even under the case where product 1 is of low value and product 2 is of high value. The reason is as follows: since a first-period consumer always searches product 1 first (if she ever searches) and the $n_1 - m$ high-cost consumers never perform the second search, the $n_1 - m$ high-cost consumers always purchase product 1, regardless of its value. This result differs from that in the base model (where each high-cost consumer randomly picks a product to search and purchase) and bears two important consequences. First, product 1 has a higher probability of being the bestseller compared to product 2, even under the case where both products have the same value. Consequently, sales ranking information may become less informative, which we shall elaborate in the next subsection. Second, since the high-cost consumers' purchasing actions are perfectly predictable to the second-period consumers, the latter ones are also able to deduce from the sales volume the purchasing actions of all the *low* search cost consumers. Thus, compared to that in the base model, sales volume information can become more informative about the product values, as we shall detail in the next subsection.

Another interesting observation from Proposition S.7 is that, conditional on some realizations of the product values (e.g., the realizations specified in parts (i) and (ii) of Proposition S.7), the first-period sales is deterministic. This is in stark contrast with the results in the base model and, as we shall show, has important implications on learning through sales volume information.

Before proceeding to the second-period analysis, we note that, when $m = 0$, none of the consumers performs the second search in the first period and, thus, the second-period consumers cannot derive any information about product values from either sales ranking or volume of the first period. Hence hereafter we assume $m > 0$.

SC.2. Second Period

Given first-period consumers' optimal search and purchasing strategy, we investigate consumers' optimal search and purchase strategy in the second period where the first-period sales information is publicized. Separate analysis for sales ranking and volume information are provided below.

Ranking Information

We start by considering the case where the platform releases sales ranking information. Let $\pi_1^r(i)$ denote the posterior belief that product i is of high value when product i has a higher sales ranking. There are two different cases depending on the values of n_1 and m.

Case 1 $n_1 - m > m$: In this case the number of high-cost consumers exceeds that of the low-cost consumers. As all of the high-cost consumers purchase product 1, it follows that product 1 always has higher sales ranking than product 2, regardless of the purchasing decisions of the m low-cost consumers. In this case, the posterior beliefs are such that $\pi_1^r(1) = p_1$ and $\pi_1^r(2) = p_2$.

Case 2 $n_1 - m \leq m$: In this case it is possible for product 2 to have higher sales ranking than product 1. Let $\pi_1^r(1)$ (resp. $\pi_1^r(2)$) be the belief that product 1 (resp. product 2) is of high value under ranking information when product 1 (resp. product 2) has higher sales ranking. We have the following proposition.

PROPOSITION S.8. *Assume $m > 0$. When $n_1 - m \leq m$, it is optimal for a second-period consumer to first search the product with higher sales ranking and the belief that the bestseller is of high value is higher than the prior belief, i.e., $\pi_1^r(1) \geq p_1$ and $\pi_1^r(2) \geq p_2$.*

Proposition S.8 echoes its counterpart in the base model and confirms the robustness of two key results. First, sales ranking directs the second-period consumers' first search and the product with higher sales ranking is their preferred choice for first search. To see the rationale behind this result, consider two cases: if product 1 has higher sales ranking, the belief is sustained about it more likely to be of high value than product 2. Therefore, same as the first-period consumers, the second-period consumers search product 1 first. On the other hand, if product 2 is the bestseller, the second-period consumers know that product 1 cannot be of high value (as otherwise, by Proposition S.7 all of the consumers should have purchased product 1, and, thus, product 2 cannot have higher sales ranking). Hence, it is optimal to search product 2 first.

Second, ranking information enhances the belief that the product with higher sales ranking is of high value. The intuition is similar to that under symmetric product. Specifically, if product 1 has higher sales, the second-period consumers are more confident about product 1 being of high value as the probability for product 1 being the bestseller is higher when it is of high value than when it is of low value. Similarly, given that product 2 is ranked higher, the second-period consumers' belief about product 2 being of high value is higher than the prior belief. In particular, as the $n_1 - m$ high-cost consumers always purchase product 1, the fact that product 2 has higher sales ranking than product 1 increases the belief about product 2 being of high value as the probability for product 2 overselling product 1 is much lower when product 2 is of low value than when it is of high value.

Volume Information

We now proceed to sales volume information. Recall that in the first period, all of the high-cost consumers purchase product 1. Thus, the low-cost consumers' purchasing actions in the first period can be fully deduced when sales volume information is released. Specifically, the following proposition characterizes a second-period consumer's posterior belief under different realization of first-period sales, where x denotes the first-period sales of product 1 and $k_1(x)$ and $k_2(x)$ denote the posterior beliefs that product 1 and product 2 are of high value, respectively, when the first-period sales of product 1 is realized to be x.

Proposition S.9. *Assume $m > 0$. Under sales volume information, a second-period consumer's posterior belief and optimal first-search product choice are as follows:*

(i) When the sales for product 1 is $n_1 - m$ (i.e., $x = n_1 - m$), $k_1(n_1 - m) = 0$ and $k_2(n_1 - m) > p_2$. It is optimal for the second-period consumer to first search product 2 if she decides to search;

(ii) When the sales for product 1 is strictly between $n_1 - m$ and n_1 (i.e., $n_1 - m < x < n_1$), $k_1(x) = 0$ and $k_2(x) = 0$. The second-period consumer is indifferent between first searching product 1 and first searching product 2 if she decides to search;

(iii) When the sales for product 1 is n_1 (i.e., $x = n_1$), $k_1(n_1) > p_1$ and $k_2(n_1) \leq k_1(n_1)$. It is optimal for the second-period consumer to first search product 1 if she decides to search.

Moreover, if the first search reveals a low-value product, the unsearched product is of low value for sure and none of the second-period consumers performs the second search.

Proposition S.9 implies that product asymmetry can *enhance* the informativeness of sales volume about the product values. This implication is reflected in two important aspects: first, sales volume may perfectly reveal a product's value under asymmetric products, but this never occurs when products are symmetric ex ante; and second, product asymmetry eliminates the consumers' need to perform a second search in the second period when sales volume information is available.

The first aspect is evidenced by parts (i) and (ii) of Proposition S.9, which are driven by an earlier observation (following Proposition S.7) that under asymmetric products, the sales distribution is deterministic conditional on certain product value realizations. The deterministic sales realization allows the consumers to draw a perfect inference about a product's value: for example, since product 1's first-period sales is n_1 for sure if product 1 is of high value, consumers deduce that product 1 is of low value if they observe any sales realization other than n_1. Such an inference is never possible for the case of symmetric products because under symmetric products, the sales distribution conditional on any value realization is never deterministic. In particular, recall from the base model that the case of the products sharing high values and that of sharing low values lead to the same sales distribution $G_s(x)$ and the corresponding density $g_s(x)$ is positive for any sales realization x between zero and n_1. That is why none of the possible sales realizations allows for perfect inference of product values, since sales volume never enables the consumers to eliminate the possibility of two products being of equal values and, given identical product values, it never enables them to perfectly infer the exact (common) value of the products.

The second aforementioned aspect is elaborated in the last sentence of Proposition S.9. In particular, sales volume and discovery of a low-value product in a first search jointly imply low value of the unsearched product and, thus, a second search is never worthy. This is because, as the m low-cost consumers' purchasing choices are fully deducible, the second-period consumers learn from their purchases and always search a high-value product first if two product values differ from each other. This result can be elucidated by considering three scenarios: if all of the low-cost consumers purchase product 2 (i.e., part (i) of Proposition S.9), it implies that product 1 is of low value as otherwise the low-cost consumers would have purchased product 1 through their first search and, thus, the second-period consumers first search product 2 and never search product 1; if all of the low-cost consumers purchase product 1 (i.e., part (iii) of Proposition S.9), it implies that either product 1 is of high value or both products are of low value. Hence, the second-period consumers first search product 1 and never need to search product 2 because if product 1 turns out to be of low value, so is product 2; if the low-cost consumers' purchases are split between two products (i.e., part (ii) of Proposition S.9), it implies that product 1 is of low value as otherwise all of the low-cost consumers would have purchased product 1 through their first search. It also further implies that product 2 is also of low value as otherwise all of the low-cost consumers would have purchased product 2 through their second search. Thus, each second-period consumer randomly picks a product to search and purchase.

Furthermore, as exemplified by part (ii) of Proposition S.9, the second-period consumers' belief for either product being of high value is sometimes lower than the prior belief. Hence, same as in the base model, sales volume information may hurt the platform and lead to lower expected sales in the second period when the two products are asymmetric. Proposition S.10 follows.

Proposition S.10. *There exist problem instances where sales volume information reduces the expected total sales in the second period.*

SC.3. Numerical Illustration

We now numerically illustrate the impact of sales information (either ranking or volume) on the second-period expected sales under asymmetric products. As in the base model, we assume that the search cost density is bimodal: $F(x) = (\alpha + (1-\alpha)\Phi((x-\mu)/\sigma))\mathbb{I}(x \geq 0)$, where $\Phi(\cdot)$ is the cumulative distribution function for the standard normal distribution and $\mathbb{I}(\cdot)$ is the indicator function with $\alpha = 0.08$, $\mu = 4.5$ and $\sigma = 1.5$. Other parameter values are: the total number of consumers in the two periods $n = 100$, the number of consumers in the first period $n_0 = 20$, $u_h = 4$, and $u_l = 0$. Table S.6 shows the expected sales in the second period under different sales information and various values of p_1 and p_2 with $p_1 = p_2 + \delta$.

Table S.6 Impact of δ on total expected sales in the second period: $\mu = 4.5, \sigma = 1.5, n = 100, u_h = 4, u_l = 0, \alpha = 0.08$

	$p_2 = 0.4$				$p_2 = 0.45$			
δ	*Ranking*	*Volume*	*No Info.*	*Opt. Info.*	*Ranking*	*Volume*	*No Info.*	*Opt. Info.*
0.05	20.29	18.56	9.04	Ranking	22.49	20.79	9.92	Ranking
0.1	21.41	19.83	9.92	Ranking	23.54	22.01	11.01	Ranking
0.15	22.56	21.14	11.01	Ranking	19.65	23.25	12.34	Volume
0.2	18.76	22.47	12.34	Volume	21.19	24.51	13.95	Volume
0.25	20.37	23.82	13.95	Volume	22.80	25.79	15.86	Volume

We first observe from Table S.6 that, same as in the base model, either ranking information or volume information can be optimal for the platform. Furthermore, the expected sales in the second period increases in δ under either no information or volume information. This is intuitive: a higher δ implies a higher prior belief of product 1 being of high value, and, thus, the second-period consumers are more willing to search and purchase.

Another interesting, and perhaps surprising, observation from Table S.6 is that the second-period expected sales sometimes decreases in δ when the platform releases sales ranking information. This is counter-intuitive, as it implies that the second-period consumers are more reluctant to search and purchase when product 1 becomes increasingly likely to be of high value. To see the logic behind the result, recall that under asymmetric products, in the first period the number of low-cost consumers is determined by p_2 while the number of consumers making purchases (i.e., the low-cost and high-cost ones) is determined by p_1. Therefore, when p_1 increases (due to a higher δ) and p_2 stays the same, the number of low-cost consumers remain the same and the number of high-cost consumers increases. Consequently, the proportion of consumers who make informed purchases in the first period drops and, thus, sales ranking information can become less informative, leading to lower expected sales in the second period as δ increases. Notice that this does not occur for sales volume information (as we observed, the second-period expected sales increases in δ under volume information). This is because, with sales volume information, the purchasing decisions of the high-cost consumers do not make an impact on the second-period consumers as the low-cost consumers' purchases are fully deducible.

The effects discussed so far also suggest that volume information is increasingly likely to outperform ranking information (in terms of higher second-period sales) when the expected value of the more promising product increases, as we also observe from Table S.6.

SC.4. Appendix

Proof of Lemma S.6 As product 1 is more likely to be high type than product 2, Lemma S.1 implies that it is optimal for a first-period consumer to search product 1 first. If product 1 is revealed to be of low value, then the consumer considers searching product 2. The utility of purchasing a low-value product is u_l and the expected utility of searching product 2 is $p_2u_h + (1-p_2)u_l - s$. Therefore a consumer performs the second search if and only if $s \le p_2(u_h - u_l)$. For the first search, the expected utility of first search is

$$p_1u_h + (1-p_1)\max[u_l, p_2u_h + (1-p_2)u_l - s] - s.$$

Therefore a consumer performs the first search if and only if $p_1u_h + (1-p_1)\max[u_l, p_2u_h + (1-p_2)u_l - s] - s \ge 0$, which is $s \le p_1u_h + (1-p_1)u_l$ (as $p_2 < p_1$). ☐

Proof of Proposition S.7 For the $n_1 - m$ high-cost consumers, they only search once. As it is optimal to search product 1 first, these consumers purchase product 1 for sure. For the m low-cost consumers, they also search product 1 first. If product 1 is of high value, then they purchase product 1 and will not perform the second search. If product 1 is of low value, then they search product 2. If product 2 is of high value, they purchase product 2. If product 2 is also of low value, then they randomly choose a product to purchase. The proposition is proved by summarizing the above arguments. ☐

Proof of Proposition S.8 Let s_1, s_2 be the first-period sales of product 1 and product 2 and r_1, r_2 be the event that product 1 and product 2 has higher sales ranking, respectively. First consider the case that product 1 has higher sales ranking. The belief that product 1 is of high value is

$$\begin{aligned}\pi_1^r(1) &= \Pr(u_1 = u_h | r_1) \\ &= \frac{\Pr(u_1 = u_h, r_1)}{\Pr(r_1)} \\ &= \frac{\Pr(u_1 = u_h)}{\Pr(s_1 > s_2) + \Pr(s_1 = s_2)/2} \\ &= \frac{p_1}{\Pr(s_1 > s_2) + \Pr(s_1 = s_2)/2} \\ &> p_1\end{aligned}$$

where the third equality follows from Proposition S.7. Similarly, the probability that product 2 is of high value when product 1 has higher sales ranking is

$$\begin{aligned}\pi_{-1}^r(1) &= \Pr(u_2 = u_h | r_1) \\ &= \frac{\Pr(u_2 = u_h, r_1)}{\Pr(r_1)} \\ &= \frac{\Pr(u_1 = u_h, u_2 = u_h, r_1) + \Pr(u_1 = u_l, u_2 = u_h, r_1)}{\Pr(s_1 > s_2) + \Pr(s_1 = s_2)/2} \\ &= \frac{\Pr(r_1 | u_1 = u_h, u_2 = u_h) \cdot \Pr(u_1 = u_h, u_2 = u_h) + \Pr(r_1 | u_1 = u_l, u_2 = u_h) \cdot \Pr(u_1 = u_l, u_2 = u_h)}{\Pr(s_1 > s_2) + \Pr(s_1 = s_2)/2}\end{aligned}$$

When $n_1 - m < m$,

$$\begin{aligned}&\frac{\Pr(r_1 | u_1 = u_h, u_2 = u_h) \cdot \Pr(u_1 = u_h, u_2 = u_h) + \Pr(r_1 | u_1 = u_l, u_2 = u_h) \cdot \Pr(u_1 = u_l, u_2 = u_h)}{\Pr(s_1 > s_2) + \Pr(s_1 = s_2)/2} \\ &= \frac{p_1 p_2}{\Pr(s_1 > s_2) + \Pr(s_1 = s_2)/2} \\ &< \frac{p_1}{\Pr(s_1 > s_2) + \Pr(s_1 = s_2)/2}\end{aligned}$$

when $n_1 - m = m$,

$$\begin{aligned}&\frac{\Pr(r_1 | u_1 = u_h, u_2 = u_h) \cdot \Pr(u_1 = u_h, u_2 = u_h) + \Pr(r_1 | u_1 = u_l, u_2 = u_h) \cdot \Pr(u_1 = u_l, u_2 = u_h)}{\Pr(s_1 > s_2) + \Pr(s_1 = s_2)/2} \\ &= \frac{p_1 p_2 + (1 - p_1) p_2 / 2}{\Pr(s_1 > s_2) + \Pr(s_1 = s_2)/2} \\ &< \frac{p_2}{\Pr(s_1 > s_2) + \Pr(s_1 = s_2)/2} \\ &< \frac{p_1}{\Pr(s_1 > s_2) + \Pr(s_1 = s_2)/2}\end{aligned}$$

Therefore $\pi_{-1}^r < \pi_1^r$ and it is optimal for consumers to search product 1 first.

Next consider the case where product 2 has higher sales ranking. By Proposition S.7, product 1 is of low value as otherwise it would have higher sales ranking. The belief that product 2 is of high value is

$$\begin{aligned}\pi_1^r(2) &= \Pr(u_2 = u_h | r_2) \\ &= \frac{\Pr(u_2 = u_h, r_2)}{\Pr(r_2)} \\ &= \frac{\Pr(u_2 = u_h, r_2)}{\Pr(u_2 = u_h, r_2) + \Pr(u_2 = u_l, r_2)}\end{aligned}$$

$$
\begin{aligned}
&= \frac{\Pr(u_1=u_l,u_2=u_h,r_2)+\Pr(u_1=u_h,u_2=u_h,r_2)}{\begin{pmatrix}\Pr(u_1=u_l,u_2=u_h,r_2)+\Pr(u_1=u_h,u_2=u_h,r_2)\\ +\Pr(u_1=u_l,u_2=u_l,r_2)+\Pr(u_1=u_h,u_2=u_l,r_2)\end{pmatrix}}\\
&= \frac{\Pr(u_1=u_l,u_2=u_h,r_2)}{\Pr(u_1=u_l,u_2=u_h,r_2)+\Pr(u_1=u_l,u_2=u_l,r_2)}\\
&= \frac{\Pr(r_2|u_1=u_l,u_2=u_h)\cdot\Pr(u_1=u_l,u_2=u_h)}{\Pr(r_2|u_1=u_l,u_2=u_h)\cdot\Pr(u_1=u_l,u_2=u_h)+\Pr(r_2|u_1=u_l,u_2=u_l)\cdot\Pr(u_1=u_l,u_2=u_l)}
\end{aligned}
$$

When $n_1-m<m$,

$$
\begin{aligned}
&\frac{\Pr(r_2|u_1=u_l,u_2=u_h)\cdot\Pr(u_1=u_l,u_2=u_h)}{\Pr(r_2|u_1=u_l,u_2=u_h)\cdot\Pr(u_1=u_l,u_2=u_h)+\Pr(r_2|u_1=u_l,u_2=u_l)\cdot\Pr(u_1=u_l,u_2=u_l)}\\
&= \frac{(1-p_1)p_2}{(1-p_1)p_2+(1-p_1)(1-p_2)\Pr(r_2|u_1=u_l,u_2=u_l)}\\
&> \frac{(1-p_1)p_2}{(1-p_1)p_2+(1-p_1)(1-p_2)}\\
&= p_2
\end{aligned}
$$

when $n_1-m=m$,

$$
\begin{aligned}
&\frac{\Pr(r_2|u_1=u_l,u_2=u_h)\cdot\Pr(u_1=u_l,u_2=u_h)}{\Pr(r_2|u_1=u_l,u_2=u_h)\cdot\Pr(u_1=u_l,u_2=u_h)+\Pr(r_2|u_1=u_l,u_2=u_l)\cdot\Pr(u_1=u_l,u_2=u_l)}\\
&= \frac{(1-p_1)p_2/2}{(1-p_1)p_2/2+(1-p_1)(1-p_2)\Pr(r_2|u_1=u_l,u_2=u_l)}\\
&> \frac{(1-p_1)p_2}{(1-p_1)p_2+(1-p_1)(1-p_2)}\\
&= p_2
\end{aligned}
$$

Hence it is optimal to search product 2 first when product 2 has higher sales ranking. □

Proof of Proposition S.9 First consider the case $x=n_1-m$. Recall that we have shown that when product 1 is of high value it has sales n_1. Thus, in this case product 1 is of low value. Hence, it is optimal to search product 2 only. The belief that product 2 is of high value is

$$
\begin{aligned}
k_2(n_1-m) &= \Pr(u_2=u_h|s_1=n_1-m)\\
&= \frac{\Pr(u_2=u_h,s_1=n_1-m)}{\Pr(s_1=n_1-m)}\\
&= \frac{\Pr(u_1=u_l,u_2=u_h,s_1=n_1-m)}{\Pr(u_1=u_l,u_2=u_h,s_1=n_1-m)+\Pr(u_1=u_l,u_2=u_l,s_1=n_1-m)}\\
&= \frac{(1-p_1)p_2}{(1-p_1)p_2+(1-p_1)(1-p_2)\cdot(\frac{1}{2})^m}\\
&> \frac{(1-p_1)p_2}{(1-p_1)p_2+(1-p_1)(1-p_2)}\\
&= p_2
\end{aligned}
$$

The fourth equality follows from $\Pr(u_1=u_l,u_2=u_l,s_1=n_1-m)=\Pr(u_1=u_l,u_2=u_l)\cdot\Pr(s_1=n_1-m|u_1=u_l,u_2=u_l)=(1-p_1)(1-p_2)\cdot(\frac{1}{2})^m$ as each of the m low-cost consumers purchase product 2 with probability 1/2 when both products are of low value and the overall probability for all these m consumers to purchase product 2 is $(\frac{1}{2})^m$ (recall that all the high-cost consumers purchase product 1).

Now, consider the case $n_1-m<x<n_1$. Similar to the previous case, as $x<n_1$, product 1 is of low value. If product 2 was of high value, then by Proposition S.7 it would have sales m. Therefore, product 2 is also of low value. That is, $k_1(n_1-m)=0$ and $k_2(n_1-m)=0$. In this case, there is no need to perform a second search.

We now proceed to the case $x = n_1$. By Proposition S.7, this happens only if either product 1 is of high value or both products are of low value. The belief that product 1 is of high value is

$$\begin{aligned} k_1(n_1) &= \Pr(u_1 = u_h | s_1 = n_1) \\ &= \frac{\Pr(u_1 = u_h, s_1 = n_1)}{\Pr(s_1 = n_1)} \\ &= \frac{\Pr(u_1 = u_h, s_1 = n_1)}{\Pr(u_1 = u_h, s_1 = n_1) + \Pr(u_1 = u_l, s_1 = n_1)} \\ &= \frac{p_1}{p_1 + (1-p_1)(1-p_2) \cdot (\frac{1}{2})^m} \\ &> \frac{p_1}{p_1 + (1-p_1)} \\ &= p_1 \end{aligned}$$

For the fourth equality, notice that $\Pr(u_1 = u_l, s_1 = n_1) = \Pr(u_1 = u_l, u_2 = u_h, s_1 = n_1) + \Pr(u_1 = u_l, u_2 = u_l, s_1 = n_1)$. By Proposition S.7, $\Pr(u_1 = u_l, u_2 = u_h, s_1 = n_1) = 0$ and we have $\Pr(u_1 = u_l, u_2 = u_l, s_1 = n_1) = \Pr(u_1 = u_l, u_2 = u_l) \cdot \Pr(s_1 = n_1 | u_1 = u_l, u_2 = u_l) = (1-p_1)(1-p_2) \cdot (\frac{1}{2})^m$ as each of the m low-cost consumers purchase product 1 with probability 1/2 when both products are of low value and the overall probability for all these m consumers to purchase product 1 is $(\frac{1}{2})^m$ (recall that all the high-cost consumers purchase product 1).

The belief that product 2 is of high value is

$$\begin{aligned} k_2(n_1) &= \Pr(u_2 = u_h | s_1 = n_1) \\ &= \frac{\Pr(u_2 = u_h, s_1 = n_1)}{\Pr(s_1 = n_1)} \\ &= \frac{\Pr(u_1 = u_h, u_2 = u_h, s_1 = n_1)}{\Pr(u_1 = u_h, u_2 = u_h, s_1 = n_1) + \Pr(u_1 = u_h, u_2 = u_l, s_1 = n_1) + \Pr(u_1 = u_l, u_2 = u_l, s_1 = n_1)} \\ &= \frac{p_1 p_2}{p_1 p_2 + p_1(1-p_2) + (1-p_1)(1-p_2) \cdot (\frac{1}{2})^m} \\ &= \frac{p_1 p_2}{p_1 + (1-p_1)(1-p_2) \cdot (\frac{1}{2})^m} \\ &\leq k_1(n_1) \end{aligned}$$

If product 1 is revealed to be of low value, then product 2 is also of low value as otherwise product 1 would have sales $n_1 - m$. Therefore no consumer performs the second search. □

Proof of Proposition S.10: Let S_ϕ be the expected sales in the second period when no sales information is released and S_v be the expected sales in the second period when sales volume information is released. We prove the proposition by constructing the following instance. Let $u_h = 5, u_l = 1$, $p_1 = 0.6$ and $p_2 = 0.4$. Let n_2 be the number of consumers in the second period and set $n_0 = 4$ and $n_2 = 10$. Consider a distribution F such that $F(x) = 1$ for $x \geq 2$ and $F(x) = 0.5$ for $0 \leq x < 2$. Then $n_1 = m = 4$ and the expected sales in the second period with no information is $S_\phi = 10$. If sales volume information is released, then there is a positive probability that the first-period sales for product 1 is strictly between $n_1 - m$ and n_1, under which case the sales in the second period is $n_2 \cdot F(u_l) = 5$, by Proposition S.9 (ii). It follows that the expected sales in the second period when sales volume information is released is lower than 10. So $S_v < S_\phi$ and the proposition is proved. □

SD. More Than Two Levels of Product Value

In the base model model, we focus on the setting where either product's value can be of two different levels, u_l and u_h. The bi-valued product-value (or quality) model is commonly adopted in social-learning literature (e.g., Yu et al. 2016) as it allows learning process to be captured by a single variable (i.e., probability of high quality/value), and thus facilitates the analysis. With more than two quality (or value) levels, a posterior belief needs to be evaluated for each level, which significantly complicates Bayesian learning. In the context of our model, the analysis is further complicated by the presence of multiple products and consumers' sequential product search: the former indicates that the beliefs about each value are to be updated for each product, and the latter implies that the belief updating occurs repeatedly, after each search. In particular, under a much expanded state space (consisting of beliefs for each value of each product), consumers' decisions on which product to search first and whether to perform a further search become much more complex as the number of product-value levels increases.

In this extension, we consider the case where either product's value can be one of N different levels with $N \geq 2$. More specifically, either product is of a value in the set $\{v_1, v_2, ..., v_N\}$ with $0 < v_1 < v_2 < ... < v_N$. We assume that the probability for either product taking value v_i is p_i with $\sum_{i=1}^{N} p_i = 1$, which is a common knowledge to the platform and consumers. As in the base model, we assume that the two products' values, u_1 and u_2, are independently distributed. For the sake of exposition, in this extension we let $\sum_{i=j}^{k} f(i) = 0$ for $j > k$ for any function $f(i)$.

SD.1. First Period

We first analyze consumers' search and purchasing behavior in the first period. Lemma S.7 follows.

Lemma S.7. *A consumer performs the first search if and only if $s \leq \sum_{j=1}^{N} p_j v_j$. If the first search reveals a product with value v_i, then the consumer performs the second search if and only if $s \leq \sum_{j=i+1}^{N} p_j (v_j - v_i)$.*

Lemma S.7 identifies the threshold for a first-period consumer to perform the first and the second search, respectively. The overall structure is similar to that in the base model, but as a product's value is of n different levels, the threshold for second search is more complicated.

Next, we analyze the purchasing behavior of a first-period consumer. Suppose now that $u_1 = v_i, u_2 = v_j$. If $v_i = v_j$, then the two products are symmetric and a first-period consumer purchases each product with equal probabilities. If $v_i < v_j$ and the consumer searches product 1 first, then the consumer purchases product 1 directly if $s > \sum_{k=i+1}^{N} p_k (v_k - v_i)$ and performs the second search when $s \leq \sum_{k=i+1}^{N} p_k (v_k - v_i)$. If the consumer performs the second search, then he purchases product 2. Alternatively, if the consumer searches product 2 first, then he purchases product 2 directly if $s > \sum_{k=j+1}^{N} p_k (v_k - v_j)$ and performs the second search when $s \leq \sum_{k=j+1}^{N} p_k (v_k - v_j)$. After the second search, the consumer still purchases product 2. Therefore, if $v_i < v_j$, a first-period consumer purchases product 2 for sure if $s \leq \sum_{k=i+1}^{N} p_k (v_k - v_i)$ and purchases either product with equal probability if $s > \sum_{k=i+1}^{N} p_k (v_k - v_i)$. The case where $v_i > v_j$ is symmetric.

So, for any given pair of values (v_i, v_j), the sales distribution is similar to that in the base model. Specifically, we have the following proposition.

Proposition S.11. *Define* $n_1 := \lfloor n_0 \cdot F(\sum_{j=1}^N p_j v_j)\rfloor$ *and* $m_i = \lfloor n_0 \cdot F(\sum_{j=i+1}^N p_j(v_j - v_i))\rfloor$ *for given* v_i*. Suppose the value of the two products are* $v_i \le v_j$*. For* $x \in [0, n_1]$ *and*

$$g_s(x) := Binomial(x, n_1, 1/2),$$
$$g_a^i(x) := Binomial(x - m_i, n_1 - m_i, 1/2) \text{ if } x \ge m_i, \text{ and } 0 \text{ if } x < m_i,$$

where $Binomial(x, y, p)$ *is the probability that among* y *independent trials,* x *of them succeed, where the probability of success is* p*. Let* $G_s(x)$ *and* $G_a^i(x)$ *be the cumulative distribution functions corresponding to* $g_s(x)$ *and* $g_a^i(x)$*, respectively. Let* $\bar{G}_s(x) := 1 - G_s(x)$ *and* $\bar{G}_a^i(x) := 1 - G_a^i(x)$*.*
(i) If $v_i = v_j$*, the sales of either product follows distribution* $G_s(x)$*;*
(ii) If $v_i < v_j$*, the sales of the higher-valued (resp. lower-valued) product follows distribution* $G_a^i(x)$ *(resp.* $\bar{G}_a^i(n_1 - x)$*).*

If the products share a common value, the sales of either product follows the distribution $G_s(\cdot)$. If the products have different values, the sales distribution of the higher-valued product is $G_a^i(\cdot)$, where the number of consumers performing the second search is determined by the value of the lower-valued product.

SD.2. Second Period

Next, we consider consumers' beliefs in the second period. We focus on the case where n_1 is odd. The case of even n_1 can be analyzed similarly as in the base model.

As in the base model, we assume without loss of generality that product 1 is the bestseller product. Let $H_1^v(v_i, x)$ and $H_{-1}^v(v_i, x)$ be the beliefs that product 1 and product 2 have value v_i when product 1 has sales x, respectively. Let $H_1^r(v_i)$ and $H_{-1}^r(v_i)$ be the beliefs that product 1 and product 2 have value v_i respectively, when product 1 has higher sales. Lemma S.8 follows.

Lemma S.8.

$$H_1^v(v_k, x) = \frac{g_s(x) \cdot p_k^2 + g_a^j(x) \cdot \sum_{j=1}^{k-1} p_k p_j + g_a^k(n_1 - x) \cdot p_k \cdot (\sum_{j=k+1}^N p_j)}{g_s(x) \cdot \sum_{i=1}^N p_i^2 + \sum_{i=1}^N \sum_{j=1}^{i-1} g_a^j(x) \cdot p_i p_j + \sum_{i=1}^N g_a^i(n_1 - x) \cdot p_i \cdot (\sum_{j=i+1}^N p_j)}$$
$$H_{-1}^v(v_k, x) = \frac{g_s(n_1 - x) \cdot p_k^2 + g_a^j(n_1 - x) \cdot \sum_{j=1}^{k-1} p_k p_j + g_a^k(x) \cdot p_k \cdot (\sum_{j=k+1}^N p_j)}{g_s(n_1 - x) \cdot \sum_{i=1}^N p_i^2 + \sum_{i=1}^N \sum_{j=1}^{i-1} g_a^j(n_1 - x) \cdot p_i p_j + \sum_{i=1}^N g_a^i(x) \cdot p_i \cdot (\sum_{j=i+1}^N p_j)}$$

and

$$H_1^r(v_k) = 2p_k \cdot (p_k \cdot G_s(n_1/2) + \sum_{j=1}^{k-1} \bar{G}_a^j(n_1/2) p_j + G_a^k(n_1/2) \cdot (\sum_{j=k+1}^N p_j))$$
$$H_{-1}^r(v_k) = 2p_k \cdot (p_k \cdot G_s(n_1/2) + \sum_{j=1}^{k-1} G_a^j(n_1/2) p_j + \bar{G}_a^k(n_1/2) \cdot (\sum_{j=k+1}^N p_j))$$

Next, we consider second-period consumers' beliefs after the first search. Let $H_2^v(v_i, x | u_1 = v_j)$ be the belief that product 2 has value v_i when product 1 has sales x and value v_j. Similarly, let $H_{-2}^v(v_i, x | u_2 = v_j)$ be the belief that product 1 has value v_i when product 2 has value v_j and product 1 has sales x. Let $H_2^r(v_i | u_1 = v_j)$ be the belief that product 2 has value v_i when product 1 has higher sales and value v_j and let $H_{-2}^r(v_i | u_2 = v_j)$ be the belief that product 1 has value v_i when product 2 has value v_j and product 1 has higher sales. Lemma S.9 follows.

LEMMA S.9.

$$H_2^v(v_i, x|u_1 = v_j) = \begin{cases} \frac{p_i p_j g_s(x)}{p_j^2 \cdot g_s(x) + \sum_{k=1}^{j-1} p_j p_k \cdot g_a^k(x) + \sum_{k=j+1}^{N} p_j p_k \cdot g_a^j(n_1 - x)} & \text{if } v_i = v_j, \\ \frac{p_i p_j g_a^i(x)}{p_j^2 \cdot g_s(x) + \sum_{k=1}^{j-1} p_j p_k \cdot g_a^k(x) + \sum_{k=j+1}^{N} p_j p_k \cdot g_a^j(n_1 - x)} & \text{if } v_i < v_j, \\ \frac{p_i p_j g_a^j(n_1 - x)}{p_j^2 \cdot g_s(x) + \sum_{k=1}^{j-1} p_j p_k \cdot g_a^k(x) + \sum_{k=j+1}^{N} p_j p_k \cdot g_a^j(n_1 - x)} & \text{if } v_i > v_j. \end{cases}$$

$$H_{-2}^v(v_i, x|u_2 = v_j) = \begin{cases} \frac{p_i p_j g_s(n_1 - x)}{p_j^2 \cdot g_s(n_1 - x) + \sum_{k=1}^{j-1} p_j p_k \cdot g_a^k(n_1 - x) + \sum_{k=j+1}^{N} p_j p_k \cdot g_a^j(x)} & \text{if } v_i = v_j, \\ \frac{p_i p_j g_a^i(n_1 - x)}{p_j^2 \cdot g_s(n_1 - x) + \sum_{k=1}^{j-1} p_j p_k \cdot g_a^k(n_1 - x) + \sum_{k=j+1}^{N} p_j p_k \cdot g_a^j(x)} & \text{if } v_i < v_j, \\ \frac{p_i p_j g_a^j(x)}{p_j^2 \cdot g_s(n_1 - x) + \sum_{k=1}^{j-1} p_j p_k \cdot g_a^k(n_1 - x) + \sum_{k=j+1}^{N} p_j p_k \cdot g_a^j(x)} & \text{if } v_i > v_j. \end{cases}$$

and

$$H_2^r(v_i|u_1 = v_j) = \begin{cases} \frac{p_i p_j G_s(n_1/2)}{p_j^2 \cdot G_s(n_1/2) + \sum_{k=1}^{j-1} p_j p_k \cdot \bar{G}_a^k(n_1/2) + \sum_{k=j+1}^{N} p_j p_k \cdot G_a^j(n_1/2)} & \text{if } v_i = v_j, \\ \frac{p_i p_j \bar{G}_a^i(n_1/2)}{p_j^2 \cdot G_s(n_1/2) + \sum_{k=1}^{j-1} p_j p_k \cdot \bar{G}_a^k(n_1/2) + \sum_{k=j+1}^{N} p_j p_k \cdot G_a^j(n_1/2)} & \text{if } v_i < v_j, \\ \frac{p_i p_j G_a^j(n_1/2)}{p_j^2 \cdot G_s(n_1/2) + \sum_{k=1}^{j-1} p_j p_k \cdot \bar{G}_a^k(n_1/2) + \sum_{k=j+1}^{N} p_j p_k \cdot G_a^j(n_1/2)} & \text{if } v_i > v_j. \end{cases}$$

$$H_{-2}^r(v_i|u_2 = v_j) = \begin{cases} \frac{p_i p_j G_s(n_1/2)}{p_j^2 \cdot G_s(n_1/2) + \sum_{k=1}^{j-1} p_j p_k \cdot G_a^k(n_1/2) + \sum_{k=j+1}^{N} p_j p_k \cdot \bar{G}_a^j(n_1/2)} & \text{if } v_i = v_j, \\ \frac{p_i p_j G_a^i(n_1/2)}{p_j^2 \cdot G_s(n_1/2) + \sum_{k=1}^{j-1} p_j p_k \cdot G_a^k(n_1/2) + \sum_{k=j+1}^{N} p_j p_k \cdot \bar{G}_a^j(n_1/2)} & \text{if } v_i < v_j, \\ \frac{p_i p_j \bar{G}_a^j(n_1/2)}{p_j^2 \cdot G_s(n_1/2) + \sum_{k=1}^{j-1} p_j p_k \cdot G_a^k(n_1/2) + \sum_{k=j+1}^{N} p_j p_k \cdot \bar{G}_a^j(n_1/2)} & \text{if } v_i > v_j. \end{cases}$$

Lemmas S.8 and S.9 show that when products' value is of N different levels, the second period belief updating is much more complicated than that in the base model as consumers have to form beliefs for each product to take valuation v_i. The beliefs after the first search are even more involved as consumers may perform the second search when the first search reveals a value lower than v_N, which is of $N-1$ possibilities. In contrast, in the base model a consumer only performs the second search when the first search reveals a low-value product.

Proposition S.12 shows that as in the base model, the belief under sales volume information is a mean-preserving spread of the belief under sales ranking information.

PROPOSITION S.12. *$H_1^v(v_i, x)$ is a mean preserving spread of $H_1^r(v_i), \forall 1 \le i \le N$.*

We further prove in Proposition S.13 that, under ranking information, the posterior belief for the bestseller's value is first-order stochastically higher than that for the other product.

PROPOSITION S.13. *We have $\sum_{j=1}^{N} H_{-1}^r(v_j)v_j \le \sum_{j=1}^{N} H_1^r(v_j)v_j$.*

SD.2.1. Second Period Sales In this subsection, we compare the expected sales in the second period and show that our findings in the base model remain robust.

Let S_ϕ, S_r, and S_v be the expected sales in the second period with no sales information, with sales ranking information, and with sales volume information, respectively.

Proposition S.14 shows that sales ranking information always leads to higher second-period expected sales compared to no sales information.

PROPOSITION S.14. *We have $S_r \ge S_\phi$.*

Therefore, as in the base model, it is never optimal to release no sales information and the platform always benefits from offering sales ranking information.

Next, we compare the expected sales under ranking and volume information. Proposition S.15 shows that, between ranking information and volume information, either one can lead to higher second-period sales.

Proposition S.15. *There exist instances such that $S_r > S_v$ and $S_v > S_r$.*

Thus, as in the base model, either sales ranking information and sales volume information can be optimal for the platform and our main findings remain robust when products' value is of N different levels.

SD.3. Appendix

Proof of Lemma S.7: As the two products are symmetric, assume without loss of generality that product 1 is searched first. We first prove the second part. Suppose that the first search reveals that the value of product 1 is v_i. The expected utility of not performing the second search is v_i, while the expected utility of performing the second search is

$$\begin{aligned}&\mathbb{E}[\max[v_i,u_2]]-s\\=&\sum_{j=1}^{i}p_j\cdot v_i+\sum_{j=i+1}^{N}p_jv_j-s.\end{aligned}$$

Therefore, a consumer performs the second search if and only if $\sum_{j=1}^{i}p_j\cdot v_i+\sum_{j=i+1}^{N}p_jv_j-s\geq v_i$, which is $s\leq\sum_{j=i+1}^{N}p_j(v_j-v_i)$.

Next, we prove the first part of the lemma. The utility for not performing the first search is zero, and the expected utility of performing the first search is

$$\mathbb{E}[\max[u_1,\mathbb{E}[\max[u_1,u_2]]-s]]-s$$

where $\mathbb{E}[u_1]-s$ is the value for the consumer if she decides not to perform the second search and $\mathbb{E}[\max[u_1,u_2]]-s-s$ is the expected utility for the consumer if she decides to perform the second search after observing u_1.

We start with proving the "if" part. When $s\leq\sum_{j=1}^{N}p_jv_j$, since

$$\begin{aligned}&\mathbb{E}[\max[u_1,\mathbb{E}[\max[u_1,u_2]]-s]]-s\\\geq&\,\mathbb{E}[u_1]-s\\=&\sum_{j=1}^{N}p_jv_j-s\\\geq&\,0,\end{aligned}$$

the consumer performs the first search. Next, we prove the "only if" part. Suppose $s>\sum_{j=1}^{N}p_jv_j$, then as $\sum_{j=1}^{N}p_jv_j\geq\sum_{j=i+1}^{N}p_j(v_j-v_i),\forall i$, no consumer with $s>\sum_{j=1}^{N}p_jv_j$ performs the second search and

$$\begin{aligned}&\mathbb{E}[\max[u_1,\mathbb{E}[\max[u_1,u_2]]-s]]-s\\=&\,\mathbb{E}[u_1]-s\\=&\sum_{j=1}^{N}p_jv_j-s\\<&\,0.\end{aligned}$$

Therefore, a consumer with search cost $s>\sum_{j=1}^{N}p_jv_j$ does not perform the first search and the lemma is proved. □

Proof of Lemma S.8:

$$H_1^v(v_i,x)=\Pr(u_1=v_i|X_1=x)=\frac{\Pr(u_1=v_i,X_1=x)}{\Pr(X_1=x)}$$

We have

$$\begin{aligned}\Pr(X_1 = x) =& \sum_{i=1}^{N}\sum_{j=1}^{N}(g_s(x)\cdot \mathbb{I}[v_i = v_j] + g_a^j(x)\cdot \mathbb{I}[v_i > v_j] + g_a^i(n_1 - x)\cdot \mathbb{I}[v_i < v_j]) \\ =& g_s(x)\cdot \sum_{i=1}^{N} p_i^2 + \sum_{i=1}^{N}\sum_{j=1}^{i-1} g_a^j(x)\cdot p_i p_j + \sum_{i=1}^{N} g_a^i(n_1 - x)\cdot p_i \cdot (\sum_{j=i+1}^{N} p_j)\end{aligned}$$

So

$$H_1^v(v_k, x) = \frac{g_s(x)\cdot p_k^2 + \sum_{j=1}^{k-1} p_k p_j g_a^j(x) + g_a^k(n_1 - x)\cdot p_k \cdot (\sum_{j=k+1}^{N} p_j)}{g_s(x)\cdot \sum_{i=1}^{N} p_i^2 + \sum_{i=1}^{N}\sum_{j=1}^{i-1} g_a^j(x)\cdot p_i p_j + \sum_{i=1}^{N} g_a^i(n_1 - x)\cdot p_i \cdot (\sum_{j=i+1}^{N} p_j)}$$

By a similar derivation we have

$$H_{-1}^v(v_k, x) = \frac{g_s(n_1 - x)\cdot p_k^2 + \sum_{j=1}^{k-1} p_k p_j g_a^j(n_1 - x) + g_a^k(x)\cdot p_k \cdot (\sum_{j=k+1}^{N} p_j)}{g_s(n_1 - x)\cdot \sum_{i=1}^{N} p_i^2 + \sum_{i=1}^{N}\sum_{j=1}^{i-1} g_a^j(n_1 - x)\cdot p_i p_j + \sum_{i=1}^{N} g_a^i(x)\cdot p_i \cdot (\sum_{j=i+1}^{N} p_j)}$$

For ranking information, we have

$$\begin{aligned}H_1^r(v_k) =& \frac{G_s(n_1/2)\cdot p_k^2 + \sum_{j=1}^{k-1} \bar{G}_a^j(n_1/2)\cdot p_k p_j + G_a^k(n_1/2)\cdot p_k \cdot (\sum_{j=k+1}^{N} p_j)}{1/2} \\ =& 2p_k \cdot (p_k \cdot G_s(n_1/2) + \sum_{j=1}^{k-1} \bar{G}_a^j(n_1/2) p_j + G_a^k(n_1/2)\cdot (\sum_{j=k+1}^{N} p_j)),\end{aligned}$$

and a similar derivation gives

$$H_{-1}^r(v_k) = 2p_k \cdot (p_k \cdot G_s(n_1/2) + \sum_{j=1}^{k-1} G_a^j(n_1/2) p_j + \bar{G}_a^k(n_1/2)\cdot (\sum_{j=k+1}^{N} p_j))$$

□

Proof of Lemma S.9: We have

$$\begin{aligned}& H_2^v(v_i, x|u_1 = v_j) = \Pr(u_2 = v_i | X_1 = x, u_1 = v_j) \\ =& \frac{\Pr(u_2 = v_i, X_1 = x, u_1 = v_j)}{\Pr(X_1 = x, u_1 = v_j)} \\ =& \frac{\Pr(u_2 = v_i, X_1 = x, u_1 = v_j)}{p_j^2 \cdot g_s(x) + \sum_{k=1}^{j-1} p_j p_k \cdot g_a^k(x) + \sum_{k=j+1}^{N} p_j p_k \cdot g_a^j(n_1 - x)}\end{aligned}$$

and we have

$$\begin{aligned}& \Pr(u_2 = v_i, X_1 = x, u_1 = v_j) \\ =& \Pr(u_2 = v_i, u_1 = v_j)\cdot \Pr(X_1 = x | u_2 = v_i, u_1 = v_j) \\ =& p_i p_j \cdot \Pr(X_1 = x | u_2 = v_i, u_1 = v_j).\end{aligned}$$

If $v_i = v_j$, $\Pr(X_1 = x|u_2 = v_i, u_1 = v_j) = g_s(x)$; if $v_i < v_j$, $\Pr(X_1 = x|u_2 = v_i, u_1 = v_j) = g_a^i(x)$; and if $v_i > v_j$, $\Pr(X_1 = x|u_2 = v_i, u_1 = v_j) = g_a^j(n_1 - x)$. So,

$$H_2^v(v_i, x|u_1 = v_j) = \begin{cases} \frac{p_i p_j g_s(x)}{p_j^2 \cdot g_s(x) + \sum_{k=1}^{j-1} p_j p_k \cdot g_a^k(x) + \sum_{k=j+1}^{N} p_j p_k \cdot g_a^j(n_1 - x)} & \text{if } v_i = v_j, \\ \frac{p_i p_j g_a^i(x)}{p_j^2 \cdot g_s(x) + \sum_{k=1}^{j-1} p_j p_k \cdot g_a^k(x) + \sum_{k=j+1}^{N} p_j p_k \cdot g_a^j(n_1 - x)} & \text{if } v_i < v_j, \\ \frac{p_i p_j g_a^j(n_1 - x)}{p_j^2 \cdot g_s(x) + \sum_{k=1}^{j-1} p_j p_k \cdot g_a^k(x) + \sum_{k=j+1}^{N} p_j p_k \cdot g_a^j(n_1 - x)} & \text{if } v_i > v_j. \end{cases}$$

By symmetry, we have

$$H^v_{-2}(v_i, x|u_2 = v_j) = H^v_2(v_i, n_1 - x|u_1 = v_j) = \begin{cases} \frac{p_i p_j g_s(n_1-x)}{p_j^2 \cdot g_s(n_1-x) + \sum_{k=1}^{j-1} p_j p_k \cdot g_a^k(n_1-x) + \sum_{k=j+1}^N p_j p_k \cdot g_a^j(x)} & \text{if } v_i = v_j, \\ \frac{p_i p_j g_a^i(n_1-x)}{p_j^2 \cdot g_s(n_1-x) + \sum_{k=1}^{j-1} p_j p_k \cdot g_a^k(n_1-x) + \sum_{k=j+1}^N p_j p_k \cdot g_a^j(x)} & \text{if } v_i < v_j, \\ \frac{p_i p_j g_a^j(x)}{p_j^2 \cdot g_s(n_1-x) + \sum_{k=1}^{j-1} p_j p_k \cdot g_a^k(n_1-x) + \sum_{k=j+1}^N p_j p_k \cdot g_a^j(x)} & \text{if } v_i > v_j. \end{cases}$$

Then, we have

$$\begin{aligned} &H^r_2(v_i|u_1 = v_j) = \Pr(u_2 = v_i|X_1 \geq n_1/2, u_1 = v_j) \\ =&\frac{\Pr(u_2 = v_i, X_1 \geq n_1/2, u_1 = v_j)}{\Pr(X_1 \geq n_1/2, u_1 = v_j)} \\ =&\frac{\Pr(u_2 = v_i, X_1 \geq n_1/2, u_1 = v_j)}{p_j^2 \cdot G_s(n_1/2) + \sum_{k=1}^{j-1} p_j p_k \cdot \bar{G}_a^k(n_1/2) + \sum_{k=j+1}^N p_j p_k \cdot G_a^j(n_1/2)} \end{aligned}$$

and

$$\begin{aligned} &\Pr(u_2 = v_i, X_1 \geq n_1/2, u_1 = v_j) \\ =&\Pr(u_2 = v_i, u_1 = v_j) \cdot \Pr(X_1 \geq n_1/2|u_2 = v_i, u_1 = v_j) \\ =&p_i p_j \cdot \Pr(X_1 \geq n_1/2|u_2 = v_i, u_1 = v_j). \end{aligned}$$

If $v_i = v_j$, $\Pr(X_1 \geq n_1/2|u_2 = v_i, u_1 = v_j) = G_s(n_1/2)$; if $v_i < v_j$, $\Pr(X_1 \geq n_1/2|u_2 = v_i, u_1 = v_j) = \bar{G}_a^i(n_1/2)$; and if $v_i > v_j$, $\Pr(X_1 \geq n_1/2|u_2 = v_i, u_1 = v_j) = G_a^j(n_1/2)$. So,

$$H^r_2(v_i|u_1 = v_j) = \begin{cases} \frac{p_i p_j G_s(n_1/2)}{p_j^2 \cdot G_s(n_1/2) + \sum_{k=1}^{j-1} p_j p_k \cdot \bar{G}_a^k(n_1/2) + \sum_{k=j+1}^N p_j p_k \cdot G_a^j(n_1/2)} & \text{if } v_i = v_j, \\ \frac{p_i p_j \bar{G}_a^i(n_1/2)}{p_j^2 \cdot G_s(n_1/2) + \sum_{k=1}^{j-1} p_j p_k \cdot \bar{G}_a^k(n_1/2) + \sum_{k=j+1}^N p_j p_k \cdot G_a^j(n_1/2)} & \text{if } v_i < v_j, \\ \frac{p_i p_j G_a^j(n_1/2)}{p_j^2 \cdot G_s(n_1/2) + \sum_{k=1}^{j-1} p_j p_k \cdot \bar{G}_a^k(n_1/2) + \sum_{k=j+1}^N p_j p_k \cdot G_a^j(n_1/2)} & \text{if } v_i > v_j. \end{cases}$$

By a similar argument, we have

$$\begin{aligned} &H^r_{-2}(v_i|u_2 = v_j) = \Pr(u_1 = v_i|X_1 \geq n_1/2, u_2 = v_j) \\ =&\frac{\Pr(u_2 = v_j, X_1 \geq n_1/2, u_1 = v_i)}{\Pr(X_1 \geq n_1/2, u_2 = v_j)} \\ =&\frac{\Pr(u_2 = v_j, X_1 \geq n_1/2, u_1 = v_i)}{p_j^2 \cdot G_s(n_1/2) + \sum_{k=1}^{j-1} p_j p_k \cdot G_a^k(n_1/2) + \sum_{k=j+1}^N p_j p_k \cdot \bar{G}_a^j(n_1/2)} \end{aligned}$$

and

$$\begin{aligned} &\Pr(u_2 = v_j, X_1 \geq n_1/2, u_1 = v_i) \\ =&\Pr(u_2 = v_j, u_1 = v_i) \cdot \Pr(X_1 \geq n_1/2|u_2 = v_j, u_1 = v_i) \\ =&p_i p_j \cdot \Pr(X_1 \geq n_1/2|u_2 = v_j, u_1 = v_o). \end{aligned}$$

If $v_i = v_j$, $\Pr(X_1 \geq n_1/2|u_2 = v_j, u_1 = v_i) = G_s(n_1/2)$; if $v_i < v_j$, $\Pr(X_1 \geq n_1/2|u_2 = v_j, u_1 = v_i) = G_a^i(n_1/2)$; and if $v_i > v_j$, $\Pr(X_1 \geq n_1/2|u_2 = v_j, u_1 = v_i) = \bar{G}_a^j(n_1/2)$. Hence,

$$H^r_{-2}(v_i|u_2 = v_j) = \begin{cases} \frac{p_i p_j G_s(n_1/2)}{p_j^2 \cdot G_s(n_1/2) + \sum_{k=1}^{j-1} p_j p_k \cdot G_a^k(n_1/2) + \sum_{k=j+1}^N p_j p_k \cdot \bar{G}_a^j(n_1/2)} & \text{if } v_i = v_j, \\ \frac{p_i p_j G_a^i(n_1/2)}{p_j^2 \cdot G_s(n_1/2) + \sum_{k=1}^{j-1} p_j p_k \cdot G_a^k(n_1/2) + \sum_{k=j+1}^N p_j p_k \cdot \bar{G}_a^j(n_1/2)} & \text{if } v_i < v_j, \\ \frac{p_i p_j \bar{G}_a^j(n_1/2)}{p_j^2 \cdot G_s(n_1/2) + \sum_{k=1}^{j-1} p_j p_k \cdot G_a^k(n_1/2) + \sum_{k=j+1}^N p_j p_k \cdot \bar{G}_a^j(n_1/2)} & \text{if } v_i > v_j. \end{cases}$$

□

Proof of Proposition S.12: It suffices to show that $\mathbb{E}_{X_1 \geq \frac{n_1}{2}}[H_1^v(v_k, X_1)] = H_1^r(v_k), \forall 1 \leq i \leq N$. We have

$$\begin{aligned}
&\mathbb{E}_{X_1 \geq \frac{n_1}{2}}[H_1^v(v_k, X_1)] \\
=&\mathbb{E}_{X_1 \geq \frac{n_1}{2}}\left[\frac{g_s(X_1)\cdot p_k^2 + g_a^j(x)\cdot \sum_{j=1}^{k-1} p_k p_j + g_a^k(n_1 - X_1)\cdot p_k \cdot (\sum_{j=k+1}^N p_j)}{g_s(X_1)\cdot \sum_{i=1}^N p_i^2 + \sum_{i=1}^N \sum_{j=1}^{k-1} g_a^j(X_1)\cdot p_i p_j + \sum_{i=1}^N g_a^i(n_1 - X_1)\cdot p_i \cdot (\sum_{j=i+1}^N p_j)}\right] \\
=& \sum_{x=n_1/2}^{n_1} \frac{g_s(x)\cdot p_k^2 + g_a^j(x)\cdot \sum_{j=1}^{i} p_k p_j + g_a^k(n_1 - x)\cdot p_k \cdot (\sum_{j=k+1}^N p_j)}{g_s(x)\cdot \sum_{i=1}^N p_i^2 + \sum_{i=1}^N \sum_{j=1}^{i} g_a^j(x)\cdot p_i p_j + \sum_{i=1}^N g_a^i(n_1 - x)\cdot p_i \cdot (\sum_{j=i+1}^N p_j)} \\
&\cdot 2(g_s(x)\cdot \sum_{i=1}^N p_i^2 + \sum_{i=1}^N \sum_{j=1}^{i} g_a^j(x)\cdot p_i p_j + \sum_{i=1}^N g_a^i(n_1 - x)\cdot p_i \cdot (\sum_{j=i+1}^N p_j)) \\
=&2 \sum_{x=n_1/2}^{n_1} p_k \cdot (p_k \cdot g_s(x) + \sum_{j=1}^{k-1} p_j \cdot g_a^j(x) + \sum_{j=k+1}^N p_j \cdot g_a^k(n_1 - x)) \\
=&2p_k \cdot (p_k \cdot G_s(n_1/2) + \sum_{j=1}^{k-1} p_j \cdot \bar{G}_a^j(n_1/2) + \sum_{j=i+1}^N p_j \cdot G_a^k(n_1/2)) \\
=&H_1^r(v_k)
\end{aligned}$$

for all $1 \leq i \leq N$, which concludes the proof. □

Proof of Proposition S.13: By the property of first-order stochastic dominance, it suffices to prove

$$\sum_{k=1}^i H_{-1}^r(v_k) \geq \sum_{k=1}^i H_1^r(v_k), \forall 1 \leq i \leq N$$

which is

$$\begin{aligned}
&\sum_{k=1}^i 2p_k \cdot (p_k \cdot G_s(n_1/2) + \sum_{j=1}^{k-1} G_a^j(n_1/2) p_j + \bar{G}_a^k(n_1/2)\cdot (\sum_{j=k+1}^N p_j)) \\
\geq &\sum_{k=1}^i 2p_k \cdot (p_k \cdot G_s(n_1/2) + \sum_{j=1}^{k-1} \bar{G}_a^j(n_1/2) p_j + G_a^k(n_1/2)\cdot (\sum_{j=k+1}^N p_j)), \forall i
\end{aligned}$$

We have

$$\begin{aligned}
&\sum_{k=1}^i H_{-1}^r(v_k) - \sum_{k=1}^i H_1^r(v_k) \\
=&\sum_{k=1}^i 2p_k \cdot (p_k \cdot G_s(n_1/2) + \sum_{j=1}^{k-1} G_a^j(n_1/2) p_j + \bar{G}_a^k(n_1/2)\cdot (\sum_{j=k+1}^N p_j)) \\
&- \sum_{k=1}^i 2p_k \cdot (p_k \cdot G_s(n_1/2) + \sum_{j=1}^{k-1} \bar{G}_a^j(n_1/2) p_j + G_a^k(n_1/2)\cdot (\sum_{j=k+1}^N p_j)) \\
=&\sum_{k=1}^i (\sum_{j=1}^{k-1} G_a^j(n_1/2) p_j + \bar{G}_a^k(n_1/2)\cdot (\sum_{j=k+1}^N p_j) - \sum_{j=1}^{k-1} \bar{G}_a^j(n_1/2) p_j - G_a^k(n_1/2)\cdot (\sum_{j=k+1}^N p_j))
\end{aligned}$$

As $\bar{G}_a^j(n_1/2) \geq 1/2 \geq G_a^j(n_1/2), \forall j$, it follows that $\sum_{j=1}^{k-1} G_a^j(n_1/2) p_j + \bar{G}_a^k(n_1/2)\cdot (\sum_{j=k+1}^N p_j) - \sum_{j=1}^{k-1} \bar{G}_a^j(n_1/2) p_j - G_a^k(n_1/2)\cdot (\sum_{j=k+1}^N p_j)$ is first positive and then negative (as k increases). Therefore, $\sum_{k=1}^i H_{-1}^r(v_k) \geq \sum_{k=1}^i H_1^r(v_k)$ first increases then decreases. As $\sum_{k=1}^N H_{-1}^r(v_k) = \sum_{k=1}^N H_1^r(v_k) = 1$, it must be that $\sum_{k=1}^i H_{-1}^r(v_k) \geq \sum_{k=1}^i H_1^r(v_k), \forall 1 \leq i \leq N$. So the proposition is proved. □

Proof of Proposition S.14: Recall that n_2 is the total number of consumers in the second period. When there is no sales information, $S_\phi = n_2 \cdot F(\sum_{j=1}^N p_j v_j)$. When sales ranking information is released, second-period consumers with search cost lower than $\sum_{j=1}^N H_1^r(v_j) v_j$ always perform the first search. Therefore, $S_r \geq$

$n_2 \cdot F(\sum_{j=1}^N H_1^r(v_j)v_j)$. As F is a cumulative distribution function and is increasing, it suffices to show that $\sum_{j=1}^N p_j v_j \le \sum_{j=1}^N H_1^r(v_j)v_j$. By the property of first-order stochastic dominance, it suffices to show

$$\sum_{k=1}^{i} p_k \ge \sum_{k=1}^{i} H_1^r(v_k), \forall i$$

which is

$$\sum_{k=1}^{i} p_k \ge \sum_{k=1}^{i} 2p_k \cdot (p_k \cdot G_s(n_1/2) + \sum_{j=1}^{k-1} \bar{G}_a^j(n_1/2)p_j + G_a^k(n_1/2) \cdot (\sum_{j=k+1}^{N} p_j)), \forall i$$

We have

$$\begin{aligned} &2p_k \cdot (p_k \cdot G_s(n_1/2) + \sum_{j=1}^{k-1} \bar{G}_a^j(n_1/2)p_j + G_a^k(n_1/2) \cdot (\sum_{j=k+1}^{N} p_j)) \\ =&p_k \cdot (p_k + \sum_{j=1}^{k-1} p_j \cdot 2\bar{G}_a^j(n_1/2) + \sum_{j=k+1}^{N} p_j \cdot 2G_a^k(n_1/2)) \end{aligned}$$

By the result in the base model, we know that $G_a^j(n_1/2) \le 1/2, \forall j$ and $\bar{G}_a^j(n_1/2) \ge 1/2, \forall j$. As $p_k + \sum_{j=1}^{k-1} p_j + \sum_{j=k+1}^{N} p_j = 1$, $p_k + \sum_{j=1}^{k-1} p_j \cdot 2\bar{G}_a^j(n_1/2) + \sum_{j=k+1}^{N} p_j \cdot 2G_a^k(n_1/2)$ is first lower than 1 and then higher than 1 (as k increases). It follows that $p_k - H_1^r(v_k)$ is first positive and then negative (as k increases). Since $\sum_{k=1}^N p_k = \sum_{k=1}^N H_1^r(v_k) = 1$, it must be that $\sum_{k=1}^i p_k \ge \sum_{k=1}^i H_1^r(v_k), \forall 1 \le i \le N$, which concludes the proof of this proposition. □

Proof of Proposition S.15: We prove the proposition by constructing two examples.

We first show an example where $S_v > S_r$. Consider an arbitrary $0 < \epsilon < 0.1$. Define p_i as the probability that products's utility is v_i. Let $p_1 = 0.4, p_i = \frac{3}{5(N-1)}, 2 \le i \le N$. Let $v_i = 6 + \frac{(i-(N+2)/2)\epsilon}{N}, 2 \le i \le N$. The v_i is chosen so that the average value of $v_2, v_3, ...v_N$ is 6. Let $v_1 = 2$.

Consider a distribution function F such that $F(x) = 0, x < 2.4$, $F(2.4) = \epsilon/36$, $F(4.4) = \epsilon/12$, $F(4.88) = \epsilon/6$, and $F(5.36) = 2/3$. From the search cost distribution it is clear that a first-period consumer only performs the second search when the first search reveals a utility of v_1, and the consumer performs the second search if $s \le 2.4$. The expected utility of performing the first search is 4.4. Let $n_0 = \lceil 36/\epsilon \rceil$, we have $n_1 = 3, m_1 = 1, m_i = 0, 2 \le i \le N$. Consequently, we have

$$\begin{aligned} H_1^r(v_1) &= 0.28 \\ H_1^r(v_i) &= \frac{0.72}{N-1} = \frac{18}{25(N-1)}, 2 \le i \le N \end{aligned}$$

Notice that here by the construction, valuation 2 to valuation N can be considered as a "pseudo-valuation" as consumers' search and purchase behavior are exactly the same for these $N-1$ valuations. The expected value of the bestseller product is 4.88, so the expected sales under ranking information is $n_2 \cdot \epsilon/6$. The probability that the bestseller product's first-period sales is 3 is $2 \cdot (0.4^2 \cdot 1/8 + 0.6^2 \cdot 1/8 + 0.4 \cdot 0.6 \cdot 2 \cdot 1/4) = 0.37$. We have

$$\begin{aligned} H_1^v(v_1, 3) &= 0.16 \\ H_1^v(v_i, 3) &= \frac{0.84}{N-1} = \frac{21}{25(N-1)}, 2 \le i \le N \end{aligned}$$

The expected value of the bestseller product when sales is 3 is $\sum_{i=1}^N H_1^v(v_i, 3) \cdot v_i = 5.36$, so the expected sales under sales volume information is at least $0.37 \cdot n_2 \cdot F(5.36) = 0.74n_2/3$. Picking an $\epsilon < 0.1$, we have an instance where $S_r < S_v$.

Next, we show another example where that $S_v < S_r$. Consider an instance where the p's and v's are the same as in the previous example. Consider a distribution function F such that $F(x) = 0, x < 2.4$, $F(2.4) = 1/6$, $F(4.4) = 1/2$, $F(4.88) = 1$, and $F(x) = 1/2, 4.4 \leq x < 4.88$. From the search cost distribution it is clear that a first-period consumer only performs the second search when the first search reveals a value of v_1, and the consumer performs the second search if $s \leq 2.4$. The expected utility of search in the first period is 4.4. Let $n_0 = 6$, we have $n_1 = 3, m_1 = 1$, implying that the expected value of the bestseller product is 4.88. So all second-period consumers make purchase when sales ranking information is released. When sales volume information is released, there is a positive probability that the bestseller product has sales 2, in which case the expected value of the bestseller product is lower than 4.88 (as the belief under sales volume information is a mean-preserving spread of the belief under sales ranking information). Therefore the expected sales under sales volume information is lower than n_2 and in this case sales ranking information leads to higher expected second-period sales compared to sales volume information. □

SE. Endogenous Consumer Arrivals with Platform-Visiting Cost

In the base model we assume that the total number of consumers arriving to the platform, n, is exogenous and independent of bestseller information provision. This corresponds to many practical settings where consumers' cost of visiting an e-commerce platform is negligible. If, however, such a cost is substantial, consumers' platform-visiting decision becomes non-trivial and needs to take account of the bestseller information upon arrival to the platform. We now extend the model to incorporate endogenous consumer arrivals with a positive platform-visiting cost. All the other assumptions remain the same as in the base model.

Specifically, assume that there are a total of N consumers who are interested in purchasing either product and decide whether to visit a particular platform on which both products are offered for sales. Each consumer incurs a fixed cost K for visiting the platform. Thus, a consumer chooses to visit the platform if and only if the expected utility from a platform visit is no less than the cost K. Assume $K \leq p_h u_h + p_l u_l$, i.e., the visiting cost does not exceed the expected product value evaluated at the prior belief. The platform releases bestseller information (if any) after N_0 consumers make their platform-visiting decision. Same as in the base model, we refer to the time phase before the information provision as the first period and that after as the second period. Assume $N_0 F(p_h u_h + p_l u_l - K) \geq 1$ to ensure that some consumers make purchase in the first period and public learning takes place in the second period.

In the first period, a consumer visits the platform if and only if

$$p_h u_h + p_l \max(u_l, p_h u_h + p_l u_l - s) - s \geq K$$

Consider the following two cases:

- $K \leq u_l$: in this case, $p_h u_h + p_l u_l - K \geq p_h(u_h - u_l)$. Thus, consumers with search cost $s \leq p_h u_h + p_l u_l - K$ visit the platform and perform a first search. Among them, those with search cost $s \leq p_h(u_h - u_l)$ are willing to perform a second search if the first search reveals a low type. In this case, the total sales of the two products in the first period, n_1, equals $\lfloor N_0 F(p_h u_h + p_l u_l - K) \rfloor$.
- $u_l < K \leq p_h u_h + p_l u_l$: in this case, $p_h u_h + p_l u_l - K < p_h(u_h - u_l)$. Solving $p_h u_h + p_l(p_h u_h + p_l u_l - s) - s = K$, we have $s = \hat{s} := \frac{-K + p_h u_h + p_l(p_h u_h + p_l u_l)}{p_l + 1}$. It can be shown that $\hat{s} \in (p_h u_h + p_l u_l - K, p_h(u_h - u_l))$. Thus, consumers

with search cost $s \leq \hat{s}$ visit the platform and are willing to perform a second search; and those with search cost $s > \hat{s}$ does not visit the platform. In other words, all of the consumers who choose to visit the platform are willing to conduct a second search. In this case, if the product types differ, the sales of the low type is zero. Hence, a low type can never outsell a high type in the first period. Thus, learning from the bestseller information is trivial in the sense that (i) no second-period consumer will perform a second search because, if the first search of the higher-ranked product reveals a low type, it can be inferred that the lower-ranked product is also a low type; (ii) sales volume sometimes perfectly reveals whether the two product types differ from each other: specifically, if both products have positive sales, then it implies that the product types are the same. This is because no one would purchase the low type if the product types differ. In this case, the total sales of the two products in the first period, n_1, equals $\lfloor N_0 F(\hat{s}) \rfloor$. As K increases, $\hat{s}$ decreases: an increase in the visiting cost reduces the first-period arrivals and purchases, but all the purchases are informed. Note that $N_0 F(\hat{s}) > N_0 F(p_h u_h + p_l u_l - K) \geq 1$ and thus the total first-period sales is positive.

In the second period, a consumer visits the platform if and only if

$$\pi_1^t u_h + (1 - \pi_1^t) \max(u_l, \pi_2^t u_h + (1 - \pi_2^t) u_l - s) - s \geq K$$

where $\pi_1^t \geq \pi_2^t$. Consider the following two cases:

- $K \leq u_l$: in this case, $\pi_1^t u_h + (1 - \pi_1^t) u_l - K \geq \pi_2^t (u_h - u_l)$. Thus, among the remaining $N - N_0$ consumers who are interested in the products, consumers with search cost $s \leq \pi_1^t u_h + (1 - \pi_1^t) u_l - K$ visit the platform and perform a first search. Among them, those with search cost $s \leq \pi_2^t (u_h - u_l)$ are willing to perform a second search if the first search reveals a low type. In this case, the expected total sales of the two products in the second period, S_t, equals to $\mathbb{E}[(N - N_0) F(\pi_1^t u_h + (1 - \pi_1^t) u_l - K)]$. The extended model thus degenerates to the base model with parameters (u_h', u_l', n_0, n_2), where $u_h' = u_h - K$, $u_l' = u_l - K$, $n_0 = N_0$, and $n_2 = N - N_0$. Essentially, even if we endogenize the visiting decision, those who choose to visit must have decided to perform the first search upon arrival. Thus, the visiting cost does not qualitatively alter the search or purchasing decision, except that it shrinks the expected utility of the platform visit.
- $u_l < K \leq p_h u_h + p_l u_l$: in this case, following the first-period analysis, we have $\pi_2^t = 0$ for $t \in \{r, v\}$. Thus, among the remaining $N - N_0$ consumers who are interested in the products, a consumer visits the platform if and only if $s \leq \pi_1^t u_h + (1 - \pi_1^t) u_l - K$ Upon visit, the consumer searches and purchases the product with higher sales. In this case, the expected total sales of the two products in the second period, S_t, equals to $\mathbb{E}[(N - N_0) F(\pi_1^t u_h + (1 - \pi_1^t) u_l - K)]$ for $t \in \{r, v\}$. For the case of no information, $S_\phi = \mathbb{E}[(N - N_0) F(\hat{s})]$.

PROPOSITION S.16. *(i) $S_r \geq S_\phi$; (ii) π_1^r increases in K.*

Proposition S.16 is illustrated in Figure S.1. The proposition and figure highlight three important takeaways of this extension:

- First, endogenizing the number of arrivals by incorporating the visiting decision and cost does not qualitatively change the main results in the base model. In particular, consumers visit the platform if and only if they are willing to perform a first search. Thus, expected sales equals to expected arrivals.
- Second, increasing visiting cost may enhance the informativeness of the bestseller information as it reduces the volume of uninformed purchase and essentially increases the proportion of informed purchases. This effect

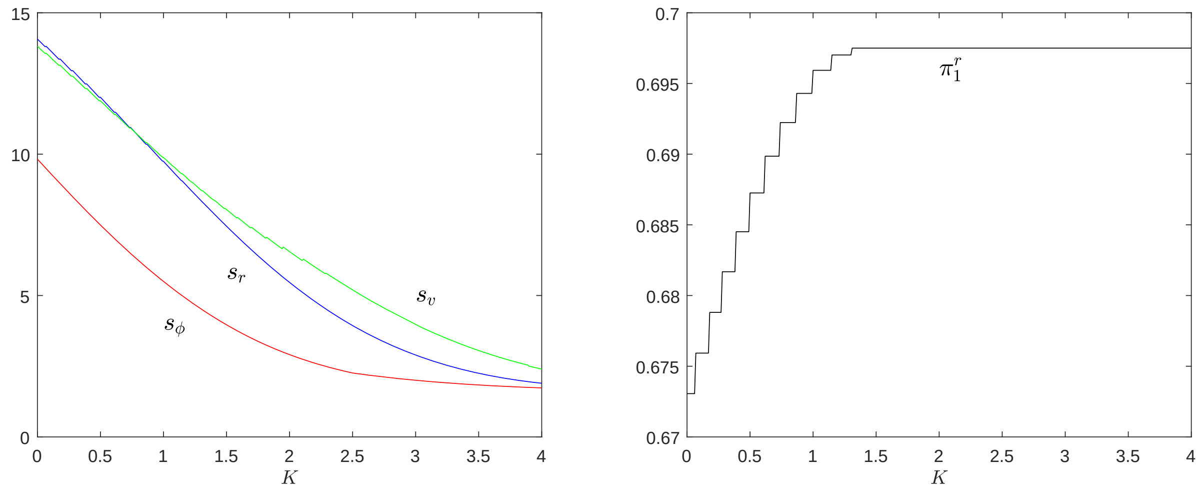

Figure S.1 Expected second-period sales (left) and π_1^r as functions of visiting cost K: $F(x) = (\alpha + (1-\alpha)\Phi((x-\mu)/\sigma))\mathbb{I}(x \geq 0)$ with $\mu = 4.5$, $\sigma = 1.5$ and $\alpha = 0.08$, $N = 100$, $N_0 = 80$, $u_h = 6.5$, $u_l = 2.5$, $p_h = 0.45$.

is evidenced by the fact that, under ranking information, consumers are more confident about the high-ranked product being of high type under a higher visiting cost, i.e., π_1^r increases in K.

• Third, the left panel of Figure S.1 suggests that increasing visiting cost may strengthen the platform's preference for volume information over ranking information, especially when the visiting cost is below u_l. The rationale is similar to that discussed in the base model: increasing K is similar to reducing u_h and u_l by the same amount, i.e., it results in a stochastically lower product value, which shifts the interval Ω to the left and, given the normal distribution of search cost, renders volume information more likely favored by the platform over ranking information.

SE.1. Appendix

Proof of Proposition S.16 (i) When $K \leq u_l$, by a similar proof as in the base model we have $\pi_1^r \geq p_h$, implying $S_r \geq S_\phi$; When $u_l < K \leq p_h u_h + p_l u_l$, by part (i) of Proposition S.22, $\pi_1^r = p_h^2 + 2p_h p_l$. In the meanwhile, recall that $\hat{s}$ satisfies $p_h u_h + p_l(p_h u_h + p_l u_l - \hat{s}) - \hat{s} = K$. These facts jointly imply $\pi_1^r u_h + (1-\pi_1^r)u_l - K \geq \hat{s}$, which further implies $S_r \geq S_\phi$.

(ii) Following a proof similar to that of Proposition S.22, π_1^r increases in K for $K \leq u_l$ (in this case m is independent of K and n_1 decreases in K) and is independent of K when $u_l < K \leq p_h u_h + p_l u_l$ (in this case $m = n_1 \geq 1$). □

SF. Independently-Distributed Search Costs

In the base model we assume that the proportion of early consumers whose search cost is less than x equals to $F(x)$. It can be considered as a fluid approximation of the setting with independently-distributed search costs when the size of consumer population is large (Yu et al. 2015). In this extension we consider an alternative setting in which each consumer's search cost is independently drawn from the search-cost distribution $F(\cdot)$, i.e., the probability that each early consumer's search cost is less than or equal to x is $F(x)$.

SF.1. Posterior Beliefs

Similar to the base model, we first derive consumers' beliefs in the second period under no information, ranking information, and volume information, respectively. Different from the base model, where the numbers of early consumers willing to search once or twice are deterministic functions of the number of early consumers n_0, these numbers become random variables when consumers' search costs are independently distributed. Specifically, let $\tilde{n}_1$ and $\tilde{m}$ denote the number of first-period consumers performing the first and second search, respectively. We have $\tilde{n}_1 \sim Binomial(n_0, F(p_h u_h + (1-p_h)u_l))$ and $\tilde{m} \sim Binomial(n_0, F(p_h(u_h - u_l)))$. Note that $\tilde{n}_1$ and $\tilde{m}$ are correlated. Specifically, the conditional distribution of $\tilde{m}$ given $\tilde{n}_1 = n_1$ is $Binomial(n_1, \frac{F(p_h(u_h-u_l))}{F(p_h u_h + p_l u_l)})$.

For $n_1 = 0, ..., n_0$ and $m = 0, ..., n_1$, let $P(n_1, m)$ denote the (joint) probability that in the first period n_1 consumers perform the first search and m consumers perform the second search. Specifically, $P(n_1, m) = \frac{n_0!}{m!(n_1-m)!(n_0-n_1)!}[F(p_h(u_h - u_l))]^m [F(p_h u_h + p_l u_l) - F(p_h(u_h - u_l))]^{n_1 - m}[1 - F(p_h u_h + p_l u_l)]^{n_0 - n_1}$. Below we derive the posterior beliefs under various types of sales information.

No Information

When there is no sales information, the posterior beliefs are the same as the prior, i.e., $\pi_1^\phi = \nu_1^\phi = \pi_2^\phi = p_h$, where, similar to in the base model, π_1^ϕ denotes a late consumer's belief that the higher-ranked product is of high value before she makes a first search, ν_1^ϕ denotes her belief that the lower-ranked product is of high value before she makes a first search, and π_2^ϕ denotes her belief that the lower-ranked product is of high value after her first search reveals a low value in the higher-ranked product.

Ranking Information

Under ranking information, let π_1^r be a late consumer's belief that the higher-ranked product is of high value before she makes a first search, and ν_1^r be her belief that the lower-ranked product is of high value before she makes a first search, and π_2^r be her belief that the lower-ranked product is of high value after her first search reveals a low value in the higher-ranked product.

For given (n_1, m) and for $x = 0, 1, ..., n_1$, define

$$g_s(x|n_1, m) := Binomial(x, n_1, 1/2),$$
$$g_a(x|n_1, m) := Binomial(x - m, n_1 - m, 1/2) \text{ if } x \geq m, \text{ and } 0 \text{ if } x < m,$$

where $Binomial(x, y, p)$ is the probability that among y independent trials, x of them succeed, where the probability of success is p. Let $G_s(x|n_1, m)$ and $G_a(x|n_1, m)$ be the cumulative distribution functions corresponding to $g_s(x|n_1, m)$ and $g_a(x|n_1, m)$, respectively. Let $\bar{G}_s(x|n_1, m) := 1 - G_s(x|n_1, m)$ and $\bar{G}_a(x|n_1, m) := 1 - G_a(x|n_1, m)$.

Given (n_1, m), define

$$\tilde{\pi}_1^r(n_1, m) = p_h^2 + 2p_h p_l(1 - G_a(\tfrac{n_1}{2}|n_1, m))$$
$$\tilde{\nu}_1^r(n_1, m) = p_h^2 + 2p_h p_l G_a(\tfrac{n_1}{2}|n_1, m)$$
$$\tilde{\pi}_2^r(n_1, m) = \frac{p_h G_a(\frac{n_1}{2}|n_1, m)}{p_h G_a(\frac{n_1}{2}|n_1, m) + p_l \bar{G}_s(\frac{n_1}{2}|n_1, m)}$$

when n_1 is odd. The definitions under even values of n_1 are similar to those in the base model and are omitted here. As the late consumers cannot observe n_1 or m, they take account of all the possible values of n_1 and m. Lemma S.10 follows.

Lemma S.10.

$$\pi_1^r = \sum_{n_1,m} P(n_1,m)\tilde{\pi}_1^r(n_1,m)$$
$$\nu_1^r = \sum_{n_1,m} P(n_1,m)\tilde{\nu}_1^r(n_1,m)$$
$$\pi_2^r = \sum_{n_1,m} P(n_1,m)\tilde{\pi}_2^r(n_1,m)$$

Volume Information

When sales volume information is publicized, the late-arriving consumers observe the sales volume of both products. As the total sales in the first period is $\tilde{n}_1$, it follows that the late consumers are aware of $\tilde{n}_1$ under volume information. Denote the realization of $\tilde{n}_1$ by n_1. On the other hand, the number of early consumers who perform the second search, $\tilde{m}$, remains uncertain. Recall that x, the sales of the higher sales ranking product, is also known to late-arriving consumers. Let $p(n_1,m,x)$ be the probability that $\tilde{n}_1 = n_1, \tilde{m} = m$ and the sales for the higher sales ranking product is x. Then for $x \geq n_1/2$,

$$\begin{aligned} p(n_1,m,x) &= \Pr(x|\tilde{n}_1 = n_1, \tilde{m} = m)\cdot \Pr(\tilde{n}_1 = n_1, \tilde{m} = m) \\ &= [(p_h^2 + p_l^2)g_s(x|n_1,m) + p_h p_l g_a(x|n_1,m) + p_h p_l g_a(n_1 - x|n_1,m)]\cdot P(n_1,m) \end{aligned}$$

Let $p(n_1,x) = \sum_{m\leq n_1} p(n_1,m,x)$. The knowledge of n_1 and x allows the late consumers to update their belief about $\tilde{m}$: for $m = 0,...,n_1$, $p(m|n_1,x) = \Pr[\tilde{m} = m|\tilde{n}_1 = n_1, x] = \frac{p(n_1,m,x)}{p(n_1,x)}$.

Under volume information, let $\pi_1^v(x,n_1)$ be a late consumer's belief that the higher-ranked product is of high value before she makes a first search, and $\nu_1^v(x,n_1)$ be her belief that the lower-ranked product is of high value before she makes a first search, and $\pi_2^v(x,n_1)$ be her belief that the lower-ranked product is of high value after her first search reveals a low value in the higher-ranked product.

For given (n_1,m), we define

$$\begin{aligned} \tilde{\pi}_1^v(x,n_1,m) &= \frac{p_h^2 g_s(x|n_1,m) + p_h p_l g_a(x|n_1,m)}{p_h^2 g_s(x|n_1,m) + p_h p_l g_a(x|n_1,m) + p_h p_l g_a(n_1-x|n_1,m) + p_l^2 g_s(x|n_1,m)} \\ \tilde{\nu}_1^v(x,n_1,m) &= \frac{p_h^2 g_s(x|n_1,m) + p_h p_l g_a(n_1-x|n_1,m)}{p_h^2 g_s(x|n_1,m) + p_h p_l g_a(x|n_1,m) + p_h p_l g_a(n_1-x|n_1,m) + p_l^2 g_s(x|n_1,m)} \\ \tilde{\pi}_2^v(x,n_1,m) &= \frac{p_h g_a(n_1-x|n_1,m)}{p_h g_a(n_1-x|n_1,m) + p_l g_s(x|n_1,m)} \end{aligned}$$

where x is the sales of the bestseller. Lemma S.11 follows.

Lemma S.11.

$$\pi_1^v(x,n_1) = \sum_{m\leq n_1} \frac{p(n_1,m,x)}{p(n_1,x)}\tilde{\pi}_1^v(x,n_1,m)$$
$$\nu_1^v(x,n_1) = \sum_{m\leq n_1} \frac{p(n_1,m,x)}{p(n_1,x)}\tilde{\nu}_1^v(x,n_1,m)$$
$$\pi_2^v(x,n_1) = \sum_{m\leq n_1} \frac{p(n_1,m,x)}{p(n_1,x)}\tilde{\pi}_2^v(x,n_1,m)$$

SF.2. Robustness of Results in the Base Model

In this section we prove that the main results in the base model remain valid in this extended model (with the statement of some results slightly modified, as highlighted in boldface below). For each result, we repeat its (modified) statement in this subsection and prove it under independently-distributed search costs in §SF.4.

LEMMA 1 *(Consumers' purchasing choices in the first period)*
(i) In the first period, consumers with search cost $s \le u_h p_h + u_l p_l$ purchase a product and consumers with higher search cost leave without buying. ***That is, the total sales of the two products in the first period is $\tilde{n}_1 \sim Binomial(n_0, F(p_h u_h + (1-p_h)u_l))$.***
(ii) If the two products' values are identical (i.e., either $u_1 = u_2 = u_h$ or $u_1 = u_2 = u_l$), a consumer who chooses to make a purchase buys either product with equal probabilities. If the two products' values are different (i.e., $u_i = u_h$ and $u_{3-i} = u_l$, $i \in \{1,2\}$), a consumer with search cost $s \le p_h\Delta$ purchases the product with high value, while a consumer with search cost $s \in (p_h\Delta, u_h p_h + u_l p_l]$ purchases the first product that she searches.

PROPOSITION 1 ***Given $n_1 \in \{0,...,n_0\}$ and $m \in \{0,...,n_1\}$***, *for $x \in [0, n_1]$,*

$$g_s(x|\boldsymbol{n_1}, \boldsymbol{m}) := Binomial(x, n_1, 1/2),$$
$$g_a(x|\boldsymbol{n_1}, \boldsymbol{m}) := Binomial(x-m, n_1-m, 1/2) \text{ if } x \ge m, \text{ and } 0 \text{ if } x < m,$$

where $Binomial(x,y,p)$ is the probability that among y independent trials, x of them succeed, where the probability of success is p. Let $G_s(x|\boldsymbol{n_1}, \boldsymbol{m})$ and $G_a(x|\boldsymbol{n_1}, \boldsymbol{m})$ be the cumulative distribution functions corresponding to $g_s(x|\boldsymbol{n_1}, \boldsymbol{m})$ and $g_a(x|\boldsymbol{n_1}, \boldsymbol{m})$, respectively. Let $\bar{G}_s(x|\boldsymbol{n_1}, \boldsymbol{m}) := 1 - G_s(x|\boldsymbol{n_1}, \boldsymbol{m})$ and $\bar{G}_a(x|\boldsymbol{n_1}, \boldsymbol{m}) := 1 - G_a(x|\boldsymbol{n_1}, \boldsymbol{m})$.
(i) If the two products' values are identical, the sales of either product follows distribution $G_s(x|\boldsymbol{n_1}, \boldsymbol{m})$, ***conditional on $\tilde{n}_1 = n_1$ and $\tilde{m} = m$****;*
(ii) If the two products' values are different, the sales of the high-value (resp. low-value) product follows distribution $G_a(x|\boldsymbol{n_1}, \boldsymbol{m})$ (resp. $\bar{G}_a(n_1 - x|\boldsymbol{n_1}, \boldsymbol{m})$), ***conditional on $\tilde{n}_1 = n_1$ and $\tilde{m} = m$****.*

PROPOSITION 2 *Under either ranking or volume information, if a consumer finds it worthwhile to search, then it is optimal for her to first search product i^*.*

LEMMA 2 $\pi_1^r \ge \nu_1^r \ge \pi_2^r$, *and* $\pi_1^v(x, \boldsymbol{n_1}) \ge \nu_1^v(x, \boldsymbol{n_1}) \ge \pi_2^v(x, \boldsymbol{n_1}), \forall x \ge n_1/2$.

LEMMA 3 *(Consumers' purchasing choices in the second period) For $t \in \{\phi, r, v\}$,*
(i) Consumers with search cost $s \le u_h \pi_1^t + u_l(1-\pi_1^t)$ purchase a product and consumers with higher search cost leave without buying. ***That is, the total sales of the two products in the second period follows $Binomial(n_2, F(\pi_1^t u_h + (1-\pi_1^t)u_l))$****;*
(ii) If a consumer's first search reveals a low type, she performs a second search if and only if her search cost is low (i.e., $s \le \pi_2^t\Delta$). Thus, if the two products' values are different, a consumer with search cost $s \le \pi_2^t\Delta$ purchases the product with high value, while a consumer with search cost $s \in (\pi_2^t\Delta, u_h\pi_1^t + u_l(1-\pi_1^t)]$ purchases the first product that she searches.

PROPOSITION 3 $\pi_1^r \ge \pi_1^\phi = p_h \ge \nu_1^r$, $\pi_2^r \le \pi_2^\phi = p_h$.

LEMMA 4 *For $i=1,2$,*
(i) $\Pr\left[u_i=u_h|u_1\neq u_2, X_i\geq \frac{n_1}{2}\right]\geq \Pr\left[u_i=u_h|u_1\neq u_2\right]$ *and* $\Pr\left[u_i=u_h|u_1=u_2, X_i\geq \frac{n_1}{2}\right]=\Pr\left[u_i=u_h|u_1=u_2\right]$*;*
(ii) $\Pr\left[u_1\neq u_2|X_i\geq \frac{n_1}{2}\right]=\Pr\left[u_1\neq u_2\right]$ *and* $\Pr\left[u_1=u_2|X_i\geq \frac{n_1}{2}\right]=\Pr\left[u_1=u_2\right]$.

PROPOSITION 4 *(i)* $\pi_2^v(x,\boldsymbol{n_1})\leq p_h$ *for all* $x\geq n_1/2$. *(ii) If* $p_h\geq \frac{1}{2}$, $\pi_1^v(x,\boldsymbol{n_1})\geq p_h$ *for any* $x\geq n_1/2$*; otherwise,* $\pi_1^v(x,\boldsymbol{n_1})\geq p_h$ *if and only if* x *is sufficiently high.*

LEMMA 5 $\pi_1^v(\frac{n_1}{2},\boldsymbol{n_1})>p_h$ *if* $p_h>\frac{1}{2}$, $\pi_1^v(\frac{n_1}{2},\boldsymbol{n_1})=p_h$ *if* $p_h=\frac{1}{2}$, *and* $\pi_1^v(\frac{n_1}{2},\boldsymbol{n_1})<p_h$ *if* $p_h<\frac{1}{2}$.

LEMMA 6
(i) Both $Pr[u_{i^*}=u_h|u_1\neq u_2, X_{i^*}=x]$ *and* $Pr[u_1\neq u_2|X_{i^*}=x]$ *increase in* x*, for* $x\geq \frac{n_1}{2}$*;*
(ii) $\pi_1^v(x,\boldsymbol{n_1})$ *increases in* x*, for* $x\geq \frac{n_1}{2}$.
(iii) $\pi_2^v(x,\boldsymbol{n_1})$ *decreases in* x*, for* $x\geq \frac{n_1}{2}$.

LEMMA 7 ***For given*** $\boldsymbol{n_1}$, $\pi_j^v(x,\boldsymbol{n_1})$ *is a mean-preserving spread of* π_j^r *for* $j=1,2$.

To evaluate the impact of sales information on the expected sales, we define the expected second-period sales as follows:

- The expected second-period sales under no information is

$$\mathbb{E}[S_\phi]=n_2F(p_hu_h+p_lu_l)$$

- The expected second-period sales under ranking information is

$$\mathbb{E}[S_r]=n_2F\left(\sum_{n_1,m}P(n_1,m)\tilde{\pi}_1^r(n_1,m)u_h+(1-\sum_{n_1,m}P(n_1,m)\tilde{\pi}_1^r(n_1,m))u_l\right)$$

- The expected second-period sales under volume information is

$$\begin{aligned}\mathbb{E}[S_v]&=\sum_{n_1}p(n_1)\sum_{x}p(x|n_1)n_2\left[F\left(\sum_{m}\frac{p(n_1,m,x)}{p(n_1,x)}\tilde{\pi}_1^v(x,n_1,m)u_h+(1-\sum_{m}\frac{p(n_1,m,x)}{p(n_1,x)}\tilde{\pi}_1^v(x,n_1,m))u_l\right)\right]\\&=\sum_{n_1}p(n_1)\sum_{x}p(x|n_1)n_2[F(\pi_1^v(x,n_1)u_h+(1-\pi_1^v(x,n_1))u_l)]\\&=\sum_{n_1,x}p(n_1,x)n_2[F(\pi_1^v(x,n_1)u_h+(1-\pi_1^v(x,n_1))u_l)]\end{aligned}$$

where $p(x|n_1)$ is the probability that bestseller product has sales x (where $x\geq n_1/2$) when there are in total n_1 sales in the first period.

PROPOSITION 6 *Compared to no information:*
(i) ranking information increases the expected second-period sales, i.e., $\mathbb{E}[\boldsymbol{S_r}]\geq\mathbb{E}[\boldsymbol{S_\phi}]$. ***In particular, there exist problem instances in which*** $\frac{\mathbb{E}[\boldsymbol{S_\phi}]}{\mathbb{E}[\boldsymbol{S_r}]}\leq\epsilon$ ***for any given*** $\epsilon\in(0,1)$***.***
(ii) volume information reduces the second-period sales if both $p_h<\frac{1}{2}$ *and the first-period sales difference is small, and increases the sales otherwise.* ***Furthermore, volume information may lead to a lower expected second-period sales, i.e., there exist problem instances in which*** $\mathbb{E}[\boldsymbol{S_v}]<\mathbb{E}[\boldsymbol{S_\phi}]$***.***

PROPOSITION 7 *Compared to ranking information:*
(i) volume information reduces the second-period sales when the first-period sales difference is small and increases the sales otherwise.
(ii) volume information may lead to a lower expected second-period sales. In particular, when the search-cost distribution $F(\cdot)$ *is convex,* $\mathbb{E}[\boldsymbol{S_r}]\leq\mathbb{E}[\boldsymbol{S_v}]$*; and when* $F(\cdot)$ *is concave,* $\mathbb{E}[\boldsymbol{S_r}]\geq\mathbb{E}[\boldsymbol{S_v}]$.

Proposition 8 *Consider a second-period consumer with search cost s.*

(i) Compared to the case where no sales information is provided, ranking information provision reduces the consumer's expected purchased value if $\pi_2^r(u_h-u_l)<s\le p_h(u_h-u_l)$;

(ii) Compared to the case where ranking information is provided, volume information provision reduces the consumer's expected purchased value if $\pi_2^v(n_1,\boldsymbol{n_1})(u_h-u_l)<s\le \pi_2^r(u_h-u_l)$.

Proposition 9 *Consider a second-period consumer with search cost s.*

(i) Compared to the case where no sales information is provided, ranking information provision increases the first-search probability and decreases the second-search probability;

(ii) Compared to the case where ranking information is provided, volume information provision decreases both search probabilities if $\pi_2^v(n_1,\boldsymbol{n_1})(u_h-u_l)<s\le\pi_2^r(u_h-u_l)$ and increase both probabilities if $\pi_1^r u_h+(1-\pi_1^r)u_l<s\le \pi_1^v(n_1,\boldsymbol{n_1})u_h+(1-\pi_1^v(n_1,\boldsymbol{n_1}))u_l$.

Proposition 10 *A consumer's expected surplus is higher under either ranking or volume information than that under no information. Furthermore, it is higher under volume information than that under ranking information.*

SF.3. Optimal Timing of Sales Information Provision

In this subsection we confirm that our (numerical) findings about the optimal timing of sales information provision remain valid under this extended model. Same as in the base model, we assume that the search cost density is bimodal: specifically, $F(x)=(\alpha+(1-\alpha)\Phi((x-\mu)/\sigma))\mathbb{I}(x\ge 0)$, where $\Phi(\cdot)$ is the cumulative distribution function for the standard normal distribution and $\mathbb{I}(\cdot)$ is the indicator function. Furthermore, we set $\alpha=0.08$ and $\mu=4.5$, same as in the base model.

Table S.7 presents the effect of increasing the number of first-period arrivals, n_0, on the expected total sales in the two periods. Same as in the base model, we observe that ranking information is increasingly likely to outperform volume information at a larger n_0 on under a stochastically higher product value.

Table S.8 illustrates the optimal value of n_0. Same as in the base model, it is observed that the larger the consumer population, the later the sales information should be released, i.e., n_0^* increases in n. In addition, the result that ranking information is more likely to be the optimal choice for the platform under either a larger consumer population or a stochastically higher product value is robust, as observed from the table.

Table S.7 Impact of n_0 on total expected sales in two periods: $\mu=4.5,\sigma=1.5,n=100,p_h=0.4,\alpha=0.08$

	$u_h=6,u_l=2$				$u_h=6.2,u_l=2.2$			
n_0	*Ranking*	*Volume*	*No Info.*	*Opt. Info.*	*Ranking*	*Volume*	*No Info.*	*Opt. Info.*
0	33.23	33.23	33.23	Tie	37.47	37.47	37.47	Tie
10	41.37	41.59	33.23	Volume	45.59	45.71	37.47	Volume
20	43.26	43.47	33.23	Volume	47.45	47.50	37.47	Volume
40	43.10	43.19	33.23	Volume	47.31	47.22	37.47	Ranking
60	40.67	40.67	33.23	Ranking	44.92	44.76	37.47	Ranking
80	37.21	37.18	33.23	Ranking	41.47	41.35	37.47	Ranking
100	33.23	33.23	33.23	Tie	37.47	37.47	37.47	Tie

Table S.8 Impact of n on total expected sales in two periods: $\mu=4.5, \sigma=1.5, p_h=0.4, \alpha=0.08$

	$u_h=6, u_l=2$						$u_h=6.2, u_l=2.2$					
n	*Ranking*		*Volume*		*Opt. Info.*		*Ranking*		*Volume*		*Opt. Info.*	
	n_0^*	*Sales*	n_0^*	*Sales*	n_0^*	*Type*	n_0^*	*Sales*	n_0^*	*Sales*	n_0^*	*Type*
20	7	7.62	7	7.65	7	Volume	7	8.47	7	8.49	7	Volume
50	16	20.47	16	20.56	16	Volume	16	22.57	15	22.61	15	Volume
100	28	43.62	28	43.78	28	Volume	28	47.80	27	47.80	28	Ranking
200	47	92.99	45	93.14	45	Volume	48	101.43	46	101.05	48	Ranking
300	61	144.33	59	144.31	61	Ranking	63	157.14	59	156.13	63	Ranking

SF.4. Appendix

Proof of Lemma S.10 Below we prove the equality for π_1^r. The proof of ν_1^r and π_2^r is similar and thus omitted. Denote the product with higher sales as i^* and the sales of product i as X_i.

$$\begin{aligned}\pi_1^r &= \Pr[u_1=u_h|X_1 \geq X_2] \\ &= \sum_{n_1,m} P(n_1,m)\Pr[u_1=u_h|X_1\geq X_2, \tilde{n}_1=n_1, \tilde{m}=m] \\ &= \sum_{n_1,m} P(n_1,m)\tilde{\pi}_1^r(n_1,m) \quad \square\end{aligned}$$

Proof of Lemma S.11 Below we prove the equality for $\pi_1^v(x,n_1)$. The proof of the other two equalities is similar and thus omitted. Let $p(m|n_1)$ be the probability that m consumers perform the second search in the first period given that n_1 consumers perform the first search in the first period. Then for $x \geq n_1/2$,

$$\begin{aligned}\pi_1^v(x,n_1) &= \Pr[u_1=u_h|X_1+X_{3-1}=n_1, X_1=x] \\ &= \sum_m \Pr[\tilde{m}=m|\tilde{n}_1=n_1, X_1=x]\cdot\Pr[u_1=u_h|X_1=x,\tilde{n}_1=n_1,\tilde{m}=m] \\ &= \sum_m \frac{\Pr[\tilde{m}=m,\tilde{n}_1=n_1,X_1=x]}{\Pr[\tilde{n}_1=n_1,X_1=x]}\cdot\Pr[u_1=u_h|X_1=x,\tilde{n}_1=n_1,\tilde{m}=m] \\ &= \sum_m \frac{p(n_1,m,x)}{p(n_1,x)}\tilde{\pi}_1^v(n_1,m,x)\end{aligned}$$

□

Proof of Lemma 1 The first part of (i) and (ii) can be proved by a similar proof to that of Lemma 1 in the base model. To see the second part of (i), it suffices to notice that each consumer has search cost smaller than $p_hu_h+(1-p_h)u_l$ independently with probability $F(p_hu_h+(1-p_h)u_l)$. □

Proof of Proposition 1 Since the proposition is based on given n_1 and m, the proof is the same as the proof for Proposition 1 in the base model. □

Proof of Proposition 2 This Lemma holds as the proof for Proposition 2 in the base model applies for all value of n_1 and m with $m \leq n_1$. □

Proof of Lemma 2 For the first part, it suffices to prove that $\tilde{\pi}_1^r(n_1,m)\geq\tilde{\nu}_1^r(n_1,m)\geq\tilde{\pi}_2^r(n_1,m)$ for all n_1,m. This is true by our proof for Lemma 3 in the base model as it applies to all value of n_1 and m with $m\leq n_1$. For the second part, it suffices to prove $\tilde{\pi}_1^v(x,n_1,m)\geq\tilde{\nu}_1^v(x,n_1,m)\geq\tilde{\pi}_2^v(x,n_1,m), \forall x\geq n_1/2$, which is true again by Lemma 3 in the main paper. □

Proof of Lemma 3 The first part of (i) and (ii) follows similar proof of Lemma 3 in the base model. The second part of (i) follows from the fact that each consumer's search cost is independently drawn. □

Proof of Proposition 3 This proposition holds as $\tilde{\pi}_1^r(n_1,m) \geq \pi_1^{\phi} = p_h \geq \tilde{\nu}_1^r(n_1,m)$, $\tilde{\pi}_2^r(n_1,m) \leq \pi_2^{\phi} = p_h$ for all n_1, m, for which the proof follows that of Proposition 3 in the base model. □

Proof of Lemma 4 This Lemma continues to hold as Lemma 4 in the base model holds for any given pairs of n_1, m with $m \leq n_1$ and the overall probability is simply an average value of corresponding probability with given value of n_1 and m. For instance, $\Pr[u_1 = u_h | u_1 \neq u_2, X_1 \geq \frac{n_1}{2}] = \sum_{n_1,m} P(n_1,m)\Pr[u_1 = u_h | u_1 \neq u_2, X_1 \geq \frac{n_1}{2}, \tilde{n}_1 = n_1, \tilde{m} = m]$ where $\Pr[u_1 = u_h | u_1 \neq u_2, X_1 \geq \frac{n_1}{2}, \tilde{n}_1 = n_1, \tilde{m} = m]$ is the probability given that n_1 consumers perform the first search and m consumers perform the second search in the first period. □

Proof of Proposition 4 For part (i), as $\tilde{\pi}_2^v(x,n_1,m) \leq p_h$ for all n_1, m, it follows that $\pi_2^v(x) \leq p_h$. For part (ii), it suffices to consider Lemma 5, which is examined below. Hence Proposition 4 holds. □

Proof of Lemma 5 Notice that under volume information n_1 is known and we have $\tilde{\pi}_1^v(\frac{n_1}{2}, n_1, m) > p_h$ for $p_h > \frac{1}{2}$, $\tilde{\pi}_1^v(\frac{n_1}{2}, n_1, m) = p_h$ for $p_h = \frac{1}{2}$, and $\tilde{\pi}_1^v(\frac{n_1}{2}, n_1, m) < p_h$ for $p_h < \frac{1}{2}$ given all $m \geq 1$ (by Lemma 5 in the base model). If $m = 0$, then all beliefs reduces to p_h. By the fact that $\pi_1^v(\frac{n_1}{2}, n_1)$ is an average of $\tilde{\pi}_1^v(\frac{n_1}{2}, n_1, m)$, this Lemma still holds. □

Proof of Lemma 6 Again it suffices to consider $\tilde{\pi}_1^v(x,n_1,m)$. (ii) and (iii) follows from the facts that $\tilde{\pi}_1^v(x,n_1,m)$ increases in x and $\tilde{\pi}_2^v(x,n_1,m)$ decreases in x for all m by Lemma 6 in the base model. Part (i) follows similar reasoning. □

Proof of Lemma 7 We have

$$\pi_1^v(x,n_1) = \sum_m \frac{p(n_1,m,x)}{p(n_1,x)} \tilde{\pi}_1^v(x,n_1,m)$$

and

$$\pi_1^r = \sum_{n_1,m} P(n_1,m)\tilde{\pi}_1^r(n_1,m)$$

Then

$$\begin{aligned}
\mathbb{E}_{[x,n_1]}[\pi_1^v(x,n_1)] &= \mathbb{E}_{x,n_1} \sum_m \frac{p(n_1,m,x)}{p(n_1,x)} \tilde{\pi}_1^v(x,n_1,m) \\
&= \sum_{n_1,x} p(n_1,x) \sum_m \frac{p(n_1,m,x)}{p(n_1,x)} \tilde{\pi}_1^v(x,n_1,m) \\
&= \sum_{n_1,m,x} p(n_1,m,x)\tilde{\pi}_1^v(x,n_1,m) \\
&= \sum_{n_1,m} P(n_1,m) \sum_x p(x|n_1,m)\tilde{\pi}_1^v(x,n_1,m) \\
&= \sum_{n_1,m} P(n_1,m)\tilde{\pi}_1^r(n_1,m)
\end{aligned}$$

where $p(x|n_1,m) = \frac{p(n_1,m,x)}{P(n_1,m)}$ is the probability that the higher sales product have sales x given that there are in total n_1 consumers perform the first search and m consumers is willing to perform the second search when the first search reveals a low type product in the first period. The last equality follows from the proof for mean preserving spread in the main model.

The proof for π_2^r is similar and the proof is complete. □

Proof of Proposition 6 The first part of (i) follows directly from previous result on π_1^r and p_h.

For the second part of (i), we construct the following example. Let $p_h = 0.6, u_h = 6, u_l = 2$. Thus, $p_h u_h + p_l u_l = 4.4$ and $p_h(u_h - u_l) = 2.4$. Let $\epsilon < 1$ be given. Consider a distribution function F such that $F(2.4) = \epsilon/6$,

$F(4.4)=\epsilon/2$, and $F(\tau_r)=3/4$, where τ_r is defined later. Let $n_0=\lceil 6/\epsilon\rceil$, the probability that $m=0$ is clearly less than 1/2 for any $\epsilon<1$. Let $\bar{G}=G_a(n_1/2|\lceil 6/\epsilon\rceil,1)$, so $\bar{G}$ is the lowest value of $G_a(n_1/2)$ for all realizations of n_1,m when $m\neq 0$. Let $t_r=\tilde{\pi}_1^r(\lceil 6/\epsilon\rceil,1)$, so t_r is the lowest belief that the bestseller product is of high value when $m\neq 0$. We have

$$\pi_1^r=\sum_{n_1,m}P(n_1,m)\tilde{\pi}_1^r(n_1,m)\geq t_r/2+p_h/2>p_h$$

Let $\tau_r=(t_r/2+p_h/2)u_h+(1-t_r/2-p_h/2)u_l$. Notice that $\tau_r>4.4$ as $t_r/2+p_h/2>p_h$. It follows that

$$\frac{\mathbb{E}[S_\phi]}{\mathbb{E}[S_r]}=\frac{n_2F(p_hu_h+p_lu_l)}{n_2F(\pi_1^ru_h+(1-\pi_1^r)u_l)}\geq\frac{F(p_hu_h+p_lu_l)}{F(\tau_r)}=\frac{2\epsilon}{3}<\epsilon$$

The first part of (ii) can be proved similarly as its counterpart in the base model. To see that $\mathbb{E}[S_v]$ can be strictly less than $\mathbb{E}[S_\phi]$, consider the same example constructed in the base model. In that example, volume information can never lead to higher sales than no information, regardless of the realized value of n_1,m,x. Moreover, there exists a realization of n_1,m,x such that the sales under volume information is lower than the sales under no sales information. As this realization occurs with a positive probability, it follows that $\mathbb{E}[S_v]<\mathbb{E}[S_\phi]$ in that example. □

Proof of Proposition 7 Part (i) follows similar proof for Proposition 7 in the base model. For the first part of part (ii),

$$\begin{aligned}\mathbb{E}[S_r]&=n_2F(\sum_{n_1,m}P(n_1,m)\tilde{\pi}_1^r(n_1,m)u_h+(1-\sum_{n_1,m}P(n_1,m)\tilde{\pi}_1^r(n_1,m))u_l)\\&=n_2F(u_l+\sum_{n_1,m}P(n_1,m)\tilde{\pi}_1^r(n_1,m)(u_h-u_l))\\&=n_2F(u_l+\sum_{n_1,m}P(n_1,m)\sum_x p(x|n_1,m)\tilde{\pi}_1^v(x,n_1,m)(u_h-u_l))\\&=n_2F(u_l+\sum_{n_1,m,x}p(n_1,m,x)\tilde{\pi}_1^v(x,n_1,m)(u_h-u_l))\end{aligned}$$

and

$$\begin{aligned}\mathbb{E}[S_v]&=n_2\mathbb{E}_{[n_1,x]}[F(\sum_m\frac{p(n_1,m,x)}{p(n_1,x)}\tilde{\pi}_1^v(x,n_1,m)u_h+(1-\sum_m\frac{p(n_1,m,x)}{p(n_1,x)}\tilde{\pi}_1^v(x,n_1,m))u_l)]\\&=n_2\mathbb{E}_{[n_1,x]}[F(u_l+\sum_m\frac{p(n_1,m,x)}{p(n_1,x)}\tilde{\pi}_1^v(x,n_1,m)(u_h-u_l))]\\&=n_2\sum_{n_1,x}p(n_1,x)[F(u_l+\sum_m\frac{p(n_1,m,x)}{p(n_1,x)}\tilde{\pi}_1^v(x,n_1,m)(u_h-u_l))]\end{aligned}$$

The result follows by Jensen's inequality.

□

Proof of Propositions 8 and 9 The proof is similar to those of Propositions 8 and 9 in the base model. □

Proof of Proposition 10 The proof is similar to that of Proposition 10 in the base model, by substituting $\pi_1^v(x)$ (resp. $\pi_2^v(x)$) with $\pi_1^v(x,n_1)$ (resp. $\pi_2^v(x,n_1)$). □

SG. Platform's Alternative Objectives

In the base model we focus on the platform's objective of maximizing the total expected product sales. This corresponds to the prevailing practice whereby platforms collect revenue in the form of commissions, which are usually proportional to product sales. Such an objective of the platform is also commonly considered in prior studies on platform operations (e.g., Küçükgül et al. 2022). In this section we examine two alternative objectives of the platform.

SG.1. Welfare-Maximizing Platform

Some prior studies (e.g., Derakhshan et al. 2022) consider platforms' consumer-centric objective of maximizing consumer welfare. In our setting, as implied by Proposition 10, a welfare-maximizing platform always provides bestseller information and prefers volume information over ranking information.

SG.2. Platform's Preference For Selling Higher-Value Product

In light of certain long-term interests, some platforms may prefer selling high-value products over low-value ones. We now consider this alternative objective of the platform and examine the situation where the platform enjoys a higher benefit from selling a higher-value product and aims to maximize its total expected benefit.

Specifically, let ξ_h and ξ_l denote the platform's benefit of selling one unit of high-value and low-value product, respectively, where $\xi_h > \xi_l$. To evaluate the impact of bestseller information on the platform's total expected benefit, we focus on the second period since the bestseller information is only available in the second period and thus is irrelevant to the platform's benefit in the first period.

Impact of Ranking Information on Platform's Expected Benefit

Given the posterior beliefs π_1^t and π_2^t, $t \in \{\phi, r\}$, the platform's expected second-period benefit is:

$$\begin{aligned}\Xi_2^t &= \{F(\pi_2^t(u_h - u_l))(\pi_1^t + (1-\pi_1^t)\pi_2^t) + (F(u_h\pi_1^t + u_l(1-\pi_1^t)) - F(\pi_2^t(u_h - u_l)))\pi_1^t\}\xi_h \\ &\quad + \{F(\pi_2^t(u_h - u_l))(1-\pi_1^t)(1-\pi_2^t) + (F(u_h\pi_1^t + u_l(1-\pi_1^t)) - F(\pi_2^t(u_h - u_l)))(1-\pi_1^t)\}\xi_l \\ &= F(\pi_2^t(u_h - u_l))((\pi_1^t + (1-\pi_1^t)\pi_2^t)\xi_h + (1-\pi_1^t)(1-\pi_2^t)\xi_l) + (F(u_h\pi_1^t + u_l(1-\pi_1^t)) - F(\pi_2^t(u_h - u_l)))(\pi_1^t\xi_h + (1-\pi_1^t)\xi_l)\end{aligned}$$

In general, the platform benefits more from a purchase made by a consumer who has low search cost and is willing to search both products (i.e., $s \le \pi_2^t(p_h - p_l)$), than from a purchase made by a consumer with medium search costs and willing to search only once (i.e., $\pi_2^t(p_h - p_l) < s \le u_h\pi_1^t + u_l(1-\pi_1^t)$). This is because consumers in the former segment always purchase a high type, if any, while those in the latter segment may purchase a low type even if there is a high type.

Due to ranking information, the former segment of consumers shrinks (i.e., $\pi_2^r \le p_h$), which tends to lower the platform's expected benefit. In the meanwhile, the latter segment expands (i.e., $u_h\pi_1^r + u_l(1-\pi_1^r) - \pi_2^r(u_h - u_l) \ge u_h p_h + u_l(1-p_h) - p_h(u_h - u_l)$) and so does the aggregate of the two segments (i.e., $\pi_1^r \ge p_h$). Furthermore, the chance of the latter segment purchasing a high type also increases due to ranking information (i.e., $\pi_1^r \ge p_h$), while that of the former segment remains unchanged (because $\pi_1^t + (1-\pi_1^t)\pi_2^t = p_h^2 + 2p_h p_l$ for $t \in \{\phi, r\}$). These effects lead to an increase of the expected sales and tend to improve the platform's expected benefit. With the two opposing forces, the overall effect of the ranking information on the platform's expected benefit is ambiguous. Nevertheless, we note from the following (S.9) that, when the gap between the two marginal benefits (i.e., $\xi_h - \xi_l$) is sufficiently small, the sales-enhancing effect dominates, and the platform's expected benefit increases due to sales ranking information.

$$\lim_{\xi_h \to \xi_l} \Xi_2^t = F(u_h\pi_1^t + u_l(1-\pi_1^t))\xi_l \tag{S.9}$$

Furthermore, if the search cost follows a uniform distribution with an upper bound greater than u_h, we prove that ranking information enhances the platform's expected benefit under any given ξ_h and ξ_l.

Proposition S.17. *Assume that the search cost follows a standard uniform distribution, i.e., $F(x)=x$ for $x\in[0,1]$ and that $u_h<1$ (as stated in the model section, we assume that the upper bound of the search cost, $\bar{s}$, is greater than u_h). Ranking information increases the platform's expected benefit in the second period.*

Impact of Volume Information on Platform's Expected Benefit

As for the volume information, given the posterior beliefs, $\pi_1^v(\cdot)$ and $\pi_2^v(\cdot)$, the platform's expected second-period benefit is

$$\begin{aligned}\Xi_2^v=&\ \mathbb{E}[F(\pi_2^v(x)(u_h-u_l))((\pi_1^v(x)+(1-\pi_1^v(x))\pi_2^v(x))\xi_h+(1-\pi_1^v(x))(1-\pi_2^v(x))\xi_l)\\&+(F(u_h\pi_1^v(x)+u_l(1-\pi_1^v(x)))-F(\pi_2^v(x)(u_h-u_l)))(\pi_1^v(x)\xi_h+(1-\pi_1^v(x))\xi_l)]\\=&\ \mathbb{E}[F(\pi_2^v(x)(u_h-u_l))(1-\pi_1^v(x))\pi_2^v(x)(\xi_h-\xi_l)+F(u_h\pi_1^v(x)+u_l(1-\pi_1^v(x)))(\pi_1^v(x)(\xi_h-\xi_l)+\xi_l)]\end{aligned}$$

Thus,

$$\frac{d\Xi_2^v}{d\xi_h}=\mathbb{E}[F(\pi_2^v(x)(u_h-u_l))(1-\pi_1^v(x))\pi_2^v(x)+F(u_h\pi_1^v(x)+u_l(1-\pi_1^v(x)))\pi_1^v(x)]$$

Recall:

$$\begin{aligned}\Xi_2^\phi&=F(p_h(u_h-u_l))p_hp_l(\xi_h-\xi_l)+F(u_hp_h+u_lp_l)(p_h\xi_h+(1-p_h)\xi_l)\\\frac{d\Xi_2^\phi}{d\xi_h}&=F(p_h(u_h-u_l))p_hp_l+F(u_hp_h+u_lp_l)p_h\end{aligned}$$

When $\frac{d\Xi_2^v}{d\xi_h}<\frac{d\Xi_2^\phi}{d\xi_h}$ (as exemplified with an instance[20]), $\Xi_2^v-\Xi_2^\phi$ decreases in ξ_h for given ξ_l. Thus, volume information can be more likely to be detrimental to the platform when the platform prefers selling higher-value product (compared to the case when it does not hold such a preference). This finding substantiates the result that volume information can be detrimental to the platform.

SG.3. Appendix

Proof of Proposition S.17 Notice that we have

$$\Xi_2^\phi=(F(p_h(u_h-u_l))p_hp_l+F(u_hp_h+u_l(1-p_h))(p_h^2+p_hp_l))(\xi_h-\xi_l)+F(u_hp_h+u_l(1-p_h))\xi_l \tag{S.10}$$

and

$$\begin{aligned}\Xi_2^r=&\{F(\pi_2^r(u_h-u_l))(\pi_1^r+(1-\pi_1^r)\pi_2^r)+(F(u_h\pi_1^r+u_l(1-\pi_1^r))-F(\pi_2^r(u_h-u_l)))\pi_1^r\}\xi_h\\&+\{F(\pi_2^r(u_h-u_l))(1-\pi_1^r)(1-\pi_2^r)+(F(u_h\pi_1^r+u_l(1-\pi_1^r))-F(\pi_2^r(u_h-u_l)))(1-\pi_1^r)\}\xi_l\\=&\{F(\pi_2^r(u_h-u_l))(p_h^2+2p_hp_l)+(F(u_h\pi_1^r+u_l(1-\pi_1^r))-F(\pi_2^r(u_h-u_l)))\pi_1^r\}\xi_h\\&+\{F(\pi_2^r(u_h-u_l))p_l^2+(F(u_h\pi_1^r+u_l(1-\pi_1^r))-F(\pi_2^r(u_h-u_l)))(1-\pi_1^r)\}\xi_l\\\geq&\{F(\pi_2^r(u_h-u_l))(p_h^2+2p_hp_l)+(F(u_h\pi_1^r+u_l(1-\pi_1^r))-F(\pi_2^r(u_h-u_l)))p_h\}\xi_h\\&+\{F(\pi_2^r(u_h-u_l))p_l^2+(F(u_h\pi_1^r+u_l(1-\pi_1^r))-F(\pi_2^r(u_h-u_l)))p_l\}\xi_l\\=&F(\pi_2^r(u_h-u_l))((p_h^2+2p_hp_l)\xi_h+p_l^2\xi_l)+(F(u_h\pi_1^r+u_l(1-\pi_1^r))-F(\pi_2^r(u_h-u_l)))(p_h\xi_h+p_l\xi_l)\\=&(F(\pi_2^r(u_h-u_l))p_hp_l+F(u_h\pi_1^r+u_l(1-\pi_1^r))(p_h^2+p_hp_l))(\xi_h-\xi_l)+F(u_h\pi_1^r+u_l(1-\pi_1^r))\xi_l\end{aligned}$$

[20] We exemplify that $\mathbb{E}[F(\pi_2^v(x)(u_h-u_l))(1-\pi_1^v(x))\pi_2^v(x)+F(u_h\pi_1^v(x)+u_l(1-\pi_1^v(x)))\pi_1^v(x)]$ can be strictly less than $F(p_h(u_h-u_l))p_hp_l+F(u_hp_h+u_lp_l)p_h$ by constructing a specific instance. Consider an example with $u_h=2,u_l=1$, then $u_h\pi_1^t+u_l(1-\pi_1^t)=\pi_1^t+1$ and $\pi_1^t\Delta=\pi_1^t$. Let $n_0=20$, $p_h=0.4,p_l=0.6$ and consider a F with probability mass $f(0.4)=0.1,f(1.4)=0.4,f(2)=0.5$. It follows that $n_1=10$ and $m=2$, Straightforward calculation shows that $\mathbb{E}[F(\pi_2^v(x)(u_h-u_l))(1-\pi_1^v(x))\pi_2^v(x)+F(u_h\pi_1^v(x)+u_l(1-\pi_1^v(x)))\pi_1^v(x)]=0.2223$ and $F(p_h(u_h-u_l))p_hp_l+F(u_hp_h+u_lp_l)p_h=0.2240$.

The first equality follows from $\pi_1^r+(1-\pi_1^r)\pi_2^r=p_h^2+2p_hp_l$. The first inequality follows from $p_h\le\pi_1^r$ and $\xi_l<\xi_h$.

Therefore, it suffices to show

$$\begin{aligned}&(F(\pi_2^r(u_h-u_l))p_hp_l+F(u_h\pi_1^r+u_l(1-\pi_1^r))(p_h^2+p_hp_l))(\xi_h-\xi_l)+F(u_h\pi_1^r+u_l(1-\pi_1^r))\xi_l\\ \ge&(F(p_h(u_h-u_l))p_hp_l+F(u_hp_h+u_l(1-p_h))(p_h^2+p_hp_l))(\xi_h-\xi_l)+F(u_hp_h+u_l(1-p_h))\xi_l\end{aligned}$$

Since $\pi_1^r\ge p_h$, it suffices to show $\Delta_b\ge 0$, where

$$\Delta_b:=F(\pi_2^r(u_h-u_l))p_hp_l+F(u_h\pi_1^r+u_l(1-\pi_1^r))(p_h^2+p_hp_l)-F(p_h(u_h-u_l))p_hp_l-F(u_hp_h+u_l(1-p_h))(p_h^2+p_hp_l)$$

Since $0\le\pi_2^r(u_h-u_l)\le p_h(u_h-u_l)\le u_hp_h+u_l(1-p_h)\le u_h\pi_1^r+u_l(1-\pi_1^r)\le u_h<1$, we note that, given $F(x)=x$ for $x\in[0,1]$,

$$\Delta_b=\pi_2^r(u_h-u_l)p_hp_l+(u_h\pi_1^r+u_l(1-\pi_1^r))(p_h^2+p_hp_l)-p_h(u_h-u_l)p_hp_l-(u_hp_h+u_l(1-p_h))(p_h^2+p_hp_l)$$

Recall that if n_1 is odd, $\pi_1^r=p_h^2+2p_hp_l(1-G_a(\frac{n_1}{2}))$ and $\pi_2^r=\frac{p_h}{p_h+p_l/(2G_a(\frac{n_1}{2}))}$, where $0\le G_a(\frac{n_1}{2})\le\frac12$; and if n_1 is even, $\pi_1^r=p_h^2+2p_hp_l(1-G_a(\frac{n_1}{2})+g_a(\frac{n_1}{2})/2)$ and $\pi_2^r=\frac{p_h}{p_h+p_l/(2(G_a(\frac{n_1}{2})-g_a(\frac{n_1}{2})/2))}$, where $0\le G_a(\frac{n_1}{2})-g_a(\frac{n_1}{2})/2\le\frac12$. Next we consider two cases to show $\Delta_b\ge0$:

- n_1 is odd:

$$\begin{aligned}\Delta_b=&((p_h^2+2p_h(1-p_h)(1-G_a(\tfrac{n_1}{2})))(u_h-u_l)+u_l)(p_h^2+p_h(1-p_h))\\&+\frac{p_hG_a(\frac{n_1}{2})}{p_hG_a(\frac{n_1}{2})+\frac12(1-p_h)}(u_h-u_l)p_h(1-p_h)\\&-p_h(u_h-u_l)p_h(1-p_h)-(p_h(u_h-u_l)+u_l)(p_h^2+p_h(1-p_h))\\=&2G_a(\frac{n_1}{2})p_h^3(u_h-u_l)(1-p_h)\frac{1-2G_a(\frac{n_1}{2})}{1-(1-2G_a(\frac{n_1}{2}))p_h}\ge0\end{aligned}$$

where the inequality is due to $0\le G_a(\frac{n_1}{2})\le\frac12$.

- n_1 is even:

$$\begin{aligned}\Delta_b=&((p_h^2+2p_h(1-p_h)(1-G_a(\tfrac{n_1}{2})+g_a(\tfrac{n_1}{2})/2))(u_h-u_l)+u_l)(p_h^2+p_h(1-p_h))\\&+\frac{p_h(2G_a(\frac{n_1}{2})-g_a(\frac{n_1}{2}))}{p_h(2G_a(\frac{n_1}{2})-g_a(\frac{n_1}{2}))+(1-p_h)}(u_h-u_l)p_h(1-p_h)\\&-p_h(u_h-u_l)p_h(1-p_h)-(p_h(u_h-u_l)+u_l)(p_h^2+p_h(1-p_h))\\=&-p_h^3(u_h-u_l)\frac{1-p_h}{1-p_h(1-2G_a(\frac{n_1}{2})+g_a(\frac{n_1}{2}))}\left(2G_a\left(\frac{n_1}{2}\right)-g_a\left(\frac{n_1}{2}\right)-1\right)\left(2G_a\left(\frac{n_1}{2}\right)-g_a\left(\frac{n_1}{2}\right)\right)\end{aligned}$$

where the inequality follows from the fact $0\le G_a(\frac{n_1}{2})-g_a(\frac{n_1}{2})/2\le\frac12$. □

SH. First Best

Thus far our analysis takes the perspective of self-interested consumers and platform. A natural benchmark is the first-best scenario (see, e.g., Glazer et al. 2021) in which a welfare-maximizing principal is aware of each arriving consumer's search cost, decides whether each consumer should search and, if so, which product to search first and whether to search the other product, and observes all the search results. This (hypothetical) scenario allows us to identify the highest possible welfare and to discuss inefficiencies due to self interests or other factors.

We formulate the principal's decision problem as a stochastic dynamic program. Specifically, let ξ_1 and ξ_2 denote the principal's current beliefs about the two products' values, i.e., ξ_1 (resp. ξ_2) is the probability of product 1 (resp. product 2) being of high value. Let s be the search cost of a newly-arriving consumer. The set (ξ_1, ξ_2, s) represents the states of the decision problem. Given (ξ_1, ξ_2, s), the principal decides whether the consumer with search cost s should search and, if so, which product to search first and whether to search the other product. The principal's objective is to maximize the expected total surplus of all of the n consumers. For $k = 1, ..., n$, let $\Phi_k(\xi_1, \xi_2, s)$ denote the principal's expected optimal surplus-to-go from the k-th consumer to the n-th consumer. We have:

$$\Phi_n(\xi_1,\xi_2,s) = \max(\underbrace{0}_{\text{search neither product}}, \underbrace{-s+\xi_1 u_h + (1-\xi_1)\max(u_l, \xi_2 u_h + (1-\xi_2)u_l - s)}_{\text{first search product 1}}, \underbrace{-s+\xi_2 u_h + (1-\xi_2)\max(u_l, \xi_1 u_h + (1-\xi_1)u_l - s)}_{\text{first search product 2}})$$

and for $k = 1, ..., n-1$,

$$\Phi_k(\xi_1,\xi_2,s) = \max(\underbrace{\mathbb{E}_{s'}[\Phi_{k+1}(\xi_1,\xi_2,s')]}_{\text{search neither product}}, \underbrace{\phi_k(\xi_1,\xi_2,s,1)}_{\text{first search product 1}}, \underbrace{\phi_k(\xi_1,\xi_2,s,2)}_{\text{first search product 2}})$$

where s' denotes the search cost of the next arriving consumer and follows the distribution $F(\cdot)$, and $\phi_k(\xi_1, \xi_2, s, i)$ denotes the principal's expected optimal surplus-to-go from the k-th consumer to the n-th consumer when the k-th consumer first searches product $i \in \{1, 2\}$. Specifically,

$$\begin{aligned} &\phi_k(\xi_1,\xi_2,s,1) \\ =& -s + \xi_1 \underbrace{(u_h + \mathbb{E}_{s'}[\Phi_{k+1}(1,\xi_2,s')])}_{\text{first search reveals a high type}} \\ &+ (1-\xi_1) \underbrace{\max(\underbrace{u_l + \mathbb{E}_{s'}[\Phi_{k+1}(0,\xi_2,s')]}_{\text{no second search}}, -s + \xi_2(\underbrace{u_h + \mathbb{E}_{s'}[\Phi_{k+1}(0,1,s')]}_{\text{second search reveals a high type}}) + (1-\xi_2)(\underbrace{u_l + \mathbb{E}_{s'}[\Phi_{k+1}(0,0,s')]}_{\text{second search reveals a low type}}))}_{\text{first search reveals a low type}} \end{aligned}$$

and, similarly,

$$\begin{aligned} \phi_k(\xi_1,\xi_2,s,2) =& -s + \xi_2(u_h + \mathbb{E}_{s'}[\Phi_{k+1}(\xi_1,1,s')]) \\ &+ (1-\xi_2)\max(u_l + \mathbb{E}_{s'}[\Phi_{k+1}(\xi_1,0,s')], -s + \xi_1(u_h + \mathbb{E}_{s'}[\Phi_{k+1}(1,0,s')]) + (1-\xi_1)(u_l + \mathbb{E}_{s'}[\Phi_{k+1}(0,0,s')])) \end{aligned}$$

As the principal's prior belief is either product being of high value with probability p_h, the optimal total expected surplus from the n consumers is $\mathbb{E}_s[\Phi_1(p_h, p_h, s)]$.

For notational convenience, define:

$$U_h := \mathbb{E}_{s'}[\max(u_h - s', 0)],\ U_l := \mathbb{E}_{s'}[\max(u_l - s', 0)]$$

Lemma S.12, Lemma S.13, and Corollary S.2 below characterize the surplus-to-go function $\Phi_k(\xi_1, \xi_2, s)$ for a few states.

Lemma S.12.

$$\Phi_k(\xi_1,\xi_2,s) = \max(u_h - s, 0) + (n-k)U_h, \ \textit{ if } \xi_1 = 1 \textit{ or } \xi_2 = 1 \tag{S.11}$$

$$\Phi_k(0,0,s) = \max(u_l - s, 0) + (n-k)U_l \tag{S.12}$$

Lemma S.13.

$$\Phi_k(\xi_1,0,s)=\max(\mathbb{E}_{s'}[\Phi_{k+1}(\xi_1,0,s')],-s+\xi_1(u_h+(n-k)U_h)+(1-\xi_1)(u_l+(n-k)U_l)),\ \textit{for}\ k=1,...,n-1 \quad (S.13)$$

$$\Phi_n(\xi_1,0,s)=\max(0,-s+\xi_1 u_h+(1-\xi_1)u_l) \quad (S.14)$$

$$\mathbb{E}_s[\Phi_k(\xi_1,0,s)]\le \xi_1(n+1-k)U_h+(1-\xi_1)(n+1-k)U_l,\ \textit{for}\ k=1,...,n \quad (S.15)$$

Corollary S.2.

$$\Phi_k(0,\xi_2,s)=\max(\mathbb{E}_{s'}[\Phi_{k+1}(0,\xi_2,s')],-s+\xi_2(u_h+(n-k)U_h)+(1-\xi_2)(u_l+(n-k)U_l)),\ \textit{for}\ k=1,...,n-1$$

$$\Phi_n(0,\xi_2,s)=\max(0,-s+\xi_2 u_h+(1-\xi_2)u_l)$$

$$\mathbb{E}_s[\Phi_k(0,\xi_2,s)]\le \xi_2(n+1-k)U_h+(1-\xi_2)(n+1-k)U_l,\ \textit{for}\ k=1,...,n$$

Now we derive $\Phi_k(p_h,p_h,s)$. Due to symmetry of the two products, $\phi_k(p_h,p_h,s,1)=\phi_k(p_h,p_h,s,2)$. Furthermore, by definition,

$$\begin{aligned}\phi_k(p_h,p_h,s,1)=&-s+p_h(u_h+(n-k)U_h)\\&+(1-p_h)\max(u_l+\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')],-s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l))\end{aligned}$$

Hence,

$$\begin{aligned}&\Phi_k(p_h,p_h,s)\\&=\max(\mathbb{E}_{s'}[\Phi_{k+1}(p_h,p_h,s')],\phi_k(p_h,p_h,s,1))\\&=\max(\mathbb{E}_{s'}[\Phi_{k+1}(p_h,p_h,s')],-s+p_h(u_h+(n-k)U_h)\\&\quad+(1-p_h)\max(u_l+\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')],-s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l))\end{aligned} \quad (S.16)$$

The following lemma characterizes $\Phi_k(p_h,p_h,s)$.

Lemma S.14.

$$\mathbb{E}_{s'}[\Phi_k(p_h,p_h,s')]\le(2-p_h)\mathbb{E}_{s'}[\Phi_k(0,p_h,s')]-(1-p_h)(n+1-k)U_l+u_l \quad (S.17)$$

Based on the results so far, Theorem S.1 presents the principal's optimal strategy in the first-best scenario.

Theorem S.1. *(First-Best Solution) Given the current belief $\xi_1,\xi_2\in\{0,p_h,1\}$ and the kth-arriving consumer's search cost s, the principal's optimal strategy in the first-best scenario is as follows:*

- *If $\xi_i=1$, $i\in\{1,2\}$, instruct the consumer to search and purchase product i if $s\le u_h$ and to leave without any search or purchase if $s>u_h$;*
- *If $\xi_1=\xi_2=0$, instruct the consumer to search and purchase a randomly-picked product if $s\le u_l$ and to leave without any search or purchase if $s>u_l$;*
- *If $\xi_i=0$ and $\xi_{3-i}=p_h$, $i\in\{1,2\}$, instruct the consumer to search and purchase product $3-i$ if $s\le\overline{\zeta}_k$ and to leave without any search or purchase if $s>\overline{\zeta}_k$, where $\overline{\zeta}_k=p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l)-\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')]$;*
- *If $\xi_1=\xi_2=p_h$, there exist two thresholds, $\underline{\tau}_k$ and $\overline{\tau}_k$, such that $\underline{\tau}_k\le\overline{\tau}_k$ and:*

— *if* $s \leq \underline{\tau}_k$, *instruct the consumer to first search a randomly-picked product and purchase it if it is a high type; if the first search reveals a low type, instruct the consumer to search the other product and to purchase a high type if any and to purchase either product otherwise;*

— *if* $\underline{\tau}_k < s \leq \overline{\tau}_k$, *instruct the consumer to search and purchase a randomly-picked product;*

— *if* $s > \overline{\tau}_k$, *instruct the consumer to leave without any search or purchase.*

Specifically,

$$\underline{\tau}_k = p_h(u_h + (n-k)U_h) + (1-p_h)(u_l + (n-k)U_l) - \mathbb{E}_{s'}[\Phi_{k+1}(0, p_h, s')] - u_l$$

$$\overline{\tau}_k = p_h(u_h + (n-k)U_h) + (1-p_h)(u_l + \mathbb{E}_{s'}[\Phi_{k+1}(0, p_h, s')]) - \mathbb{E}_{s'}[\Phi_{k+1}(p_h, p_h, s')]$$

As shown in Theorem S.1, the socially-optimal search and purchasing strategy is similar, structure-wise, to that of an individual consumer. The learning dynamics, however, is fundamentally different due to the principal's welfare-maximizing objective and central-planning capability. Specifically, different from an individual consumer who makes independent decisions based on his/her own search cost, a principal takes account of the search costs of the current and future arrivals. In particular, starting from the prior belief (p_h, p_h), it essentially tackles two optimal stopping problems: wait (i.e., instruct all the arriving consumers to leave without search or purchase) until the search cost of a newly-arriving consumer (say, the k-th arrival) is below $\overline{\tau}_k$, in which case it instructs consumer k to do a first search. If the first search reveals a low type, the principal examines whether consumer k's search cost is below $\underline{\tau}_k$: if so, it guides the consumer to conduct a second search; otherwise, it instructs consumer k to buy the searched product and wait until another consumer (say, the i-th arrival, with $i > k$) with search cost below $\overline{\zeta}_i$ arrives. Subsequently, the principal optimally asks consumer i to search the other product.

As aforementioned, the first-best solution is critically determined by the search thresholds $(\overline{\tau}_k, \underline{\tau}_k, \overline{\zeta}_k)$, which are also solutions to the optimal stopping problems. Because of the principal's objective to maximize the total welfare, its incentive to explore the product values through searches differs from that of an individual consumer, which is reflected in the comparison of the search thresholds in Proposition S.18 below.

Proposition S.18. *The search thresholds in the first-best solution are greater than or equal to their counterparts in an individual consumer's search problem whereby the prior belief for either product being of high value equals* p_h. *Specifically,* $\overline{\zeta}_k \geq p_h u_h + (1-p_h)u_l$, $\underline{\tau}_k \geq p_h(u_h - u_l)$, *and* $\overline{\tau}_k \geq p_h u_h + (1-p_h)u_l$. *Furthermore,* $0 < \underline{\tau}_k < \overline{\zeta}_k$.

Proposition S.18 indicates that the welfare-maximizing principal has a stronger motivation to investigate the product values than individual consumers. This result applies for both the first search and the second search. In particular, the principal may instruct some consumer to search even if the consumer's expected surplus from the search is negative, in the hope of obtaining positive surplus from consumers arriving in the future. Conversely, individual consumers, due to self interests, are less willing to search than the principal, leading to efficiency loss under decentralized decision making.

In addition to self interests, efficiency losses in practical applications may also be attributed to other factors. For example, while the first-best scenario assumes that the principal is perfectly aware of each arrival's search cost and search results, in practice consumers are privately informed about their search cost and search results. Furthermore, due to search-cost heterogeneity, the search results cannot be (fully) deduced from consumers' purchases even if the latter are observable. On top of these, in practice early-arriving consumers' purchasing decisions may not be observable to later arrivals, who are often given access, periodically, to only a proxy (e.g., sales ranking or volume) of the purchasing history. All of these factors lead to inefficiency in social learning.

To illustrate various sources of inefficiencies, consider the following example of three consumers (i.e., $n=3$). The search cost distribution is $F(x)=(\alpha+(1-\alpha)\Phi((x-\mu)/\sigma))\mathbb{I}(x\geq 0)$, where $\Phi(\cdot)$ is the cumulative distribution function for the standard normal distribution, $\mathbb{I}(\cdot)$ is the indicator function, $\alpha=0.08$, $\mu=4.5$, and $\sigma=1.5$. Furthermore, $u_h=6$, $u_l=2$, and $p_h=0.6$. We will compare consumer welfare in three cases: first best, real-time provision of volume information as in §SP, and the base model where volume information is released only once and after the first consumer makes purchasing decision (i.e., $n_0=1$). We focus on the volume information for the inefficiency analysis to preclude the welfare loss due to lower granularity in the ranking information.

We first obtain the search thresholds for the first consumer under the three cases. For the first-best case, the first consumer performs the first search if and only if $s\leq 5.3651$ and the second search if and only if $s\leq 2.9788$. The search thresholds in the other two cases are the same: specifically, the first consumer performs the first search if and only if $s\leq p_hu_h+p_lu_l=4.4$ and performs the second search if and only if $s\leq p_h(u_h-u_l)=2.4$.

Consider the following three scenarios, differing in the first consumer's search cost, denoted by s_I. In each scenario we discuss the inefficiencies in the second and third cases by comparing them with the first-best case.

- $s_I\leq 2.4$: For this scenario, the first consumer is willing (instructed) to perform two searches in all the three cases and purchases a high value product if there is any. Thus, consumer 1's welfare is the same across the three cases. The next two consumers' expected welfare, however, differ in the three cases. For the first-best case, as the first consumer is willing to perform two searches, the principal's learning about the product values is "perfect" after the first consumer's search(es). That is, either a high-value product is revealed, or the principal knows that both products are of low value. Since the next two consumers follow the principal's purchasing instruction, they achieve their highest-possible expected welfare. This does not occur in the other two cases as the product values remain uncertain and the next two consumers may either incur additional search cost or give up searching when there is a high-value product, both leading to welfare loss. This exemplifies loss of efficiency due to incomplete information, i.e., early consumers' search results are unobservable to late consumers.
- $2.4<s_I\leq 2.9788$: For this scenario, the first consumer is willing (instructed) to perform the first search in all the three cases, but, conditional on the first search revealing a low value, consumer 1 is instructed to search the other product in the first-best case but is unwilling to perform the second search in the other two cases. Consequently, in the first-best case the first consumer's action reveals information about the product values, which is exploited by the platform and increases the expected welfare of later consumers. For the other two cases, as the first consumer's purchase is uninformative, the remaining consumers cannot learn from her observed action. This efficiency loss is due to the first consumer's self interest under decentralized decision making.
- $4.4<s_I\leq 5.3651$: For this scenario, the first consumer is instructed to perform the first search in the first-best case but is unwilling to search at all in the other two cases. Hence, learning occurs in the first best but not in the other cases. Welfare is higher in the first best due to two effects: first, consumer 1 is dictated to search in the first best, which allows for information revelation and collection; and second, the collected information is "shared" with later consumers through the principal's instruction, enabling learning and improving welfare.

The three scenarios exemplify two primary sources of inefficiencies: decentralized decision making and incomplete information about prior search results. To examine the welfare implication of information provision frequency, we further compare the second and the third cases. We find that, in this example, the aggregate consumer surplus is higher under more frequent information provision (it is 3.0051 in the real-time case and 2.9622 in the base model). The difference in aggregate surplus is due to a higher expected surplus for the third consumer in the real-time case as the first two consumers' expected surplus are the same in the two cases. The third consumer obtains a higher surplus in the real-time case because she is able to observe the purchasing action of the second consumer. In particular, we note that consumer 3's expected surplus conditioning on sales realization $(2, 0)$ is higher than that in the base model. This is because an observation of the first two consumers purchasing the same product enhances the likelihood that the bestseller product is of high value as the second consumer chooses to purchase the same product as the first consumer after observing the purchasing action of the latter.

To summarize, real-time information provision may improve consumer welfare from the level in the base model because of consumers' better informed search and purchasing decisions. In particular, notice that each arriving consumer engages in both public learning and private learning, and his/her purchasing decisions reflect the up-to-date sales information and his/her own private information acquired through product searches. Therefore, real-time sales information represents accumulated outcomes of the public and private learning by all the earlier arrivals, while those under the base model only reflect the early arrivals' private learning outcomes. Specifically, under real-time information, later-arriving consumers are able to rationalize the earlier-arriving ones' purchasing and search decisions, and deduce the sales information that the earlier arrivals face and in turn their search outcomes. Essentially, the later arrivals can leverage both the public and the private learning conducted by their predecessors. Hence, the real-time sales information may be more indicative of the product values compared to the base model, and thus result in higher consumer welfare.

We observe that the aforementioned results and insights are robust in all of the problem instances that we tested. As in Table S.9 below, the aggregate consumer surplus decreases from the first-best case to the real-time case, and then from the real-time case to the base model. An interesting observation from Table S.9 is that the welfare difference first increases and then decreases in p_h. That is, efficiency loss seems aggravated as the product values become more uncertain ex ante. Intuitively, search and purchasing decisions may be more involved under higher value uncertainty and, thus, consumers could benefit more from centralized planning and a higher frequency of sales information provision.

Table S.9 Aggregate consumer surplus: $\mu = 4.5$, $\sigma = 1.5$, $u_h = 6$, $u_l = 2$, $\alpha = 0.08$, $n = 3$, **and** $n_0 = 1$ **for the base model.**

p_h	*Expected Welfare*			*Welfare Difference*		
	First Best (FB)	*Real Time (RT)*	*Base Model (BM)*	*FB−RT*	*FB−BM*	*RT−BM*
0.2	1.1814	1.0947	1.0911	0.0867	0.0903	0.0036
0.3	1.6604	1.4399	1.4301	0.2204	0.2302	0.0098
0.4	2.2545	1.8622	1.8434	0.3923	0.4111	0.0188
0.5	2.9266	2.3806	2.3503	0.5459	0.5762	0.0303
0.6	3.6242	3.0051	2.9622	0.6190	0.6619	0.0429
0.7	4.2993	3.7255	3.6728	0.5738	0.6265	0.0526
0.8	4.9167	4.5014	4.4484	0.4153	0.4683	0.0530
0.9	5.4535	5.2599	5.2227	0.1936	0.2308	0.0372

SH.1. Appendix

Proof of Lemma S.12 Prove by induction. First observe that equations (S.11) and (S.12) hold for $k=n$ by definition of $\Phi_n(\xi_1,\xi_2,s)$. Suppose that both equations hold for $k+1$. That is,

$$\begin{aligned}\Phi_{k+1}(\xi_1,\xi_2,s) &= \max(u_h-s,0)+(n-k-1)U_h,\ \text{if } \xi_1=1 \text{ or } \xi_2=1\\ \Phi_{k+1}(0,0,s) &= \max(u_l-s,0)+(n-k-1)U_l\end{aligned}$$

It implies the following:

$$\mathbb{E}_s[\Phi_{k+1}(\xi_1,\xi_2,s)] = (n-k)U_h,\ \text{if } \xi_1=1 \text{ or } \xi_2=1 \tag{S.18}$$

$$\mathbb{E}_s[\Phi_{k+1}(0,0,s)] = (n-k)U_l \tag{S.19}$$

We prove below that equations (S.11) and (S.12) also hold for k.

• (S.11): First consider the case $\xi_1=1$ and $\xi_2\le 1$. By (S.18) and (S.19), we have

$$\begin{aligned}&\phi_k(1,\xi_2,s,1) = -s+u_h+(n-k)U_h\\ &\phi_k(1,\xi_2,s,2) = -s+\xi_2(u_h+(n-k)U_h)+(1-\xi_2)\max(u_l+(n-k)U_h,-s+\xi_1(u_h+(n-k)U_h)+(1-\xi_1)(u_l+(n-k)U_l)),\end{aligned}$$

Hence, $\phi_k(1,\xi_2,s,1)\ge\phi_k(1,\xi_2,s,2)$. Thus, by (S.18) and definition of $\Phi_k(\xi_1,\xi_2,s)$,

$$\begin{aligned}\Phi_k(1,\xi_2,s) &= \max(\mathbb{E}_{s'}[\Phi_{k+1}(1,\xi_2,s')],\phi_k(1,\xi_2,s,1),\phi_k(1,\xi_2,s,2))\\ &= \max((n-k)U_h,u_h-s+(n-k)U_h)\\ &= \max(0,u_h-s)+(n-k)U_h\end{aligned}$$

That is, (S.11) holds for k if $\xi_1=1$. Similarly, it can be shown that (S.11) holds for k if $\xi_2=1$.

• (S.12): By (S.18) and (S.19), we have

$$\begin{aligned}\phi_k(0,0,s,1) &= -s+\max(u_l+\mathbb{E}_{s'}[\Phi_{k+1}(0,0,s')],-s+u_l+\mathbb{E}_{s'}[\Phi_{k+1}(0,0,s')])\\ &= -s+\max(u_l+(n-k)U_l,-s+u_l+(n-k)U_l)\\ &= -s+u_l+(n-k)U_l\end{aligned}$$

and

$$\begin{aligned}\phi_k(0,0,s,2) &= -s+\max(u_l+\mathbb{E}_{s'}[\Phi_{k+1}(0,0,s')],-s+u_l+\mathbb{E}_{s'}[\Phi_{k+1}(0,0,s')])\\ &= -s+\max(u_l+(n-k)U_l,-s+u_l+(n-k)U_l)\\ &= -s+u_l+(n-k)U_l\end{aligned}$$

Hence,

$$\begin{aligned}\Phi_k(0,0,s) &= \max(\mathbb{E}_{s'}[\Phi_{k+1}(0,0,s')],-s+u_l+(n-k)U_l)\\ &= \max((n-k)U_l,-s+u_l+(n-k)U_l)\\ &= \max(0,u_l-s)+(n-k)U_l\end{aligned}$$

That is, (S.12) holds for k. □

Proof of Lemma S.13 We first prove the case $k=n$. By definition,

$$\begin{aligned}\Phi_n(\xi_1,0,s) &= \max(0,-s+\xi_1 u_h+(1-\xi_1)\max(u_l,u_l-s),-s+\max(u_l,\xi_1 u_h+(1-\xi_1)u_l-s))\\ &= \max(0,-s+\xi_1 u_h+(1-\xi_1)u_l,-s+\max(u_l,\xi_1 u_h+(1-\xi_1)u_l-s))\\ &= \max(0,-s+\xi_1 u_h+(1-\xi_1)u_l)\end{aligned}$$

where the last equality is due to the facts $u_l\le \xi_1 u_h+(1-\xi_1)u_l$ and $\xi_1 u_h+(1-\xi_1)u_l-s\le \xi_1 u_h+(1-\xi_1)u_l$. Thus, (S.14) holds. Furthermore, since $\mathbb{E}_{s'}[\max(x-s',0)]$ is a convex function of x,

$$\begin{aligned}\mathbb{E}_s[\Phi_n(\xi_1,0,s)] &= \mathbb{E}_s[\max(0,-s+\xi_1 u_h+(1-\xi_1)u_l)]\\ &\le \xi_1\mathbb{E}_s[\max(0,-s+u_h)]+(1-\xi_1)\mathbb{E}_s[\max(0,-s+u_l)]=\xi_1 U_h+(1-\xi_1)U_l\end{aligned}$$

Thus, (S.15) holds for $k=n$.

Now, suppose that both (S.13) and (S.15) hold for $k+1$. That is,

$$\Phi_{k+1}(\xi_1,0,s)=\max(\mathbb{E}_{s'}[\Phi_{k+2}(\xi_1,0,s')],-s+\xi_1(u_h+(n-k-1)U_h)+(1-\xi_1)(u_l+(n-k-1)U_l)) \quad \text{(S.20)}$$

$$\mathbb{E}_s[\Phi_{k+1}(\xi_1,0,s)]\le \xi_1(n-k)U_h+(1-\xi_1)(n-k)U_l \quad \text{(S.21)}$$

Next we will prove that both (S.13) and (S.15) also hold for k. By (S.18), (S.19) and (S.21), we have

$$\begin{aligned}\phi_k(\xi_1,0,s,1) &= -s+\xi_1(u_h+\mathbb{E}_{s'}[\Phi_{k+1}(1,0,s')])+(1-\xi_1)\max(u_l+\mathbb{E}_{s'}[\Phi_{k+1}(0,0,s')],-s+u_l+\mathbb{E}_{s'}[\Phi_{k+1}(0,0,s')])\\ &= -s+\xi_1(u_h+(n-k)U_h)+(1-\xi_1)(u_l+(n-k)U_l)\end{aligned}$$

and

$$\begin{aligned}\phi_k(\xi_1,0,s,2) &= -s+\max(u_l+\mathbb{E}_{s'}[\Phi_{k+1}(\xi_1,0,s')],-s+\xi_1(u_h+\mathbb{E}_{s'}[\Phi_{k+1}(1,0,s')])+(1-\xi_1)(u_l+\mathbb{E}_{s'}[\Phi_{k+1}(0,0,s')]))\\ &= -s+\max(u_l+\mathbb{E}_{s'}[\Phi_{k+1}(\xi_1,0,s')],-s+\xi_1(u_h+(n-k)U_h)+(1-\xi_1)(u_l+(n-k)U_l))\\ &\le -s+\xi_1(u_h+(n-k)U_h)+(1-\xi_1)(u_l+(n-k)U_l)\end{aligned}$$

Hence, $\phi_k(\xi_1,0,s,1)\ge\phi_k(\xi_1,0,s,2)$. It implies:

$$\begin{aligned}\Phi_k(\xi_1,0,s) &= \max(\mathbb{E}_{s'}[\Phi_{k+1}(\xi_1,0,s')],\phi_k(\xi_1,0,s,1))\\ &= \max(\mathbb{E}_{s'}[\Phi_{k+1}(\xi_1,0,s')],-s+\xi_1(u_h+(n-k)U_h)+(1-\xi_1)(u_l+(n-k)U_l))\end{aligned}$$

That is, (S.13) holds for $k+1$. Furthermore, by (S.21),

$$\begin{aligned}\mathbb{E}_s[\Phi_k(\xi_1,0,s)] &= \mathbb{E}_s[\max(\mathbb{E}_{s'}[\Phi_{k+1}(\xi_1,0,s')],-s+\xi_1(u_h+(n-k)U_h)+(1-\xi_1)(u_l+(n-k)U_l))]\\ &\le \mathbb{E}_s[\max(\xi_1(n-k)U_h+(1-\xi_1)(n-k)U_l,-s+\xi_1(u_h+(n-k)U_h)+(1-\xi_1)(u_l+(n-k)U_l))]\\ &= \mathbb{E}_s[\max(0,-s+\xi_1 u_h+(1-\xi_1)u_l)]+\xi_1(n-k)U_h+(1-\xi_1)(n-k)U_l\\ &\le \xi_1\mathbb{E}_s[\max(0,-s+u_h)]+(1-\xi_1)\mathbb{E}_s[\max(0,-s+u_l)]+\xi_1(n-k)U_h+(1-\xi_1)(n-k)U_l\\ &= \xi_1(n-k+1)U_h+(1-\xi_1)(n-k+1)U_l\end{aligned}$$

That is, (S.15) also holds for $k+1$. □

Proof of Corollary S.2 It follows from Lemma S.13 and the symmetry of the products. □

Proof of Lemma S.14 We prove (S.17) by induction. When $k=n$, by definition,

$$\Phi_n(p_h,p_h,s)=\max(0,-s+p_hu_h+(1-p_h)\max(u_l,p_hu_h+(1-p_h)u_l-s))$$
$$\Phi_n(0,p_h,s)=\max(0,-s+p_hu_h+(1-p_h)u_l)$$

That is to show:

$$\begin{aligned}&\mathbb{E}_s[\max(0,-s+p_hu_h+(1-p_h)\max(u_l,p_hu_h+(1-p_h)u_l-s))]\\ \le\ &(2-p_h)\mathbb{E}_s[\max(0,-s+p_hu_h+(1-p_h)u_l)]-(1-p_h)\mathbb{E}_s[\max(0,u_l-s)]+u_l\end{aligned}$$

We prove this by showing that, for each given s,

$$\begin{aligned}&\max(0,-s+p_hu_h+(1-p_h)\max(u_l,p_hu_h+(1-p_h)u_l-s))\\ \le\ &(2-p_h)\max(0,-s+p_hu_h+(1-p_h)u_l)-(1-p_h)\max(0,u_l-s)+u_l\end{aligned}$$

Let

$$\begin{aligned}Y(s)=&\max(0,-s+p_hu_h+(1-p_h)\max(u_l,p_hu_h+(1-p_h)u_l-s))\\ &-((2-p_h)\max(0,-s+p_hu_h+(1-p_h)u_l)-(1-p_h)\max(0,u_l-s)+u_l)\end{aligned}$$

Consider the following three cases:

- $s\le p_h(u_h-u_l)$

$$\begin{aligned}Y(s)&=-s+p_hu_h+(1-p_h)(p_hu_h+(1-p_h)u_l-s)-((2-p_h)(-s+p_hu_h+(1-p_h)u_l)+u_l)+(1-p_h)\max(u_l-s,0)\\ &=u_l(p_h-2)+(1-p_h)\max(u_l-s,0)\\ &=\max(-u_l-s(1-p_h),u_l(p_h-2))\le 0\end{aligned}$$

- $p_h(u_h-u_l)<s\le p_hu_h+(1-p_h)u_l$

$$\begin{aligned}Y(s)&=(-s+p_hu_h+(1-p_h)u_l)-((2-p_h)(-s+p_hu_h+(1-p_h)u_l)+u_l)+(1-p_h)\max(u_l-s,0)\\ &=s-2u_l+p_h^2u_h-p_h^2u_l-sp_h-p_hu_h+2p_hu_l+(1-p_h)\max(u_l-s,0)\\ &=\max(-(1-p_h)p_hu_h-p_h^2u_l-(1-p_h)u_l,s(1-p_h)-2u_l+p_h^2u_h-p_h^2u_l-p_hu_h+2p_hu_l)\end{aligned}$$

where

$$s(1-p_h)-2u_l+p_h^2u_h-p_h^2u_l-p_hu_h+2p_hu_l\le(p_hu_h+(1-p_h)u_l)(1-p_h)-2u_l+p_h^2u_h-p_h^2u_l-p_hu_h+2p_hu_l=-u_l$$

Hence, $Y(s)<0$ as it equals to the maximum of two negative terms.

- $p_hu_h+(1-p_h)u_l<s$

$$Y(s)=(1-p_h)\max(0,u_l-s)-u_l<0$$

Therefore, (S.17) holds for $k=n$.

Now, suppose that (S.17) holds for $k+1$. That is,

$$\mathbb{E}_{s'}[\Phi_{k+1}(p_h,p_h,s')]\le(2-p_h)\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')]-(1-p_h)(n-k)U_l+u_l \tag{S.22}$$

Next we prove that (S.17) holds for k. First, recall (S.16):

$$\begin{aligned}
&\mathbb{E}_s[\Phi_k(p_h,p_h,s)]\\
=&\mathbb{E}_s[\max(\mathbb{E}_{s'}[\Phi_{k+1}(p_h,p_h,s')],-s+p_h(u_h+(n-k)U_h)\\
&+(1-p_h)\max(u_l+\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')],-s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l))]
\end{aligned}$$

To prove (S.17), it suffices to show the following three inequalities:

$$\begin{aligned}
&\mathbb{E}_{s'}[\Phi_{k+1}(p_h,p_h,s')]\\
\le&(2-p_h)\mathbb{E}_{s'}[\Phi_k(0,p_h,s')]-(1-p_h)(n+1-k)U_l+u_l \qquad \text{(S.23)}\\
&\mathbb{E}_s[-s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')])]\\
\le&(2-p_h)\mathbb{E}_{s'}[\Phi_k(0,p_h,s')]-(1-p_h)(n+1-k)U_l+u_l \qquad \text{(S.24)}\\
&\mathbb{E}_s[-s+p_h(u_h+(n-k)U_h)+(1-p_h)(-s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l))]\\
\le&(2-p_h)\mathbb{E}_{s'}[\Phi_k(0,p_h,s')]-(1-p_h)(n+1-k)U_l+u_l \qquad \text{(S.25)}
\end{aligned}$$

In preparation, by Corollary S.2,

$$\mathbb{E}_s[\Phi_k(0,p_h,s)]=\mathbb{E}_s[\max(\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')],-s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l))]$$

Hence,

$$\mathbb{E}_s[\Phi_k(0,p_h,s)]\ge\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')] \qquad \text{(S.26)}$$

$$\mathbb{E}_s[\Phi_k(0,p_h,s)]\ge\mathbb{E}_s[-s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l)] \qquad \text{(S.27)}$$

Next we prove (S.23) through (S.25):

- (S.23):

$$\begin{aligned}
&\mathbb{E}_{s'}[\Phi_{k+1}(p_h,p_h,s')]-((2-p_h)\mathbb{E}_{s'}[\Phi_k(0,p_h,s')]-(1-p_h)(n+1-k)U_l+u_l)\\
\le&(2-p_h)\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')]-(1-p_h)(n-k)U_l+u_l-((2-p_h)\mathbb{E}_{s'}[\Phi_k(0,p_h,s')]-(1-p_h)(n+1-k)U_l+u_l)\\
=&(2-p_h)(\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')]-\mathbb{E}_{s'}[\Phi_k(0,p_h,s')])-(1-p_h)U_l\le 0
\end{aligned}$$

where the first inequality is by (S.22) and the second inequality is by (S.26).

- (S.24):

$$\begin{aligned}
&\mathbb{E}_s[-s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')])]-((2-p_h)\mathbb{E}_{s'}[\Phi_k(0,p_h,s')]-(1-p_h)(n+1-k)U_l+u_l)\\
\le&\mathbb{E}_s[-s]+p_h(u_h+(n-k)U_h)+(1-p_h)u_l-\mathbb{E}_{s'}[\Phi_k(0,p_h,s')]+(1-p_h)(n+1-k)U_l-u_l\\
=&\mathbb{E}_s[-s]+p_h(u_h-u_l+(n-k)U_h)-\mathbb{E}_{s'}[\Phi_k(0,p_h,s')]+(1-p_h)(n+1-k)U_l\\
\le&\mathbb{E}_s[-s]+p_h(u_h-u_l+(n-k)U_h)+(1-p_h)(n+1-k)U_l-\mathbb{E}_s[-s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l)]\\
=&(1-p_h)U_l-u_l\le 0
\end{aligned}$$

where the first inequality is by (S.26), the second inequality is by (S.27), and the last inequality follows because $U_l=\mathbb{E}_s[\max(u_l-s,0)]<u_l$.

• (S.25):

$$\begin{aligned}
&\mathbb{E}_s[-s+p_h(u_h+(n-k)U_h)+(1-p_h)(-s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l))] \\
&\quad -((2-p_h)\mathbb{E}_{s'}[\Phi_k(0,p_h,s')]-(1-p_h)(n+1-k)U_l+u_l) \\
&\leq \mathbb{E}_s[-s+p_h(u_h+(n-k)U_h)+(1-p_h)(-s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l))] \\
&\quad +(1-p_h)(n+1-k)U_l-u_l-(2-p_h)\mathbb{E}_s[-s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l)] \\
&= U_l-u_l-U_lp_h-(1-p_h)u_l<0
\end{aligned}$$

where the first inequality is by (S.27).

As we have proved (S.23) through (S.25), we conclude that (S.17) holds for k. This completes the proof. □

Proof of Theorem S.1 The first three bullet points are implied by Lemmas S.12 and S.13. For the last bullet point, by (S.16), it is optimal for the principal to instruct the consumer to do the first search if and only if

$$\begin{aligned}
&\mathbb{E}_{s'}[\Phi_{k+1}(p_h,p_h,s')] \\
&\leq -s+p_h(u_h+(n-k)U_h)+(1-p_h)\max(u_l+\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')],-s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l))
\end{aligned}$$

and to do the second search (if the first search reveals a low type) if and only if

$$u_l+\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')]\leq -s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l)$$

To prove the claimed structure of the optimal strategy, it suffices to prove:

$$\begin{aligned}
&s>p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')])-\mathbb{E}_{s'}[\Phi_{k+1}(p_h,p_h,s')] \\
\Rightarrow\, &s>p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l)-\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')]-u_l
\end{aligned}$$

A sufficient condition for it is:

$$\begin{aligned}
&p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')])-\mathbb{E}_{s'}[\Phi_{k+1}(p_h,p_h,s')] \\
\geq\, &p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l)-\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')]-u_l
\end{aligned}$$

which is equivalent to

$$\mathbb{E}_{s'}[\Phi_{k+1}(p_h,p_h,s')]\leq (2-p_h)\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')]-(1-p_h)(n-k)U_l+u_l$$

That is:

$$\mathbb{E}_{s'}[\Phi_k(p_h,p_h,s')]\leq (2-p_h)\mathbb{E}_{s'}[\Phi_k(0,p_h,s')]-(1-p_h)(n+1-k)U_l+u_l$$

This inequality is proved as in Lemma S.14. □

Lemma S.15.

$$p_h(n-k+1)U_h+(1-p_h)\mathbb{E}_{s'}[\Phi_k(0,p_h,s')]-\mathbb{E}_{s'}[\Phi_k(p_h,p_h,s')]\geq 0 \tag{S.28}$$

Proof of Lemma S.15 We prove (S.28) by induction. When $k = n$, by definition,

$$\Phi_n(p_h, p_h, s) = \max(0, -s + p_h u_h + (1-p_h)\max(u_l, p_h u_h + (1-p_h)u_l - s))$$
$$\Phi_n(0, p_h, s) = \max(0, -s + p_h u_h + (1-p_h)u_l)$$

That is to show:

$$\mathbb{E}_s[\max(0, -s + p_h u_h + (1-p_h)\max(u_l, p_h u_h + (1-p_h)u_l - s))]$$
$$\le (1-p_h)\mathbb{E}_s[\max(0, -s + p_h u_h + (1-p_h)u_l)] + p_h \mathbb{E}_s[\max(0, u_h - s)]$$

We prove this by showing that, for each given s,

$$\max(0, -s + p_h u_h + (1-p_h)\max(u_l, p_h u_h + (1-p_h)u_l - s))$$
$$\le (1-p_h)\max(0, -s + p_h u_h + (1-p_h)u_l) + p_h \max(0, u_h - s)$$

Let

$$X(s) = \max(0, -s + p_h u_h + (1-p_h)\max(u_l, p_h u_h + (1-p_h)u_l - s))$$
$$-((1-p_h)\max(0, -s + p_h u_h + (1-p_h)u_l) + p_h \max(0, u_h - s))$$

Consider the following three cases:

- $s \le p_h(u_h - u_l)$

$$X(s) = -s + p_h u_h + (1-p_h)(p_h u_h + (1-p_h)u_l - s) - (1-p_h)(-s + p_h u_h + (1-p_h)u_l) - p_h(u_h - s)$$
$$= -s(1-p_h) < 0$$

- $p_h(u_h - u_l) < s \le p_h u_h + (1-p_h)u_l$

$$X(s) = (-s + p_h u_h + (1-p_h)u_l) - (1-p_h)(-s + p_h u_h + (1-p_h)u_l) - p_h(u_h - s)$$
$$= p_h(-s + p_h u_h + (1-p_h)u_l) - p_h(u_h - s)$$
$$= -p_h(1-p_h)(u_h - u_l) < 0$$

- $p_h u_h + (1-p_h)u_l < s$

$$X(s) = -p_h \max(0, u_h - s) < 0$$

Therefore, (S.28) holds for $k = n$.

Now suppose that (S.28) holds for $k+1$, i.e.,

$$\mathbb{E}_{s'}[\Phi_{k+1}(p_h, p_h, s')] \le p_h(n-k)U_h + (1-p_h)\mathbb{E}_{s'}[\Phi_{k+1}(0, p_h, s')] \tag{S.29}$$

Next we show that (S.28) holds for k. In preparation, recall:

$$\Phi_k(p_h, p_h, s)$$
$$= \max(\mathbb{E}_{s'}[\Phi_{k+1}(p_h, p_h, s')], \phi_k(p_h, p_h, s, 1))$$
$$= \max(\mathbb{E}_{s'}[\Phi_{k+1}(p_h, p_h, s')], -s + p_h(u_h + (n-k)U_h)$$
$$+ (1-p_h)\max(u_l + \mathbb{E}_{s'}[\Phi_{k+1}(0, p_h, s')], -s + p_h(u_h + (n-k)U_h) + (1-p_h)(u_l + (n-k)U_l)) \tag{S.30}$$

and

$$\Phi_k(0,p_h,s)=\max(\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')],-s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l)),$$

which implies

$$\mathbb{E}_s[\Phi_k(0,p_h,s)]\geq\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')] \tag{S.31}$$

$$\mathbb{E}_s[\Phi_k(0,p_h,s)]\geq\mathbb{E}_s[-s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l))] \tag{S.32}$$

To show

$$\mathbb{E}_{s'}[\Phi_k(p_h,p_h,s')]\leq p_h(n-k+1)U_h+(1-p_h)\mathbb{E}_{s'}[\Phi_k(0,p_h,s')],$$

by (S.30), it suffices to prove

$$\begin{aligned}&\mathbb{E}_{s'}[\Phi_{k+1}(p_h,p_h,s')]\\ \leq\ & p_h(n-k+1)U_h+(1-p_h)\mathbb{E}_{s'}[\Phi_k(0,p_h,s')] \end{aligned} \tag{S.33}$$

$$\begin{aligned}&\mathbb{E}_s[-s]+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')])\\ \leq\ & p_h(n-k+1)U_h+(1-p_h)\mathbb{E}_{s'}[\Phi_k(0,p_h,s')] \end{aligned} \tag{S.34}$$

$$\begin{aligned}&\mathbb{E}_s[-s+p_h(u_h+(n-k)U_h)+(1-p_h)(-s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l))]\\ \leq\ & p_h(n-k+1)U_h+(1-p_h)\mathbb{E}_{s'}[\Phi_k(0,p_h,s')] \end{aligned} \tag{S.35}$$

Next we prove (S.33) through (S.35).
(S.33): The inequality follows from (S.29) and (S.31).
(S.34):

$$\begin{aligned}&\mathbb{E}_s[-s]+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')])-p_h(n-k+1)U_h-(1-p_h)\mathbb{E}_{s'}[\Phi_k(0,p_h,s')]\\ =\ &\mathbb{E}_s[-s]+p_h(u_h-U_h)+(1-p_h)u_l+(1-p_h)(\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')]-\mathbb{E}_{s'}[\Phi_k(0,p_h,s')])\\ =\ &\mathbb{E}_s[-s]+p_h(u_h-U_h)+(1-p_h)u_l\\ &-(1-p_h)\mathbb{E}_s[\max(0,-s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l)-\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')])]\\ \leq\ &\mathbb{E}_s[-s]+p_h(u_h-U_h)+(1-p_h)u_l-(1-p_h)\mathbb{E}_s[\max(0,-s+p_hu_h+(1-p_h)u_l)]\\ =\ &\mathbb{E}_s[-s+p_hu_h+(1-p_h)u_l]-p_h\mathbb{E}_s[\max(u_h-s,0)]-(1-p_h)\mathbb{E}_s[\max(0,-s+p_hu_h+(1-p_h)u_l)]\\ \leq\ &\mathbb{E}_s[-s+p_hu_h+(1-p_h)u_l]-\mathbb{E}_s[\max(0,-s+p_hu_h+(1-p_h)u_l)]\leq 0\end{aligned}$$

where the first inequality is by Corollary S.2.
(S.35):

$$\begin{aligned}&\mathbb{E}_s[-s+p_h(u_h+(n-k)U_h)+(1-p_h)(-s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l))]\\ &-p_h(n-k+1)U_h-(1-p_h)\mathbb{E}_{s'}[\Phi_k(0,p_h,s')]\\ \leq\ &\mathbb{E}_s[-s+p_h(u_h+(n-k)U_h)+(1-p_h)(-s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l))]\\ &-p_h(n-k+1)U_h-(1-p_h)\mathbb{E}_s[-s+p_h(u_h+(n-k)U_h)+(1-p_h)(u_l+(n-k)U_l))]\\ =\ &\mathbb{E}_s[-(1-p_h)s+p_h(u_h-s)-p_h\max(u_h-s,0)]\leq 0\end{aligned}$$

where the first inequality is by (S.32).

As we have proved (S.33) through (S.35), we conclude that (S.28) holds for k. This completes the proof. □

Proof of Proposition S.18 Both $\overline{\zeta}_k \geq p_h u_h + (1-p_h)u_l$ and $\underline{\tau}_k \geq p_h(u_h - u_l)$ follow from the inequality in Corollary S.2. To show $\overline{\tau}_k \geq p_h u_h + (1-p_h)u_l$, it suffices to show:

$$p_h(n-k)U_h + (1-p_h)\mathbb{E}_{s'}[\Phi_{k+1}(0,p_h,s')] - \mathbb{E}_{s'}[\Phi_{k+1}(p_h,p_h,s')] \geq 0$$

and the inequality follows from Lemma S.15.

Furthermore, $\underline{\tau}_k > 0$ is by the inequality in Corollary S.2 and $\underline{\tau}_k \leq \overline{\zeta}_k$ follows immediately from their definitions. □

SI. General Sales Signals

So far we have focused on two specific forms of sales signals, i.e., sales ranking and sales volume. There can be other signals about sales. Assume that the platform commits to a signaling mechanism at the beginning of the horizon. The mechanism maps each possible realization of the first-period sales volumes to a signal. Since the two products' first-period sales add up to n_1, it suffices to consider the realization of the first-period sales volume of product 1, i.e., X_1.

We focus on public signaling, where the mapping is independent of a consumer's search cost and, at the beginning of the second period, a single signal is generated and sent to all the second-period consumers. Furthermore, assume that the platform commits to a partitioning strategy of the interval $[0,n_1]$, which supersets the domain of the first-period product-1 sales. (Candogan 2022 examines public signaling of a single state and shows that the optimal public signaling can be derived based on interval partitioning.) Specifically, the interval $[0,n_1]$ is partitioned into k non-overlapping sub-intervals (except that two sub-intervals may overlap at $\frac{n_1}{2}$, see below). In the second period, the platform publicly announces in which sub-interval the first-period sales of product 1, X_1, lies. This interval-partitioning signaling mechanism subsumes ranking and volume information as special cases: for ranking, the partitioning is $[0,\frac{n_1}{2}]\cup[\frac{n_1}{2},n_1]$, and for volume, it is such that each sub-interval contains exactly one integer.

As the products are ex ante symmetric, the partitioning is symmetric with respect to $\frac{n_1}{2}$. Thus, the partitioning takes one of the following two forms: for $k\in\mathbb{N}$ and $0 =: \delta_0 < \delta_1 < \delta_2 < \dots < \delta_k < \delta_{k+1} := \frac{n_1}{2}$,

(P1) One of the intervals contains $\frac{n_1}{2}$ as an interior point:

$$\Big[0,\frac{n_1}{2}-\delta_k\Big)\cup\Big[\frac{n_1}{2}-\delta_k,\frac{n_1}{2}-\delta_{k-1}\Big)\cup\dots\cup\Big[\frac{n_1}{2}-\delta_2,\frac{n_1}{2}-\delta_1\Big)\cup\Big[\frac{n_1}{2}-\delta_1,\frac{n_1}{2}+\delta_1\Big]$$
$$\cup\Big(\frac{n_1}{2}+\delta_1,\frac{n_1}{2}+\delta_2\Big]\cup\dots\cup\Big(\frac{n_1}{2}+\delta_{k-1},\frac{n_1}{2}+\delta_k\Big]\cup\Big(\frac{n_1}{2}+\delta_k,n_1\Big]$$

(P2) None of the intervals contains $\frac{n_1}{2}$ as an interior point:

$$\Big[0,\frac{n_1}{2}-\delta_k\Big)\cup\Big[\frac{n_1}{2}-\delta_k,\frac{n_1}{2}-\delta_{k-1}\Big)\cup\dots\cup\Big[\frac{n_1}{2}-\delta_2,\frac{n_1}{2}-\delta_1\Big)\cup\Big[\frac{n_1}{2}-\delta_1,\frac{n_1}{2}\Big]\cup\Big[\frac{n_1}{2},\frac{n_1}{2}+\delta_1\Big]$$
$$\cup\Big(\frac{n_1}{2}+\delta_1,\frac{n_1}{2}+\delta_2\Big]\cup\dots\cup\Big(\frac{n_1}{2}+\delta_{k-1},\frac{n_1}{2}+\delta_k\Big]\cup\Big(\frac{n_1}{2}+\delta_k,n_1\Big]$$

Specifically, for (P2), if n_1 is an even number and $X_1=\frac{n_1}{2}$, then the platform announces that X_1 lies in $\big[\frac{n_1}{2},\frac{n_1}{2}+\delta_1\big]$ with probability $\frac{1}{2}$ and X_1 lies in $\big[\frac{n_1}{2}-\delta_1,\frac{n_1}{2}\big]$ with probability $\frac{1}{2}$.

We first show in Lemma S.16 that (P1) is dominated by (P2) from the viewpoint of the platform.

LEMMA S.16. *From the platform's perspective, the partitioning (P1) is outperformed by the partitioning (P2). That is, the expected second-period total sales under (P1) is no greater than that under (P2).*

By Lemma S.16, the platform is better off providing at least the bestseller ranking information, i.e., upon receiving the message, the second-period consumers become aware of which product generates higher sales in the first period.

To facilitate further analysis, next we prove that, given the partitioning (P2), conditional on X_1 in any interval, the first-search threshold is no lower than the second-search threshold. Specifically, for $l \in \{0,1,...,k\}$, denote by Event E-l the event $X_1 \in \left(\frac{n_1}{2}+\delta_l, \frac{n_1}{2}+\delta_{l+1}\right]$ with $\delta_l < \delta_{l+1}$. Let $\Delta_l G_s = \sum_{t \in \left(\frac{n_1}{2}+\delta_l, \frac{n_1}{2}+\delta_{l+1}\right], t \in \mathbb{N}} g_s(t)$, $\Delta_l G_a = \sum_{t \in \left(\frac{n_1}{2}+\delta_l, \frac{n_1}{2}+\delta_{l+1}\right], t \in \mathbb{N}} g_a(t)$, and $\Delta_{-l} G_a = \sum_{t \in \left[\frac{n_1}{2}-\delta_{l+1}, \frac{n_1}{2}-\delta_l\right), t \in \mathbb{N}} g_a(t)$. The first-search and second-search thresholds are:

$$
\begin{aligned}
\alpha_1^{E-l} &:= \mathbb{P}[u_1 = u_h | X_1 \in (\tfrac{n_1}{2}+\delta_l, \tfrac{n_1}{2}+\delta_{l+1}]] \\
&= \frac{p_h^2 \Delta_l G_s + p_h p_l \Delta_l G_a}{(p_h^2+p_l^2)\Delta_l G_s + p_h p_l \Delta_l G_a + p_h p_l \Delta_{-l} G_a} \\
&= \frac{p_h}{p_h + p_l \frac{p_l \Delta_l G_s + p_h \Delta_{-l} G_a}{p_h \Delta_l G_s + p_l \Delta_l G_a}} \\
\beta_2^{E-l} &:= \mathbb{P}[u_2 = u_h | X_1 \in (\tfrac{n_1}{2}+\delta_l, \tfrac{n_1}{2}+\delta_{l+1}], u_1 = u_l] \\
&= \frac{p_h p_l \Delta_{-l} G_a}{p_l^2 \Delta_l G_s + p_h p_l \Delta_{-l} G_a} \\
&= \frac{p_h}{p_l \frac{\Delta_l G_s}{\Delta_{-l} G_a} + p_h}
\end{aligned}
$$

We have:

LEMMA S.17. $\alpha_1^{E-l} \geq \beta_2^{E-l}$.

Lemma S.17 indicates that, given any public message about X_1 in an interval, the total second-period sales is determined by α_1^{E-l}.

For expositional convenience, we focus on the case where n_1 is odd in the analysis below. The case where n_1 is even can be proved similarly. By product symmetry, the platform's expected total sales in the second period is:

$$
s_p := 2n_2 \sum_{l=0}^{k} \sum_{x \in (\frac{n_1}{2}+\delta_l, \frac{n_1}{2}+\delta_{l+1}], x \in \mathbb{N}} F(\mathbb{E}\mathbb{U}(\alpha_1^{E-l}))((p_h^2+p_l^2) g_s(x) + p_h p_l g_a(x) + p_h p_l g_a(n_1 - x))
$$

where the subscript p stands for interval-$p$artitioning.

PROPOSITION S.19. *Among all the public signaling mechanisms based on sales-interval partitioning, ranking (volume) information is optimal if the search-cost distribution $F(\cdot)$ is concave (convex).*

The results jointly imply that, given the public signaling mechanism based on interval-partitioning of the first-period sales, the platform optimally provides at least the ranking information. Furthermore, ranking information is optimal under concave $F(\cdot)$, while volume is optimal under convex $F(\cdot)$. These findings generalize and strengthen the insights in the base model.

SI.1. Appendix

SI.1.1. Properties of $G_a(\cdot)$ and $G_s(\cdot)$

By the analysis in the base model, we have the following properties:

$$\begin{aligned}
g_a(\frac{n_1}{2}) &\leq g_s(\frac{n_1}{2}) \text{ (Proposition 4)} \\
\frac{g_a(x)}{g_s(x)} & \text{ increases in } x \text{ for all } x \text{ (Proposition 4)} \\
g_a(x) &\leq g_s(x) \text{ for } x \leq \frac{n_1}{2} \text{ (Proposition 4)} \\
g_a(n_1-x) &\leq g_s(n_1-x) \text{ for } x \geq \frac{n_1}{2} \\
g_a(x) &> g_a(n_1-x) \text{ for } x \geq \frac{n_1}{2} \text{ (Lemma 2)} \\
g_s(x) &= g_s(n_1-x) \\
G_s(x) &= 1-G_s(n_1-x) \\
G_s(n_1-x) &\geq G_a(n_1-x) \text{ for } x \geq \frac{n_1}{2} \\
G_a(x) &\leq G_s(x) \text{ for } x \leq \frac{n_1}{2} \\
G_a(x) &\leq G_s(x) \leq G_s(\frac{n_1}{2}) = \frac{1}{2} \text{ for } x \leq \frac{n_1}{2}
\end{aligned}$$

LEMMA S.18. *$G_a(x) \leq G_s(x) \leq \bar{G}_a(n_1-x)$ for all $0 \leq x \leq n_1$.*

Proof of Lemma S.18 We first show $G_a(x) \leq \bar{G}_a(n_1-x)$. Since $g_a(x) > g_a(n_1-x)$ for $x \geq \frac{n_1}{2}$, we have:

$$\begin{aligned}
\sum_{t=0}^{x} g_a(t) &< \sum_{t=n_1-x}^{n_1} g_a(t) \text{ for } x \leq \frac{n_1}{2} \\
\sum_{t=x}^{n_1} g_a(t) &> \sum_{t=x}^{n_1} g_a(n_1-t) \text{ for } x \geq \frac{n_1}{2}
\end{aligned}$$

The first inequality implies $G_a(x) < \bar{G}_a(n_1-x)$ for $x \leq \frac{n_1}{2}$, and the second inequality implies $\bar{G}_a(x) > \sum_{t'=0}^{n_1-x} g_a(t') = G_a(n_1-x)$ for $x \geq \frac{n_1}{2}$, which further implies $G_a(x) < \bar{G}_a(n_1-x)$ for $x \geq \frac{n_1}{2}$.

Now we show $G_a(x) \leq G_s(x)$. Since $g_a(x) \leq g_s(x)$ for $x \leq \frac{n_1}{2}$, we have: $G_a(x) = \sum_{t=0}^{x} g_a(t) \leq \sum_{t=0}^{x} g_s(t) = G_s(x)$ for $x \leq \frac{n_1}{2}$. Suppose $G_a(x) > G_s(x)$ for some $x_0 > \frac{n_1}{2}$. Then there exists some $y_0 \in (\frac{n_1}{2}, x_0]$ such that $g_a(y_0) > g_s(y_0)$. Since $\frac{g_a(x)}{g_s(x)}$ increases in x for all x, $g_a(x) > g_s(x)$ for all $x > y_0$. In particular, $g_a(x) > g_s(x)$ for all $x > x_0$. Thus, $G_a(n_1) = G_a(x_0) + \sum_{t=x_0}^{n_1} g_a(t) > G_s(x_0) + \sum_{t=x_0}^{n_1} g_s(t) = G_s(n_1)$. This, however, contradicts with the fact $G_a(n_1) = G_s(n_1) = 1$.

Last, we show $G_s(x) \leq \bar{G}_a(n_1-x)$. This follows from the facts $G_s(x) = \bar{G}_s(n_1-x)$ and $G_a(x) \leq G_s(x)$. □

Recall that $G_a(x)$ ($\bar{G}_a(n_1-x)$) is the sales distribution of H-value (L-value) product when the two products have different values, and $G_s(x)$ is the sales distribution of either product when the products are of the same value. Hence, Lemma S.18 implies that these sales distributions are first-order stochastically ordered.

SI.1.2. Proofs

Proof of Lemma S.16 Note that (P1) and (P2) coincide with each other except that the interval $\left[\frac{n_1}{2}-\delta_1, \frac{n_1}{2}+\delta_1\right]$ in (P1) is substituted by $\left[\frac{n_1}{2}-\delta_1, \frac{n_1}{2}\right] \cup \left[\frac{n_1}{2}, \frac{n_1}{2}+\delta_1\right]$ in (P2). Thus, to prove the lemma, it suffices to prove that a public message of $X_1 \in \left[\frac{n_1}{2}-\delta_1, \frac{n_1}{2}+\delta_1\right]$ generates (weakly) lower sales than a public message of $X_1 \in \left[\frac{n_1}{2}, \frac{n_1}{2}+\delta_1\right]$. By problem symmetry, this also implies that a public message of $X_1 \in \left[\frac{n_1}{2}-\delta_1, \frac{n_1}{2}+\delta_1\right]$ generates (weakly) lower sales than a public message of $X_1 \in \left[\frac{n_1}{2}-\delta_1, \frac{n_1}{2}\right]$.

To this end, we examine the posterior beliefs under two events: $X_1 \in \left[\frac{n_1}{2}-\delta, \frac{n_1}{2}+\delta\right]$ and $X_1 \in \left[\frac{n_1}{2}, \frac{n_1}{2}+\delta\right]$. Under either event, we show that the first-search threshold is no lower than the second-search threshold, and thus, the total sales is determined by the first-search threshold. Here, given the sales information, the first-search threshold is determined by the posterior belief about the bestseller being of high value (if the bestseller is unknown, then it is determined by the posterior belief about a randomly-picked product being of high value) and the second-search threshold is determined by the posterior belief about the non-searched product being of high value after a first search. Subsequently, we show that the the first-search threshold under the first event is no lower than that under the second event, which then completes the proof.

- Event I: $X_1 \in \left[\frac{n_1}{2}-\delta, \frac{n_1}{2}+\delta\right]$

Let $\Delta G_s = \sum_{t\in[\frac{n_1}{2}-\delta,\frac{n_1}{2}+\delta],t\in\mathbb{N}} g_s(t)$ and $\Delta G_a = \sum_{t\in[\frac{n_1}{2}-\delta,\frac{n_1}{2}+\delta],t\in\mathbb{N}} g_s(t)$.

$$\begin{aligned}
\alpha_1^I &:= \mathbb{P}[u_1=u_h|X_1\in[\tfrac{n_1}{2}-\delta,\tfrac{n_1}{2}+\delta)] \\
&= \frac{p_h^2\Delta G_s + p_h p_l \Delta G_a}{(p_h^2+p_l^2)\Delta G_s + 2p_h p_l \Delta G_a} \\
&= \frac{p_h}{p_h + p_l \frac{p_l\Delta G_s + p_h\Delta G_a}{p_h\Delta G_s + p_l \Delta G_a}} \\
\beta_2^I &:= \mathbb{P}[u_2=u_h|X_1\in[\tfrac{n_1}{2}-\delta,\tfrac{n_1}{2}+\delta], u_1=u_l] \\
&= \frac{p_h p_l \Delta G_a}{p_l^2\Delta G_s + p_h p_l \Delta G_a} \\
&= \frac{p_h}{p_l\frac{\Delta G_s}{\Delta G_a} + p_h}
\end{aligned}$$

We have:

$$\frac{p_l\Delta G_s + p_h \Delta G_a}{p_h \Delta G_s + p_l \Delta G_a} - \frac{\Delta G_s}{\Delta G_a} = \frac{p_h((\Delta G_a)^2 - (\Delta G_s)^2)}{(p_h\Delta G_s + p_l\Delta G_a)\Delta G_a}$$

where $\Delta G_a - \Delta G_s = \sum_{t\in[\frac{n_1}{2}-\delta,\frac{n_1}{2}+\delta],t\in\mathbb{N}} (g_a(t)-g_s(t)) := W(\delta)$. Since $g_a(x) > g_a(n_1-x)$ for $x \geq \frac{n_1}{2}$ and $g_s(x) = g_s(n_1-x)$, $W(\delta)$ increases in δ. Furthermore, $W(0) = g_a(\frac{n_1}{2}) - g_s(\frac{n_1}{2}) \leq 0$ and $W(\frac{n_1}{2}) = G_a(n_1) - G_s(n_1) = 0$. Thus, $W(\delta) \leq 0$ for all $\delta \in [0, \frac{n_1}{2}]$. Hence, $\alpha_1^I(s) \geq \beta_2^I(s)$.

- Event II: $X_1 \in \left[\frac{n_1}{2}, \frac{n_1}{2}+\delta\right]$

Let $\Delta_1 G_s = \sum_{t\in[\frac{n_1}{2},\frac{n_1}{2}+\delta],t\in\mathbb{N}} g_s(t)$, $\Delta_1 G_a = \sum_{t\in[\frac{n_1}{2},\frac{n_1}{2}+\delta],t\in\mathbb{N}} g_a(t)$, and $\Delta_{-1} G_a = \sum_{t\in[\frac{n_1}{2}-\delta,\frac{n_1}{2}],t\in\mathbb{N}} g_a(t)$.

$$\begin{aligned}
\alpha_1^{II} &:= \mathbb{P}[u_1=u_h|X_1\in[\tfrac{n_1}{2},\tfrac{n_1}{2}+\delta]] \\
&= \frac{p_h^2\Delta_1 G_s + p_h p_l \Delta_1 G_a}{(p_h^2+p_l^2)\Delta_1 G_s + p_h p_l \Delta_1 G_a + p_h p_l \Delta_{-1} G_a} \\
&= \frac{p_h}{p_h + p_l \frac{p_l\Delta_1 G_s + p_h\Delta_{-1} G_a}{p_h\Delta_1 G_s + p_l \Delta_1 G_a}} \\
\beta_2^{II} &:= \mathbb{P}[u_2=u_h|X_1\in[\tfrac{n_1}{2},\tfrac{n_1}{2}+\delta], u_1=u_l] \\
&= \frac{p_h p_l \Delta_{-1} G_a}{p_l^2\Delta_1 G_s + p_h p_l \Delta_{-1} G_a} \\
&= \frac{p_h}{p_l\frac{\Delta_1 G_s}{\Delta_{-1} G_a} + p_h}
\end{aligned}$$

We have:

$$\frac{p_l\Delta_1 G_s + p_h \Delta_{-1} G_a}{p_h \Delta_1 G_s + p_l \Delta_1 G_a} - \frac{\Delta_1 G_s}{\Delta_{-1} G_a} = \frac{p_l\Delta_1 G_s(\Delta_{-1}G_a - \Delta_1 G_a) + p_h((\Delta_{-1} G_a)^2 - (\Delta_1 G_s)^2)}{(p_h\Delta_1 G_s + p_l\Delta_1 G_a)\Delta_{-1} G_a}$$

where $\Delta_{-1}G_a - \Delta_1 G_a = \sum_{t\in[\frac{n_1}{2}-\delta,\frac{n_1}{2}],t\in\mathbb{N}} g_a(t) - \sum_{t\in[\frac{n_1}{2},\frac{n_1}{2}+\delta],t\in\mathbb{N}} g_a(t) < 0$ since $g_a(x) > g_a(n_1-x)$ for $x \geq \frac{n_1}{2}$. Meanwhile, $\Delta_{-1}G_a - \Delta_1 G_s = \sum_{t\in[\frac{n_1}{2}-\delta,\frac{n_1}{2}],t\in\mathbb{N}} (g_a(t) - g_s(t)) < 0$ since $g_s(x) = g_s(n_1-x)$ and $g_a(x) \leq g_s(x)$ for $x \leq \frac{n_1}{2}$. Hence, $\alpha_1^{II}(s) \geq \beta_2^{II}(s)$.

- Now we compare $\alpha_1^I(s)$ and $\alpha_1^{II}(s)$:

$$
\begin{aligned}
&\frac{p_l(G_s(\frac{n_1}{2}+\delta) - G_s(\frac{n_1}{2}-\delta)) + p_h(G_a(\frac{n_1}{2}+\delta) - G_a(\frac{n_1}{2}-\delta))}{p_h(G_s(\frac{n_1}{2}+\delta) - G_s(\frac{n_1}{2}-\delta)) + p_l(G_a(\frac{n_1}{2}+\delta) - G_a(\frac{n_1}{2}-\delta))} \\
&- \frac{p_l(G_s(\frac{n_1}{2}+\delta) - G_s(\frac{n_1}{2})) + p_h(\bar{G}_a(\frac{n_1}{2}-\delta) - \bar{G}_a(\frac{n_1}{2}))}{p_h(G_s(\frac{n_1}{2}+\delta) - G_s(\frac{n_1}{2})) + p_l(G_a(\frac{n_1}{2}+\delta) - G_a(\frac{n_1}{2}))} \\
=&\frac{p_l\Delta G_s + p_h\Delta G_a}{p_h\Delta G_s + p_l\Delta G_a} - \frac{p_l\Delta_1 G_s + p_h\Delta_{-1}G_a}{p_h\Delta_1 G_s + p_l\Delta_1 G_a} \\
=&\frac{p_l^2(\Delta_1 G_a\Delta G_s - \Delta G_a\Delta_1 G_s) + p_h^2(\Delta G_a\Delta_1 G_s - \Delta_{-1}G_a\Delta G_s) + p_h p_l\Delta G_a(\Delta_1 G_a - \Delta_{-1}G_a)}{(p_h\Delta G_s + p_l\Delta G_a)(p_h\Delta_1 G_s + p_l\Delta_1 G_a)}
\end{aligned}
$$

where, since $g_a(x) > g_a(n_1-x)$ for $x \geq \frac{n_1}{2}$, we have:

$$
\begin{aligned}
\Delta_1 G_a\Delta G_s - \Delta G_a\Delta_1 G_s =& \sum_{t\in[\frac{n_1}{2},\frac{n_1}{2}+\delta],t\in\mathbb{N}} g_a(t)\cdot \sum_{t\in[\frac{n_1}{2}-\delta,\frac{n_1}{2}+\delta],t\in\mathbb{N}} g_s(t) - \sum_{t\in[\frac{n_1}{2}-\delta,\frac{n_1}{2}+\delta],t\in\mathbb{N}} g_a(t)\cdot \sum_{t\in[\frac{n_1}{2},\frac{n_1}{2}+\delta],t\in\mathbb{N}} g_s(t) \\
=& \left\{2\sum_{t\in[\frac{n_1}{2},\frac{n_1}{2}+\delta],t\in\mathbb{N}} g_a(t) - \sum_{t\in[\frac{n_1}{2}-\delta,\frac{n_1}{2}+\delta],t\in\mathbb{N}} g_a(t)\right\}\cdot \sum_{t\in[\frac{n_1}{2},\frac{n_1}{2}+\delta],t\in\mathbb{N}} g_s(t) \\
>& 0 \\
\Delta G_a\Delta_1 G_s - \Delta_{-1}G_a\Delta G_s =& \sum_{t\in(\frac{n_1}{2}-\delta,\frac{n_1}{2}+\delta],t\in\mathbb{N}} g_a(t)\cdot \sum_{t\in[\frac{n_1}{2},\frac{n_1}{2}+\delta],t\in\mathbb{N}} g_s(t) - \sum_{t\in(\frac{n_1}{2}-\delta,\frac{n_1}{2}],t\in\mathbb{N}} g_a(t)\cdot \sum_{t\in[\frac{n_1}{2}-\delta,\frac{n_1}{2}+\delta],t\in\mathbb{N}} g_s(t) \\
=& \{\sum_{t\in(\frac{n_1}{2}-\delta,\frac{n_1}{2}+\delta],t\in\mathbb{N}} g_a(t) - 2\sum_{t\in(\frac{n_1}{2}-\delta,\frac{n_1}{2}],t\in\mathbb{N}} g_a(t)\}\cdot \sum_{t\in[\frac{n_1}{2}-\delta,\frac{n_1}{2}+\delta],t\in\mathbb{N}} g_s(t) \\
>& 0 \\
\Delta_1 G_a - \Delta_{-1}G_a =& \sum_{t\in[\frac{n_1}{2},\frac{n_1}{2}+\delta],t\in\mathbb{N}} g_a(t) - \sum_{t\in[\frac{n_1}{2}-\delta,\frac{n_1}{2}],t\in\mathbb{N}} g_a(t) > 0
\end{aligned}
$$

Hence, $\alpha_1^I(s) < \alpha_1^{II}(s)$. □

Proof of Lemma S.17

$$
\alpha_1^{E-l} - \beta_2^{E-l} = \frac{p_l\Delta_l G_s + p_h\Delta_{-l}G_a}{p_h\Delta_l G_s + p_l\Delta_l G_a} - \frac{\Delta_l G_s}{\Delta_{-l}G_a} = \frac{p_l\Delta_l G_s(\Delta_{-l}G_a - \Delta_l G_a) + p_h((\Delta_{-l}G_a)^2 - (\Delta_l G_s)^2)}{(p_h\Delta_l G_s + p_l\Delta_l G_a)\Delta_{-l}G_a}
$$

where $\Delta_{-l}G_a - \Delta_l G_a = \sum_{t\in[\frac{n_1}{2}-\delta_{l+1},\frac{n_1}{2}-\delta_l),t\in\mathbb{N}} g_a(t) - \sum_{t\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}],t\in\mathbb{N}} g_a(t) < 0$ since $g_a(x) > g_a(n_1-x)$ for $x \geq \frac{n_1}{2}$. Meanwhile, $\Delta_{-l}G_a - \Delta_l G_s = \sum_{t\in[\frac{n_1}{2}-\delta_{l+1},\frac{n_1}{2}-\delta_l),t\in\mathbb{N}} (g_a(t) - g_s(t)) < 0$ since $g_s(x) = g_s(n_1-x)$ and $g_a(x) \leq g_s(x)$ for $x \leq \frac{n_1}{2}$. Hence, $\alpha_1^{E-l} \geq \beta_2^{E-l}$. □

Proof of Proposition S.19 We first prove that under the public signaling mechanism based on interval partitioning, the expectation of the posterior about the bestseller being of high value equals π_1^r:

$$
\begin{aligned}
&2\sum_{l=0}^{k}\sum_{x\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}],x\in\mathbb{N}} \alpha_1^{E-l}((p_h^2+p_l^2)g_s(x) + p_h p_l g_a(x) + p_h p_l g_a(n_1-x)) \\
=&2\sum_{l=0}^{k}\alpha_1^{E-l}\sum_{x\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}],x\in\mathbb{N}} ((p_h^2+p_l^2)g_s(x) + p_h p_l g_a(x) + p_h p_l g_a(n_1-x)) \\
=&2\sum_{l=0}^{k}\alpha_1^{E-l}((p_h^2+p_l^2)\Delta_l G_s + p_h p_l\Delta_l G_a + p_h p_l\Delta_{-l}G_a)
\end{aligned}
$$

$$= 2\sum_{l=0}^{k} \frac{p_h^2\Delta_l G_s + p_h p_l \Delta_l G_a}{(p_h^2+p_l^2)\Delta_l G_s + p_h p_l \Delta_l G_a + p_h p_l \Delta_{-l} G_a}((p_h^2+p_l^2)\Delta_l G_s + p_h p_l \Delta_l G_a + p_h p_l \Delta_{-l} G_a)$$
$$= 2\sum_{l=0}^{k} p_h^2\Delta_l G_s + p_h p_l \Delta_l G_a$$
$$= 2p_h^2(1-G_s(\frac{n_1}{2})) + 2p_h p_l(1-G_a(\frac{n_1}{2}))$$
$$= \pi_1^r$$

Hence, when $F(\cdot)$ is concave,

$$2n_2 \textstyle\sum_{l=0}^{k}\sum_{x\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}],x\in\mathbb{N}} F(\mathbb{EU}(\alpha_1^{E-l}))((p_h^2+p_l^2)g_s(x)+p_h p_l g_a(x)+p_h p_l g_a(n_1-x))$$
$$\le n_2 F\left(\mathbb{EU}\left(2\textstyle\sum_{l=0}^{k}\sum_{x\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}],x\in\mathbb{N}} \alpha_1^{E-l}((p_h^2+p_l^2)g_s(x)+p_h p_l g_a(x)+p_h p_l g_a(n_1-x))\right)\right)$$
$$= n_2 F\left(\mathbb{EU}\left(\pi_1^r\right)\right)$$
$$= S_r$$

where the inequality follows from the facts that $F(\cdot)$ is concave and $2\sum_{l=0}^{k}\sum_{x\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}],x\in\mathbb{N}}((p_h^2+p_l^2)g_s(x)+p_h p_l g_a(x)+p_h p_l g_a(n_1-x))=1$.

Furthermore, recall the posterior under volume information: for $x \ge \frac{n_1}{2}$,

$$\pi_1^v(x) = \mathbb{P}[u_1=u_h|X_1=x] = \frac{p_h^2 g_s(x)+p_h p_l g_a(x)}{(p_h^2+p_l^2)g_s(x)+p_h p_l g_a(x)+p_h p_l g_a(n_1-x)}$$

Thus,

$$\sum_{x\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}],x\in\mathbb{N}} \pi_1^v(x)\frac{(p_h^2+p_l^2)g_s(x)+p_h p_l g_a(x)+p_h p_l g_a(n_1-x)}{\mathbb{P}[X_1\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}]]}$$
$$= \sum_{x\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}],x\in\mathbb{N}} \frac{p_h^2 g_s(x)+p_h p_l g_a(x)}{(p_h^2+p_l^2)g_s(x)+p_h p_l g_a(x)+p_h p_l g_a(n_1-x)}\frac{(p_h^2+p_l^2)g_s(x)+p_h p_l g_a(x)+p_h p_l g_a(n_1-x)}{\mathbb{P}[X_1\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}]]}$$
$$= \sum_{x\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}],x\in\mathbb{N}} \frac{p_h^2 g_s(x)+p_h p_l g_a(x)}{\mathbb{P}[X_1\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}]]}$$
$$= \frac{\sum_{x\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}],x\in\mathbb{N}}[p_h^2 g_s(x)+p_h p_l g_a(x)]}{\mathbb{P}[X_1\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}]]}$$
$$= \alpha_1^{E-l}$$

Hence, when $F(\cdot)$ is convex,

$$2n_2 \textstyle\sum_{l=0}^{k}\sum_{x\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}],x\in\mathbb{N}} F(\mathbb{EU}(\alpha_1^{E-l}))((p_h^2+p_l^2)g_s(x)+p_h p_l g_a(x)+p_h p_l g_a(n_1-x))$$
$$= 2n_2 \textstyle\sum_{l=0}^{k} F(\mathbb{EU}(\alpha_1^{E-l}))\mathbb{P}[X_1\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}]]$$
$$= 2n_2 \textstyle\sum_{l=0}^{k} F\left(\sum_{x\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}],x\in\mathbb{N}} \mathbb{EU}(\pi_1^v(x))\frac{(p_h^2+p_l^2)g_s(x)+p_h p_l g_a(x)+p_h p_l g_a(n_1-x)}{\mathbb{P}[X_1\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}]]}\right)\mathbb{P}[X_1\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}]]$$
$$\le 2n_2 \textstyle\sum_{l=0}^{k}\sum_{x\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}],x\in\mathbb{N}} F\left(\mathbb{EU}(\pi_1^v(x))\right)\frac{(p_h^2+p_l^2)g_s(x)+p_h p_l g_a(x)+p_h p_l g_a(n_1-x)}{\mathbb{P}[X_1\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}]]}\mathbb{P}[X_1\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}]]$$
$$= 2n_2 \textstyle\sum_{l=0}^{k}\sum_{x\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}],x\in\mathbb{N}} F\left(\mathbb{EU}(\pi_1^v(x))\right)((p_h^2+p_l^2)g_s(x)+p_h p_l g_a(x)+p_h p_l g_a(n_1-x))$$
$$= 2n_2 \textstyle\sum_{x\in(\frac{n_1}{2},n_1],x\in\mathbb{N}} F\left(\mathbb{EU}(\pi_1^v(x))\right)((p_h^2+p_l^2)g_s(x)+p_h p_l g_a(x)+p_h p_l g_a(n_1-x))$$
$$= S_v$$

where the inequality is by the convexity of $F(\cdot)$ and $\sum_{x\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}],x\in\mathbb{N}} \frac{(p_h^2+p_l^2)g_s(x)+p_h p_l g_a(x)+p_h p_l g_a(n_1-x)}{\mathbb{P}[X_1\in(\frac{n_1}{2}+\delta_l,\frac{n_1}{2}+\delta_{l+1}]]} = 1$.

□

SJ. Sales Milestone for Information Provision

In the base model we assume that bestseller information is released after n_0 consumers have arrived, where n_0 is the platform's decision. Alternatively, the platform may choose to release bestseller information when a certain sales milestone is achieved. We examine this alternative scenario in this extension. Specifically, assume that consumers sequentially arrive to the platform according to a general stochastic process and bestseller information (if any) is publicized when the products' total sales reaches a threshold n_1, where n_1 is chosen by the platform. In this case, the first (second) period is defined as the period of time before (after) the sales milestone is reached.

We first note that, in the base model, by Lemma 1 (i), the total sales reaching a milestone n_1 is equivalent to the total arrivals reaching a milestone n_0, where n_1 and n_0 satisfy $n_1 = \lfloor n_0 F(u_h p_h + u_l p_l) \rfloor$. This is because the proportion of consumers who perform a search (and thus make a purchase) is $F(u_h p_h + u_l p_l)$.

Now, consider the situation where each consumer's search cost is independently drawn from the search-cost distribution $F(\cdot)$, as in §SF. In this case, the proportion of consumers who make a purchase is a random variable, and, thus, there no longer exists a deterministic mapping between sales milestone and arrival milestone. Nevertheless, similar to that in §SF, we show that our main results remain valid under this alternative setting.

SJ.1. Posterior Beliefs

Similar to the base model, we first derive consumers' beliefs in the second period under no information, ranking information, and volume information, respectively. Note that the number of first-period consumers performing the first search is the sales milestone, i.e., n_1. Let $\tilde{m}$ denote the number of first-period consumers performing the second search. Since consumers' search costs are independently distributed, $\tilde{m} \sim Binomial(n_1, \frac{F(p_h(u_h-u_l))}{F(p_h u_h + p_l u_l)})$. For $m = 0, ..., n_1$, let $P(m|n_1)$ denote the probability that in the first period m consumers perform the second search. Specifically, $P(m|n_1) = \binom{n_1}{m} \left(\frac{F(p_h(u_h-u_l))}{F(p_h u_h + p_l u_l)}\right)^m \left(1 - \frac{F(p_h(u_h-u_l))}{F(p_h u_h + p_l u_l)}\right)^{n_1-m}$. Below we derive the posterior beliefs under various types of sales information.

No Information

When there is no sales information, the posterior beliefs are the same as the prior, i.e., $\pi_1^\phi = \nu_1^\phi = \pi_2^\phi = p_h$, where, similar to in the base model, π_1^ϕ denotes a late consumer's belief that the higher-ranked product is of high value before she makes a first search, ν_1^ϕ denotes her belief that the lower-ranked product is of high value before she makes a first search, and π_2^ϕ denotes her belief that the lower-ranked product is of high value after her first search reveals a low value in the higher-ranked product.

Ranking Information

Under ranking information, let π_1^r be a late consumer's belief that the higher-ranked product is of high value before she makes a first search, and ν_1^r be her belief that the lower-ranked product is of high value before she makes a first search, and π_2^r be her belief that the lower-ranked product is of high value after her first search reveals a low value in the higher-ranked product.

For given (n_1, m) and for $x = 0, 1, ..., n_1$, define

$$g_s(x|n_1, m) := Binomial(x, n_1, 1/2),$$

$$g_a(x|n_1, m) := Binomial(x - m, n_1 - m, 1/2) \text{ if } x \geq m, \text{ and } 0 \text{ if } x < m,$$

where $Binomial(x,y,p)$ is the probability that among y independent trials, x of them succeed, where the probability of success is p. Let $G_s(x|n_1,m)$ and $G_a(x|n_1,m)$ be the cumulative distribution functions corresponding to $g_s(x|n_1,m)$ and $g_a(x|n_1,m)$, respectively. Let $\bar{G}_s(x|n_1,m) := 1-G_s(x|n_1,m)$ and $\bar{G}_a(x|n_1,m) := 1-G_a(x|n_1,m)$.

Given (n_1,m), define

$$\tilde{\pi}_1^r(n_1,m) = p_h^2 + 2p_hp_l(1-G_a(\tfrac{n_1}{2}|n_1,m))$$
$$\tilde{\nu}_1^r(n_1,m) = p_h^2 + 2p_hp_lG_a(\tfrac{n_1}{2}|n_1,m)$$
$$\tilde{\pi}_2^r(n_1,m) = \frac{p_hG_a(\frac{n_1}{2}|n_1,m)}{p_hG_a(\frac{n_1}{2}|n_1,m)+p_l\bar{G}_s(\frac{n_1}{2}|n_1,m)}$$

when n_1 is odd. The definitions under even values of n_1 are similar to those in the base model and are omitted here. As the late consumers cannot observe m, they take account of all the possible values of m. Lemma S.19 follows.

Lemma S.19. *$\pi_1^r = \sum_m P(m|n_1)\tilde{\pi}_1^r(n_1,m)$, $\nu_1^r = \sum_m P(m|n_1)\tilde{\nu}_1^r(n_1,m)$, $\pi_2^r = \sum_m P(m|n_1)\tilde{\pi}_2^r(n_1,m)$.*

Volume Information

When sales volume information is publicized, the late-arriving consumers observe the sales volume of both products. On the other hand, the number of early consumers who perform the second search, $\tilde{m}$, remains uncertain. Given n_1, let $p(m,x|n_1)$ be the probability that $\tilde{m}=m$ and the sales for the higher sales ranking product is x. Then for $x \geq n_1/2$,

$$\begin{aligned} p(m,x|n_1) &= \Pr(x|\tilde{m}=m)\cdot\Pr(\tilde{m}=m) \\ &= [(p_h^2+p_l^2)g_s(x|n_1,m)+p_hp_lg_a(x|n_1,m)+p_hp_lg_a(n_1-x|n_1,m)]\cdot P(m|n_1) \end{aligned}$$

Let $p(x|n_1) = \sum_{m\leq n_1} p(m,x|n_1)$. The knowledge of n_1 and x allows the late consumers to update their belief about $\tilde{m}$: for $m=0,...,n_1$, $p(m|n_1,x) := \Pr[\tilde{m}=m|n_1,x] = \frac{p(m,x|n_1)}{p(x|n_1)}$.

Under volume information, let $\pi_1^v(x,n_1)$ be a late consumer's belief that the higher-ranked product is of high value before she makes a first search, and $\nu_1^v(x,n_1)$ be her belief that the lower-ranked product is of high value before she makes a first search, and $\pi_2^v(x,n_1)$ be her belief that the lower-ranked product is of high value after her first search reveals a low value in the higher-ranked product.

For given (n_1,m), we define

$$\tilde{\pi}_1^v(x,n_1,m) = \frac{p_h^2g_s(x|n_1,m)+p_hp_lg_a(x|n_1,m)}{p_h^2g_s(x|n_1,m)+p_hp_lg_a(x|n_1,m)+p_hp_lg_a(n_1-x|n_1,m)+p_l^2g_s(x|n_1,m)}$$
$$\tilde{\nu}_1^v(x,n_1,m) = \frac{p_h^2g_s(x|n_1,m)+p_hp_lg_a(n_1-x|n_1,m)}{p_h^2g_s(x|n_1,m)+p_hp_lg_a(x|n_1,m)+p_hp_lg_a(n_1-x|n_1,m)+p_l^2g_s(x|n_1,m)}$$
$$\tilde{\pi}_2^v(x,n_1,m) = \frac{p_hg_a(n_1-x|n_1,m)}{p_hg_a(n_1-x|n_1,m)+p_lg_s(x|n_1,m)}$$

where x is the sales of the bestseller. Lemma S.20 follows.

Lemma S.20.

$$\pi_1^v(x,n_1) = \sum_{m\leq n_1}\frac{p(m,x|n_1)}{p(x|n_1)}\tilde{\pi}_1^v(x,n_1,m)$$
$$\nu_1^v(x,n_1) = \sum_{m\leq n_1}\frac{p(m,x|n_1)}{p(x|n_1)}\tilde{\nu}_1^v(x,n_1,m)$$
$$\pi_2^v(x,n_1) = \sum_{m\leq n_1}\frac{p(m,x|n_1)}{p(x|n_1)}\tilde{\pi}_2^v(x,n_1,m)$$

SJ.2. Robustness of Results in the Base Model

In this section we prove that the main results in the base model remain valid in this extended model (with the statement of some results slightly modified, as highlighted in boldface below). For each result, we repeat its (modified) statement in this subsection. The proofs are similar to those in §SF.4 and thus omitted for brevity.

LEMMA 1 *(Consumers' purchasing choices in the first period)*
(i) In the first period, consumers with search cost $s \le u_h p_h + u_l p_l$ *purchase a product and consumers with higher search cost leave without buying.*
(ii) If the two products' values are identical (i.e., either $u_1 = u_2 = u_h$ *or* $u_1 = u_2 = u_l$*), a consumer who chooses to make a purchase buys either product with equal probabilities. If the two products' values are different (i.e.,* $u_i = u_h$ *and* $u_{3-i} = u_l$, $i \in \{1,2\}$*), a consumer with search cost* $s \le p_h \Delta$ *purchases the product with high value, while a consumer with search cost* $s \in (p_h\Delta, u_h p_h + u_l p_l]$ *purchases the first product that she searches.*

PROPOSITION 1 ***Given*** $\boldsymbol{n_1}$ ***and*** $\boldsymbol{m \in \{0,\dots,n_1\}}$, *for* $x \in [0, n_1]$,

$$g_s(x|\boldsymbol{n_1, m}) := Binomial(x, n_1, 1/2),$$
$$g_a(x|\boldsymbol{n_1, m}) := Binomial(x-m, n_1-m, 1/2) \text{ if } x \ge m, \text{ and } 0 \text{ if } x < m,$$

where $Binomial(x,y,p)$ *is the probability that among* y *independent trials,* x *of them succeed, where the probability of success is* p. *Let* $G_s(x|\boldsymbol{n_1, m})$ *and* $G_a(x|\boldsymbol{n_1, m})$ *be the cumulative distribution functions corresponding to* $g_s(x|\boldsymbol{n_1, m})$ *and* $g_a(x|\boldsymbol{n_1, m})$*, respectively. Let* $\bar{G}_s(x|\boldsymbol{n_1, m}) := 1 - G_s(x|\boldsymbol{n_1, m})$ *and* $\bar{G}_a(x|\boldsymbol{n_1, m}) := 1 - G_a(x|\boldsymbol{n_1, m})$.
(i) If the two products' values are identical, the sales of either product follows distribution $G_s(x|\boldsymbol{n_1, m})$, ***given*** n_1 ***and conditional on*** $\boldsymbol{\tilde{m} = m}$;
(ii) If the two products' values are different, the sales of the high-value (resp. low-value) product follows distribution $G_a(x|\boldsymbol{n_1, m})$ *(resp.* $\bar{G}_a(n_1 - x|\boldsymbol{n_1, m})$*),* ***given*** n_1 ***and conditional on*** $\boldsymbol{\tilde{m} = m}$.

PROPOSITION 2 *Under either ranking or volume information, if a consumer finds it worthwhile to search, then it is optimal for her to first search product* i^*.

LEMMA 2 $\pi_1^r \ge \nu_1^r \ge \pi_2^r$, *and* $\pi_1^v(x, \boldsymbol{n_1}) \ge \nu_1^v(x, \boldsymbol{n_1}) \ge \pi_2^v(x, \boldsymbol{n_1}), \forall x \ge n_1/2$.

LEMMA 3 *(Consumers' purchasing choices in the second period) For* $t \in \{\phi, r, v\}$,
(i) Consumers with search cost $s \le u_h \pi_1^t + u_l(1-\pi_1^t)$ *purchase a product and consumers with higher search cost leave without buying.* ***That is, for given*** n_2***, the total sales of the two products in the second period follows*** $\boldsymbol{Binomial(n_2, F(\pi_1^t u_h + (1-\pi_1^t) u_l))}$;
(ii) If a consumer's first search reveals a low type, she performs a second search if and only if her search cost is low (i.e., $s \le \pi_2^t \Delta$*). Thus, if the two products' values are different, a consumer with search cost* $s \le \pi_2^t \Delta$ *purchases the product with high value, while a consumer with search cost* $s \in (\pi_2^t\Delta, u_h\pi_1^t + u_l(1-\pi_1^t)]$ *purchases the first product that she searches.*

PROPOSITION 3 $\pi_1^r \ge \pi_1^\phi = p_h \ge \nu_1^r$, $\pi_2^r \le \pi_2^\phi = p_h$.

LEMMA 4 *For* $i = 1, 2$,
(i) $\Pr\left[u_i = u_h | u_1 \ne u_2, X_i \ge \frac{n_1}{2}\right] \ge \Pr\left[u_i = u_h | u_1 \ne u_2\right]$ *and* $\Pr\left[u_i = u_h | u_1 = u_2, X_i \ge \frac{n_1}{2}\right] = \Pr\left[u_i = u_h | u_1 = u_2\right]$;
(ii) $\Pr\left[u_1 \ne u_2 | X_i \ge \frac{n_1}{2}\right] = \Pr\left[u_1 \ne u_2\right]$ *and* $\Pr\left[u_1 = u_2 | X_i \ge \frac{n_1}{2}\right] = \Pr\left[u_1 = u_2\right]$.

PROPOSITION 4 *(i)* $\pi_2^v(x,\boldsymbol{n_1}) \le p_h$ *for all* $x \ge n_1/2$. *(ii) If* $p_h \ge \frac{1}{2}$, $\pi_1^v(x,\boldsymbol{n_1}) \ge p_h$ *for any* $x \ge n_1/2$*; otherwise,* $\pi_1^v(x,\boldsymbol{n_1}) \ge p_h$ *if and only if* x *is sufficiently high.*

LEMMA 5 $\pi_1^v(\frac{n_1}{2},\boldsymbol{n_1}) > p_h$ *if* $p_h > \frac{1}{2}$, $\pi_1^v(\frac{n_1}{2},\boldsymbol{n_1}) = p_h$ *if* $p_h = \frac{1}{2}$, *and* $\pi_1^v(\frac{n_1}{2},\boldsymbol{n_1}) < p_h$ *if* $p_h < \frac{1}{2}$.

LEMMA 6

(i) Both $Pr[u_{i^*} = u_h | u_1 \ne u_2, X_{i^*} = x]$ *and* $Pr[u_1 \ne u_2 | X_{i^*} = x]$ *increase in* x*, for* $x \ge \frac{n_1}{2}$*;*

(ii) $\pi_1^v(x,\boldsymbol{n_1})$ *increases in* x*, for* $x \ge \frac{n_1}{2}$.

(iii) $\pi_2^v(x,\boldsymbol{n_1})$ *decreases in* x*, for* $x \ge \frac{n_1}{2}$.

LEMMA 7 ***For given*** $\boldsymbol{n_1}$, $\pi_j^v(x,\boldsymbol{n_1})$ *is a mean-preserving spread of* π_j^r *for* $j = 1, 2$.

To evaluate the impact of sales information on the expected sales, we define the expected second-period sales. Recall that the number of consumers in the second period is $\tilde{n}_2 = n - \tilde{n}_0$. Given n_1, denote the probability of $\tilde{n}_2 = n_2$ as $\mathcal{P}(n_2|n_1)$. Note that, given n_1, $\tilde{m}$ and $\tilde{n}_2$ are independent.

- The expected second-period sales under no information is

$$\mathbb{E}[S_\phi] = \left(\sum_{n_2} n_2 \mathcal{P}(n_2|n_1)\right) F(p_h u_h + p_l u_l)$$

- The expected second-period sales under ranking information is

$$\mathbb{E}[S_r] = \left(\sum_{n_2} n_2 \mathcal{P}(n_2|n_1)\right) F\left(\sum_m P(m|n_1)\tilde{\pi}_1^r(n_1,m)u_h + (1 - \sum_{n_1,m} P(m|n_1)\tilde{\pi}_1^r(n_1,m))u_l\right)$$

- The expected second-period sales under volume information is

$$\begin{aligned}\mathbb{E}[S_v] &= \left(\sum_{n_2} n_2 \mathcal{P}(n_2|n_1)\right) \sum_x p(x|n_1) \left[F\left(\sum_m \frac{p(m,x|n_1)}{p(x|n_1)}\tilde{\pi}_1^v(x,n_1,m)u_h + (1 - \sum_m \frac{p(m,x|n_1)}{p(x|n_1)}\tilde{\pi}_1^v(x,n_1,m))u_l\right)\right] \\ &= \left(\sum_{n_2} n_2 \mathcal{P}(n_2|n_1)\right) \sum_x p(x|n_1)[F(\pi_1^v(x,n_1)u_h + (1 - \pi_1^v(x,n_1))u_l)]\end{aligned}$$

where $p(x|n_1)$ is the probability that bestseller product has sales x (where $x \ge n_1/2$) when there are in total n_1 sales in the first period.

PROPOSITION 6 *Compared to no information:*

(i) ranking information increases the expected second-period sales, i.e., $\mathbb{E}[\boldsymbol{S_r}] \ge \mathbb{E}[\boldsymbol{S_\phi}]$. ***In particular, there exist problem instances in which*** $\frac{\mathbb{E}[S_\phi]}{\mathbb{E}[S_r]} \le \epsilon$ ***for any given*** $\epsilon \in (0,1)$***.***

(ii) volume information reduces the second-period sales if both $p_h < \frac{1}{2}$ *and the first-period sales difference is small, and increases the sales otherwise.* ***Furthermore, volume information may lead to a lower expected second-period sales, i.e., there exist problem instances in which*** $\mathbb{E}[\boldsymbol{S_v}] < \mathbb{E}[\boldsymbol{S_\phi}]$.

PROPOSITION 7 *Compared to ranking information:*

(i) volume information reduces the second-period sales when the first-period sales difference is small and increases the sales otherwise.

(ii) volume information may lead to a lower expected second-period sales. In particular, when the search-cost distribution $F(\cdot)$ *is convex,* $\mathbb{E}[\boldsymbol{S_r}] \le \mathbb{E}[\boldsymbol{S_v}]$*; and when* $F(\cdot)$ *is concave,* $\mathbb{E}[\boldsymbol{S_r}] \ge \mathbb{E}[\boldsymbol{S_v}]$.

Proposition 8 *Consider a second-period consumer with search cost s.*
(i) Compared to the case where no sales information is provided, ranking information provision reduces the consumer's expected purchased value if $\pi_2^r(u_h-u_l)<s\le p_h(u_h-u_l)$*;*
(ii) Compared to the case where ranking information is provided, volume information provision reduces the consumer's expected purchased value if $\pi_2^v(n_1,\boldsymbol{n_1})(u_h-u_l)<s\le\pi_2^r(u_h-u_l)$*.*

Proposition 9 *Consider a second-period consumer with search cost s.*
(i) Compared to the case where no sales information is provided, ranking information provision increases the first-search probability and decreases the second-search probability;
(ii) Compared to the case where ranking information is provided, volume information provision decreases both search probabilities if $\pi_2^v(n_1,\boldsymbol{n_1})(u_h-u_l)<s\le\pi_2^r(u_h-u_l)$ *and increase both probabilities if* $\pi_1^r u_h+(1-\pi_1^r)u_l<s\le\pi_1^v(n_1,\boldsymbol{n_1})u_h+(1-\pi_1^v(n_1,\boldsymbol{n_1}))u_l$*.*

Proposition 10 *A consumer's expected surplus is higher under either ranking or volume information than that under no information. Furthermore, it is higher under volume information than that under ranking information.*

SK. Effects of Total Population Size on Optimal Timing of Sales Information Provision

In the base model we numerically illustrate the effects of total population size, n, on the optimal timing of sales information provision, n_0^*. In this section we prove these effects for sales ranking information and under approximations of binomial and normal distributions, as detailed below:

Approximation 1 We use normal distribution to approximate the binomial distribution G_a. Recall that from the base model we have $m=\lfloor n_0F(p_h(u_h-u_l))\rfloor$ and $n_1=\lfloor n_0F(p_hu_h+p_lu_l)\rfloor$. Here as we are using normal approximation, we drop the floor operator and assume $m=n_0F(p_h(u_h-u_l))$ and $n_1=n_0F(p_hu_h+p_lu_l)$. Notice that $G_a(\cdot)$ is the cumulative distribution function for the sales of the high-value product ranging from m to n_1 when the two products are of different values, i.e.,

$$G_a(x)=\sum_{i=0}^{x}g_a(i)=\sum_{i=m}^{x}\frac{(i-m)!(n_1-i)!}{(n_1-m)!}\cdot\frac{1}{2^{n_1-m}}$$

where $g_a(x)=\text{Binomial}(x-m,n_1-m,1/2),x\ge m$ $(g_a(x)=0,x<m)$ is the corresponding probability density function. Therefore, $G_a(\cdot)$ has mean

$$\frac{n_1+m}{2}=n_0\frac{F(p_h(u_h-u_l))+F(p_hu_h+p_lu_l)}{2}$$

and variance

$$\frac{n_1-m}{4}=n_0\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{4}.$$

Furthermore, note that

$$\frac{\frac{n_1}{2}-\frac{n_1+m}{2}}{\sqrt{\frac{n_1-m}{4}}}=-\frac{m}{\sqrt{n_1-m}}=-\sqrt{n_0}\frac{F(p_h(u_h-u_l))}{\sqrt{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}}$$

Hence, by normal approximation, we have

$$G_a(\frac{n_1}{2})\sim\Phi(-\sqrt{n_0}\frac{F(p_h(u_h-u_l))}{\sqrt{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}})$$

where $\Phi(\cdot)$ is the cumulative distribution function for the standard normal distribution.

Approximation 2 We use $\frac{\exp(-x^2/2)}{\sqrt{2\pi}x}$ to approximate $\Phi(-x)$ when x is very large, the cumulative distribution function for the standard normal distribution. To see this, notice that

$$\begin{aligned}\Phi(-x) &= \int_{-\infty}^{-x} \frac{\exp(-y^2/2)}{\sqrt{2\pi}} dy \\ &= \int_{x}^{\infty} \frac{\exp(-y^2/2)}{\sqrt{2\pi}} dy \\ &\leq \int_{x}^{\infty} \frac{y}{x} \cdot \frac{\exp(-y^2/2)}{\sqrt{2\pi}} dy \\ &= \frac{\exp(-x^2/2)}{\sqrt{2\pi}x}\end{aligned}$$

where the inequality follows from $\frac{y}{x} \geq 1$ in the interval of integration. On the other hand, $\Phi(-x) \geq \frac{\exp(-x^2/2)}{\sqrt{2\pi}}(\frac{1}{x} - \frac{1}{x^3})$ as

$$\begin{aligned}\Phi(-x) &= \int_{x}^{\infty} \frac{\exp(-y^2/2)}{\sqrt{2\pi}} dy \\ &= -\frac{\exp(-y^2/2)}{\sqrt{2\pi}y}\bigg|_x^\infty - \int_x^\infty \frac{\exp(-y^2/2)}{\sqrt{2\pi}y^2} dy \\ &= -\frac{\exp(-y^2/2)}{\sqrt{2\pi}y}\bigg|_x^\infty + \frac{\exp(-y^2/2)}{\sqrt{2\pi}y^3}\bigg|_x^\infty + 3\int_x^\infty \frac{\exp(-y^2/2)}{\sqrt{2\pi}y^4} dy \\ &= \frac{\exp(-x^2/2)}{\sqrt{2\pi}}(\frac{1}{x} - \frac{1}{x^3}) + 3\int_x^\infty \frac{\exp(-y^2/2)}{\sqrt{2\pi}y^4} dy \\ &\geq \frac{\exp(-x^2/2)}{\sqrt{2\pi}}(\frac{1}{x} - \frac{1}{x^3})\end{aligned}$$

The second equality follows from integration by part (separating $\exp(-y^2/2)$ with $1/y$):

$$\begin{aligned}&(\frac{\exp(-y^2/2)}{\sqrt{2\pi}y})' \\ =&(\frac{\exp(-y^2/2)}{\sqrt{2\pi}} \cdot \frac{1}{y})' \\ =&(-y \cdot \frac{\exp(-y^2/2)}{\sqrt{2\pi}} \cdot \frac{1}{y} - \frac{1}{y^2}\frac{\exp(-y^2/2)}{\sqrt{2\pi}}) \\ =&-\frac{\exp(-y^2/2)}{\sqrt{2\pi}} - \frac{\exp(-y^2/2)}{\sqrt{2\pi}y^2}\end{aligned}$$

and $\int_x^\infty \frac{\exp(-y^2/2)}{\sqrt{2\pi}} dy = -\frac{\exp(-y^2/2)}{\sqrt{2\pi}y}\Big|_x^\infty - \int_x^\infty \frac{\exp(-y^2/2)}{\sqrt{2\pi}y^2} dy$. The third equality follows from performing integration by part on $\int_x^\infty \frac{\exp(-y^2/2)}{\sqrt{2\pi}y^2} dy$ (separating $\exp(-y^2/2)$ with $1/y^3$):

$$\begin{aligned}&(\frac{\exp(-y^2/2)}{\sqrt{2\pi}y^3})' \\ =&(\frac{\exp(-y^2/2)}{\sqrt{2\pi}} \cdot \frac{1}{y^3})' \\ =&(-y \cdot \frac{\exp(-y^2/2)}{\sqrt{2\pi}} \cdot \frac{1}{y^3} - \frac{3}{y^4}\frac{\exp(-y^2/2)}{\sqrt{2\pi}}) \\ =&-\frac{\exp(-y^2/2)}{\sqrt{2\pi}y^2} - \frac{3\exp(-y^2/2)}{\sqrt{2\pi}y^4}\end{aligned}$$

and $\int_x^\infty \frac{\exp(-y^2/2)}{\sqrt{2\pi}y^2} dy = -\frac{\exp(-y^2/2)}{\sqrt{2\pi}y^3}\Big|_x^\infty - 3\int_x^\infty \frac{\exp(-y^2/2)}{\sqrt{2\pi}y^4} dy$.

Thus, $\frac{\exp(-x^2/2)}{\sqrt{2\pi}}(\frac{1}{x}-\frac{1}{x^3}) \leq \Phi(-x) \leq \frac{\exp(-x^2/2)}{\sqrt{2\pi}}\frac{1}{x}$. These bounds of $\Phi(-x)$ have been noted by Durrett (2019), Theorem 1.2.6, page 13. When x is very large, $\frac{1}{x}-\frac{1}{x^3} \sim \frac{1}{x}$ and we have the desired approximation.

Let n be the total number of consumers in the two periods and n_0 be the number of consumers in the first period. By the base model, the expected sales in the first period is given by (again we drop the floor operator)

$$n_1 = n_0 F(p_h u_h + p_l u_l)$$

with $m = n_0 F(p_h(u_h - u_l))$. Recall that in the base model, we define π_1^r to be the belief that the bestseller product is of high value with sales ranking information,

$$\pi_1^r = p_h^2 + 2p_h p_l(1 - G_a(\frac{n_1}{2})) = p_h(1 + p_l - 2p_l G_a(\frac{n_1}{2}))$$

where $G_a(\cdot)$ is the sales distribution of the bestseller when the two products are of different values. As n_1 is a function of n_0, it follows that π_1^r is a function of n_0 and we denote it as $\pi_1^r(n_0)$.

Given $\pi_1^r(n_0)$, the expected sales in the second period is

$$(n - n_0)F(\pi_1^r(n_0)u_h + (1 - \pi_1^r(n_0))u_l)$$

and the total expected sales in the two periods is

$$\begin{aligned} S_T(n_0, n) :=& n_0 F(p_h u_h + p_l u_l) + (n - n_0)F(\pi_1^r(n_0)u_h + (1 - \pi_1^r(n_0))u_l) \\ =& nF(\pi_1^r(n_0)u_h + (1 - \pi_1^r(n_0))u_l) - n_0(F(\pi_1^r(n_0)u_h + (1 - \pi_1^r(n_0))u_l) - F(p_h u_h + p_l u_l)) \end{aligned}$$

Let $n_0^*(n) := \arg\max_{n_0} S_T(n_0, n)$. In what follows we characterize $n_0^*(n)$. We first show that $n_0^*(n)$ increases in n.

Proposition S.20. *Under Approximation 1, $n_0^*(n)$ increases in n, i.e., $n_0^*(n_i) \geq n_0^*(n_j), \forall n_i \geq n_j$.*

Proof of Proposition S.20: First we show that $F(\pi_1^r(x)u_h + (1 - \pi_1^r(x))u_l) \geq F(\pi_1^r(y)u_h + (1 - \pi_1^r(y))u_l)$ for $x \geq y$, which is equivalent to show $\pi_1^r(x) \geq \pi_1^r(y)$ when $x \geq y$ as F is a cumulative distribution function and hence non-decreasing. Using the first approximation, $G_a(\frac{n_1}{2})$ is approximated by $\Phi(-\sqrt{n_0}\frac{F(p_h(u_h-u_l))}{\sqrt{F(p_h u_h + p_l u_l)) - F(p_h(u_h-u_l))}})$. Clearly $\Phi(-\sqrt{n_0}\frac{F(p_h(u_h-u_l))}{\sqrt{F(p_h u_h + p_l u_l)) - F(p_h(u_h-u_l))}})$ decreases in n_0 and it follows that $G_a(\frac{n_1}{2})$ decreases in n_0. As

$$\pi_1^r(n_0) = p_h^2 + 2p_h p_l(1 - G_a(\frac{n_1}{2})) = p_h(1 + p_l - 2p_l G_a(\frac{n_1}{2}))$$

$\pi_1^r(n_0)$ increases in n_0 and $\pi_1^r(x) \geq \pi_1^r(y)$ if $x \geq y$.

We prove the proposition by contradiction. Suppose the proposition is not true, then there exist $n_i > n_j$ such that $n_0^*(n_i) < n_0^*(n_j)$. By the definition of $n_0^*(n_i)$,

$$\begin{aligned} S_T(n_0^*(n_i), n_i) =& n_0^*(n_i)F(p_h u_h + p_l u_l) + (n_i - n_0^*(n_i))F(\pi_1^r(n_0^*(n_i))u_h + (1 - \pi_1^r(n_0^*(n_i)))u_l) \\ =& n_0^*(n_i)F(p_h u_h + p_l u_l) + (n_j - n_0^*(n_i))F(\pi_1^r(n_0^*(n_i))u_h + (1 - \pi_1^r(n_0^*(n_i)))u_l) \\ &+ (n_i - n_j)F(\pi_1^r(n_0^*(n_i))u_h + (1 - \pi_1^r(n_0^*(n_i)))u_l) \\ >& S_T(n_0^*(n_j), n_i) \\ =& n_0^*(n_j)F(p_h u_h + p_l u_l) + (n_i - n_0^*(n_j))F(\pi_1^r(n_0^*(n_j))u_h + (1 - \pi_1^r(n_0^*(n_j)))u_l) \\ =& n_0^*(n_j)F(p_h u_h + p_l u_l) + (n_j - n_0^*(n_j))F(\pi_1^r(n_0^*(n_j))u_h + (1 - \pi_1^r(n_0^*(n_j)))u_l) \\ &+ (n_i - n_j)F(\pi_1^r(n_0^*(n_j))u_h + (1 - \pi_1^r(n_0^*(n_j)))u_l) \end{aligned}$$

As $n_0^*(n_j) > n_0^*(n_i)$, $F(\pi_1^r(n_0^*(n_j))u_h + (1-\pi_1^r(n_0^*(n_j)))u_l) \geq F(\pi_1^r(n_0^*(n_i))u_h + (1-\pi_1^r(n_0^*(n_i)))u_l)$ and

$$-(n_i - n_j)F(\pi_1^r(n_0^*(n_i))u_h + (1-\pi_1^r(n_0^*(n_i)))u_l) \geq -(n_i - n_j)F(\pi_1^r(n_0^*(n_j))u_h + (1-\pi_1^r(n_0^*(n_j)))u_l)$$

Adding the above two inequalities together, we have

$$\begin{aligned}
&n_0^*(n_i)F(p_h u_h + p_l u_l) + (n_j - n_0^*(n_i))F(\pi_1^r(n_0^*(n_i))u_h + (1-\pi_1^r(n_0^*(n_i)))u_l)\\
&\quad + (n_i - n_j)F(\pi_1^r(n_0^*(n_i))u_h + (1-\pi_1^r(n_0^*(n_i)))u_l) - (n_i - n_j)F(\pi_1^r(n_0^*(n_i))u_h + (1-\pi_1^r(n_0^*(n_i)))u_l)\\
\geq& n_0^*(n_j)F(p_h u_h + p_l u_l) + (n_j - n_0^*(n_j))F(\pi_1^r(n_0^*(n_j))u_h + (1-\pi_1^r(n_0^*(n_j)))u_l)\\
&\quad + (n_i - n_j)F(\pi_1^r(n_0^*(n_j))u_h + (1-\pi_1^r(n_0^*(n_j)))u_l) - (n_i - n_j)F(\pi_1^r(n_0^*(n_j))u_h + (1-\pi_1^r(n_0^*(n_j)))u_l)
\end{aligned}$$

which is

$$\begin{aligned}
&n_0^*(n_i)F(p_h u_h + p_l u_l) + (n_j - n_0^*(n_i))F(\pi_1^r(n_0^*(n_i))u_h + (1-\pi_1^r(n_0^*(n_i)))u_l)\\
>& n_0^*(n_j)F(p_h u_h + p_l u_l) + (n_j - n_0^*(n_j))F(\pi_1^r(n_0^*(n_j))u_h + (1-\pi_1^r(n_0^*(n_j)))u_l)
\end{aligned}$$

which is $S_T(n_0^*(n_i), n_j) > S_T(n_0^*(n_j), n_j)$. This contradicts the optimality of $n_0^*(n_j)$ and the proposition is proved. □

It is clear that the shape of F plays an important role here. For tractability, we consider a search cost distribution F that follows uniform distribution with support $[0, c]$ with $c > u_h$ (recall that we assume $\bar{s} > u_h$)). Then

$$\begin{aligned}
S_T(n_0, n) &= nF(\pi_1^r(n_0)u_h + (1-\pi_1^r(n_0))u_l) - n_0(F(\pi_1^r(n_0)u_h + (1-\pi_1^r(n_0))u_l) - F(p_h u_h + p_l u_l))\\
&= n\frac{\pi_1^r(n_0)u_h + (1-\pi_1^r(n_0))u_l}{c} - n_0\frac{\pi_1^r(n_0)(u_h - u_l) - p_h(u_h - u_l)}{c}\\
&= \frac{1}{c}[n(u_l + \pi_1^r(n_0)(u_h - u_l)) - n_0\pi_1^r(n_0)(u_h - u_l) + n_0 p_h(u_h - u_l)]\\
&= \frac{1}{c}[nu_l + n(u_h - u_l)p_h(1 + p_l - 2p_l G_a(\frac{n_1}{2})) - n_0(u_h - u_l)p_h p_l(1 - 2G_a(\frac{n_1}{2}))]
\end{aligned}$$

where the last equality follows from the definition of $\pi_1^r(n_0)$.

LEMMA S.21. *When F follows uniform distribution with support $[0, c]$ and n goes to ∞, $n_0^*(n) \geq 1$.*

Proof of Lemma S.21: First notice that F is uniform in $[0, c]$ and is therefore continuous. Let $\bar{n}_0 = \left\lceil \frac{1}{F(p_h(u_h - u_l))} \right\rceil$ and consider a total market size $n > \bar{n}_0$. Thus, when $n_0 = \bar{n}_0$, $m = \lfloor n_0 F(p_h(u_h - u_l)) \rfloor \geq 1$. We claim that $\pi_1^r(n_0) > p_h$ when $m \geq 1$. To see this, notice that when n_1 is odd, we have

$$\pi_1^r(n_0) = p_h^2 + 2p_h p_l(1 - G_a(\frac{n_1}{2})) = p_h(p_h + 2p_l(1 - G_a(\frac{n_1}{2}))) > p_h(p_h + p_l) = p_h$$

where the inequality follows from the fact $G_a(\frac{n_1}{2}) < 1/2$ when $m \geq 1$. To see this, recall that $G_a(x)$ is the probability that the number of success for the binomial distribution with number of trials $n_1 - m$ and success probability 0.5 is less than or equal to $x - m$. So $G_a(\frac{n_1+m}{2}) = 1/2$ when $n_1 + m$ is odd and $G_a(\frac{n_1+m}{2}) - g_a(\frac{n_1+m}{2})/2 = 1/2$ when $n_1 + m$ is even. If $n_1 + m$ is odd, $G_a(\frac{n_1}{2}) = G_a(\frac{n_1-1}{2}) < G_a(\frac{n_1+1}{2}) \leq G_a(\frac{n_1+m}{2}) = 1/2$, where the first equality follows as n_1 is odd . If $n_1 + m$ is even, $G_a(\frac{n_1+m}{2}) - g_a(\frac{n_1+m}{2})/2 \geq G_a(\frac{n_1+1}{2}) - g_a(\frac{n_1+1}{2})/2 = G_a(\frac{n_1-1}{2}) + g_a(\frac{n_1+1}{2})/2 > G_a(\frac{n_1-1}{2}) = G_a(\frac{n_1}{2})$. The first inequality follows from $G_a(x) - g_a(x)/2$ strictly increases in x as

$G_a(x+1) - g_a(x+1)/2 - (G_a(x) - g_a(x)/2) = g_a(x+1)/2 + g_a(x)/2 > 0$ since $G_a(x+1) = G_a(x) + g_a(x+1)$. The first equality follows from $G_a(\frac{n_1+1}{2}) = G_a(\frac{n_1-1}{2}) + g_a(\frac{n_1+1}{2})$. The last equality follows as n_1 is odd.

Similarly, when n_1 is even,

$$\pi_1^r(n_0) = p_h^2 + 2p_h p_l(1 - G_a(\frac{n_1}{2}) + g_a(\frac{n_1}{2})/2) = p_h(p_h + 2p_l(1 - G_a(\frac{n_1}{2}) + g_a(\frac{n_1}{2})/2) > p_h(p_h + p_l) = p_h$$

where the inequalities follow from $G_a(\frac{n_1}{2}) - g_a(\frac{n_1}{2})/2 < 1/2$ when $m \geq 1$, which we prove below. Again recall that $G_a(\frac{n_1+m}{2}) = 1/2$ when $n_1 + m$ is odd and $G_a(\frac{n_1+m}{2}) - g_a(\frac{n_1+m}{2})/2 = 1/2$ when $n_1 + m$ is even. If $n_1 + m$ is odd, $G_a(\frac{n_1}{2}) - g_a(\frac{n_1}{2})/2 < G_a(\frac{n_1}{2}) \leq G_a(\frac{n_1+m}{2}) = 1/2$. If $n_1 + m$ is even, $G_a(\frac{n_1}{2}) - g_a(\frac{n_1}{2})/2 < G_a(\frac{n_1+m}{2}) - g_a(\frac{n_1+m}{2})/2 = 1/2$ as $G_a(x) - g_a(x)/2$ strictly increases in x. Therefore $\pi_1^r(n_0) > p_h$ when $m \geq 1$.

Hence $\pi_1^r(\bar{n}_0) > p_h$ and it follows that $F(\pi_1^r(\bar{n}_0)u_h + (1 - \pi_1^r(\bar{n}_0))u_l) > F(p_h u_h + p_l u_l)$. Note that

$$\begin{aligned}
&S_T(\bar{n}_0, n) - S_T(0, n) \\
=& nF(\pi_1^r(\bar{n}_0)u_h + (1 - \pi_1^r(\bar{n}_0))u_l) - n_0(F(\pi_1^r(\bar{n}_0)u_h + (1 - \pi_1^r(\bar{n}_0))u_l) - F(p_h u_h + p_l u_l)) - nF(p_h u_h + p_l u_l) \\
=& (n - \bar{n}_0)(F(\pi_1^r(\bar{n}_0)u_h + (1 - \pi_1^r(\bar{n}_0))u_l) - F(p_h u_h + p_l u_l)) \\
>& 0
\end{aligned}$$

Hence $n_0 = 0$ can not be optimal and $n_0^*(n) \geq 1$ when n is sufficiently large. □

Proposition S.21. *When F follows uniform distribution with support $[0, c]$ and n goes to ∞, $n_0^*(n) \sim O(\log n)$ under Approximations 1 and 2.*

Proof of Proposition S.21: By Lemma S.21, we focus on $n_0 \geq 1$. Notice that

$$\begin{aligned}
S_T(n_0, n) &= \frac{1}{c}[nu_l + n(u_h - u_l)p_h(1 + p_l - 2p_l G_a(\frac{n_1}{2})) - n_0(u_h - u_l)p_h p_l(1 - 2G_a(\frac{n_1}{2}))] \\
&= \frac{1}{c}[nu_l + n(u_h - u_l)p_h(1 + p_l)] + \frac{1}{c}[-n(u_h - u_l)p_h \cdot 2p_l G_a(\frac{n_1}{2}) - n_0(u_h - u_l)p_h p_l(1 - 2G_a(\frac{n_1}{2}))]
\end{aligned}$$

As our objective is $n_0^*(n)$, we focus on terms that involve n_0, i.e.,

$$\begin{aligned}
&\frac{1}{c}[-n(u_h - u_l)p_h \cdot 2p_l G_a(\frac{n_1}{2}) - n_0(u_h - u_l)p_h p_l(1 - 2G_a(\frac{n_1}{2}))] \\
=&\frac{1}{c}(u_h - u_l)p_h p_l[-2nG_a(\frac{n_1}{2}) - n_0(1 - 2G_a(\frac{n_1}{2}))]
\end{aligned}$$

We factor out the multiplier $\frac{1}{c}(u_h - u_l)p_h p_l$ and consider

$$T(n_0, n) = -2nG_a(\frac{n_1}{2}) - n_0(1 - 2G_a(\frac{n_1}{2}))$$

Note that $T(n_0, n)$ and $S_T(n_0, n)$ are maximized at the same $n_0^*(n)$ for all n.

Using the first approximation and the fact that F follows uniform distribution with support $[0, c]$, $G_a(\frac{n_1}{2})$ is approximated by

$$\Phi(-\sqrt{n_0}\frac{F(p_h(u_h - u_l)}{\sqrt{F(p_h u_h + p_l u_l)) - F(p_h(u_h - u_l))}}) = \Phi(-\frac{\sqrt{n_0}p_h(u_h - u_l)}{\sqrt{cu_l}})$$

Next, we apply the second approximation

$$\Phi(-x) \sim \frac{\exp(-x^2/2)}{\sqrt{2\pi}x}$$

$$G_a(\frac{n_1}{2}) \sim \Phi(-\frac{\sqrt{n_0}p_h(u_h-u_l)}{\sqrt{cu_l}}) \sim \frac{\exp(-p_h^2(u_h-u_l)^2 n_0/2cu_l)}{p_h(u_h-u_l)\sqrt{2\pi n_0/cu_l}}$$

and

$$\begin{aligned} T(n_0,n) &= -2nG_a(\frac{n_1}{2}) - n_0(1-2G_a(\frac{n_1}{2})) \\ &\sim -2n\cdot\frac{\exp(-p_h^2(u_h-u_l)^2 n_0/2cu_l)}{p_h(u_h-u_l)\sqrt{2\pi n_0/cu_l}} - n_0 + 2n_0\cdot\frac{\exp(-p_h^2(u_h-u_l)^2 n_0/2cu_l)}{p_h(u_h-u_l)\sqrt{2\pi n_0/cu_l}} \end{aligned}$$

We consider a continuous relaxation of $T(n_0,n)$ where we place n_0 by a continuous variable x and denote $a := p_h^2(u_h-u_l)^2/cu_l$:

$$Q(x) = (2x-2n)\frac{\exp(-ax/2)}{\sqrt{2\pi ax}} - x$$

By Lemma S.21, when n is large we must have $n_0^*(n) \geq 1$ and we focus on $x\in[1,n]$. We have

$$\begin{aligned} Q'(x) &= 2\frac{\exp(-ax/2)}{\sqrt{2\pi ax}} - (2x-2n)\frac{\exp(-ax/2)(a\sqrt{2\pi ax}+\sqrt{2\pi a/x})}{4\pi ax} - 1 \\ &= \frac{1}{\sqrt{2\pi a}}[2\frac{\exp(-ax/2)}{\sqrt{x}} - (x-n)\exp(-ax/2)(a\sqrt{x}+\sqrt{1/x})] - 1 \end{aligned}$$

When n goes to infinity,

$$Q'(1) = \frac{1}{\sqrt{2\pi a}}[2\exp(-a/2) + (n-1)\exp(-a/2)(a+1)] - 1 > 0$$

as $Q'(1)$ increases in n and

$$Q'(n) = \frac{1}{\sqrt{2\pi a}}2\frac{\exp(-an/2)}{\sqrt{n}} - 1 < 0$$

as $\frac{1}{\sqrt{2\pi a}}2\frac{\exp(-an/2)}{\sqrt{n}}$ goes to 0 when n goes to infinity. Therefore these exist at least one $x\in[1,n]$ such that $Q'(x)=0$ and we must have $Q'(x^*(n))=0$ where $x^*(n)$ maximizes $Q(x)$. Hence,

$$\exp(-ax^*(n)/2)[\frac{2}{\sqrt{x^*(n)}} - (x^*(n)-n)(a\sqrt{x^*(n)}+\sqrt{1/x^*(n)})] = \sqrt{2\pi a}$$

which is

$$\frac{2}{\sqrt{x^*(n)}} + (n-x^*(n))(a\sqrt{x^*(n)}+\sqrt{1/x^*(n)}) = \sqrt{2\pi a}\cdot\exp(ax^*(n)/2) \tag{S.36}$$

First notice that as n goes to infinity, $x^*(n)$ must also goes to infinity as otherwise LHS goes to infinity and RHS is finite and they can not equal. Next notice that $\frac{x^*(n)}{n}\to 0$ when $n\to\infty$. If this is not true, then $\frac{x^*(n)}{n}\geq c_0$ for infinitely many n for some constant $c_0>0$ (if for all $c>0$, $\frac{x^*(n)}{n}\geq c$ for only finitely many n, then for all value $c>0$, we can find a value $N(c)$ such that $\frac{x^*(n)}{n}<c$ when $n>N(c)$, which implies $\frac{x^*(n)}{n}\to 0$ when $n\to\infty$), which is

$$\frac{2}{\sqrt{x^*(n)}} + (n-x^*(n))(a\sqrt{x^*(n)}+\sqrt{1/x^*(n)}) = \sqrt{2\pi a}\cdot\exp(ax^*(n)/2) \geq \sqrt{2\pi a}\cdot\exp(ac_0 n/2)$$

for infinitely many n. Notice that $\sqrt{2\pi a}\cdot\exp(ac_0n/2)$ grows exponentially in n while $\frac{2}{\sqrt{x^*(n)}} + (n - x^*(n))(a\sqrt{x^*(n)}+\sqrt{1/x^*(n)}) \leq \frac{2}{\sqrt{x^*(n)}} + n(a\sqrt{x^*(n)}+\sqrt{1/x^*(n)}) \leq 2n + n(an+n)$ (the last inequality follows from $1/x^*(n)\leq x^*(n)\leq n$ as $x^*(n)\geq 1$) grows with rate at most $O(n^2)$. Therefore, it is impossible that

$$\frac{2}{\sqrt{x^*(n)}} + (n-x^*(n))(a\sqrt{x^*(n)}+\sqrt{1/x^*(n)}) = \sqrt{2\pi a}\cdot\exp(ax^*(n)/2) \geq \sqrt{2\pi a}\cdot\exp(ac_0 n/2)$$

for infinitely many n, which shows that $\frac{x^*(n)}{n} \to 0$ when $n \to \infty$. Now we claim that

$$\frac{2}{\sqrt{x^*(n)}} + (n - x^*(n))(a\sqrt{x^*(n)} + \sqrt{1/x^*(n)}) = O(n\sqrt{x^*(n)})$$

First notice that

$$\begin{aligned}
&\frac{2}{\sqrt{x^*(n)}} + (n - x^*(n))(a\sqrt{x^*(n)} + \sqrt{1/x^*(n)}) \\
\le&\frac{2}{\sqrt{x^*(n)}} + n(a\sqrt{x^*(n)} + \sqrt{1/x^*(n)}) \\
\le&2n + n(a\sqrt{x^*(n)} + \sqrt{x^*(n)})
\end{aligned}$$

where the last inequality follows from $1/x^*(n) \le x^*(n) \le n$ as $x^*(n) \ge 1$. As $n + n(a\sqrt{x^*(n)} + \sqrt{x^*(n)})$ grows with rate $O(n\sqrt{x^*(n)})$, $\frac{2}{\sqrt{x^*(n)}} + (n - x^*(n))(a\sqrt{x^*(n)} + \sqrt{1/x^*(n)})$ grows with rate at most $O(n\sqrt{x^*(n)})$. On the other hand,

$$\begin{aligned}
&\frac{2}{\sqrt{x^*(n)}} + (n - x^*(n))(a\sqrt{x^*(n)} + \sqrt{1/x^*(n)}) \\
\ge&n(a\sqrt{x^*(n)} + \sqrt{1/x^*(n)}) - x^*(n)(a\sqrt{x^*(n)} + \sqrt{1/x^*(n)})
\end{aligned}$$

As $\frac{x^*(n)}{n} \to 0$ when $n \to \infty$, there exist a large value N_1 such that if $n > N_1$, $x^*(n) \le n/2$. So when $n > N_1$,

$$\begin{aligned}
&\frac{2}{\sqrt{x^*(n)}} + (n - x^*(n))(a\sqrt{x^*(n)} + \sqrt{1/x^*(n)}) \\
\ge&n(a\sqrt{x^*(n)} + \sqrt{1/x^*(n)}) - x^*(n)(a\sqrt{x^*(n)} + \sqrt{1/x^*(n)}) \\
\ge&n(a\sqrt{x^*(n)} + \sqrt{1/x^*(n)})/2 \\
\ge&n(a\sqrt{x^*(n)}/2
\end{aligned}$$

So $\frac{2}{\sqrt{x^*(n)}} + (n - x^*(n))(a\sqrt{x^*(n)} + \sqrt{1/x^*(n)})$ grows with rate at least $O(n\sqrt{x^*(n)})$ and therefore

$$\frac{2}{\sqrt{x^*(n)}} + (n - x^*(n))(a\sqrt{x^*(n)} + \sqrt{1/x^*(n)}) = O(n\sqrt{x^*(n)})$$

Next we claim that $x^*(n) = O(\log n)$, which is $c_1 < \frac{x^*(n)}{\log n} < c_2$ when n goes to infinity for some constants c_1, c_2. In particular, we show that $\frac{2}{a} < \frac{x^*(n)}{\log n} < \frac{4}{a}$ when n goes to infinity, which is equivalent to show that $\frac{x^*(n)}{\log n} > \frac{4}{a}$ and $\frac{x^*(n)}{\log n} < \frac{2}{a}$ for finitely many value of n.

Suppose this is not true, then there are two possible cases. The first case is $\frac{x^*(n)}{\log n} \ge \frac{4}{a}$ for infinitely many n. Notice that $O(n\sqrt{x^*(n)}) < O(n^2)$ as we have shown $\frac{x^*(n)}{n} \to 0$ when $n \to \infty$. So

$$\frac{2}{\sqrt{x^*(n)}} + (n - x^*(n))(a\sqrt{x^*(n)} + \sqrt{1/x^*(n)})$$

grows with rate lower than $O(n^2)$. On the other hand, when $\frac{x^*(n)}{\log n} \ge \frac{4}{a}$, $\exp(ax^*(n)/2) \ge n^2$ and

$$\sqrt{2\pi a} \cdot \exp(ax^*(n)/2) \ge \sqrt{2\pi a} n^2$$

grows with rate at least $O(n^2)$. Since $\frac{x^*(n)}{\log n} \ge \frac{4}{a}$ for infinitely many n,

$$\frac{2}{\sqrt{x^*(n)}} + (n - x^*(n))(a\sqrt{x^*(n)} + \sqrt{1/x^*(n)}) = \sqrt{2\pi a} \cdot \exp(ax^*(n)/2) \ge \sqrt{2\pi a} n^2$$

for infinitely many n (the equality follows from equation (S.36)), which contradicts with the finding above that $\frac{2}{\sqrt{x^*(n)}} + (n - x^*(n))(a\sqrt{x^*(n)} + \sqrt{1/x^*(n)})$ grows with rate lower than $O(n^2)$.

The second case is $\frac{x^*(n)}{\log n} \leq \frac{2}{a}$ for infinitely many n. Notice that $O(n\sqrt{x^*(n)}) > O(n)$ as we have shown $x^*(n)$ goes to infinity when n goes to infinity. Therefore

$$\frac{2}{\sqrt{x^*(n)}} + (n - x^*(n))(a\sqrt{x^*(n)} + \sqrt{1/x^*(n)})$$

grows with rate higher than $O(n)$. On the other hand, when $\frac{x^*(n)}{\log n} \leq \frac{2}{a}$, $\exp(ax^*(n)/2) \leq n$ and

$$\sqrt{2\pi a} \cdot \exp(ax^*(n)/2) \leq \sqrt{2\pi a} n$$

grows with rate at most $O(n)$. Since $\frac{x^*(n)}{\log n} \leq \frac{2}{a}$ for infinitely many n,

$$\frac{2}{\sqrt{x^*(n)}} + (n - x^*(n))(a\sqrt{x^*(n)} + \sqrt{1/x^*(n)}) = \sqrt{2\pi a} \cdot \exp(ax^*(n)/2) \leq \sqrt{2\pi a} n$$

for infinitely many n (the equality follows from equation (S.36)), which contradicts with the finding above that $\frac{2}{\sqrt{x^*(n)}} + (n - x^*(n))(a\sqrt{x^*(n)} + \sqrt{1/x^*(n)})$ grows with rate higher than $O(n)$.

Therefore we must have $x^*(n) = O(\log n)$ and therefore $n_0^*(n) = O(\log n)$. □

SL. Consumers' Strategic Waiting for Bestseller Information

Thus far we have assumed that consumers make their search and purchasing decisions upon their arrival at the platform. In practice some sophisticated consumers may strategically postpone their decisions until after bestseller information is available. Such strategic waiting behavior is captured in Yu et al. (2016) for consumer-generated review information. In practice firms may face a mixture of strategic and non-strategic (i.e., myopic) consumers (Kremer et al. 2017) and empirical studies (e.g., Li et al. 2014 and Mak et al. 2014) find that a considerable proportion of consumers are non-strategic. Thus, we assume non-strategic consumers in the base model to disentangle the effects of bestseller information on sales from those of strategic consumer waiting. In this section we extend the base model to incorporate strategic consumer waiting for bestseller information.

First note that, because of consumers' strategic behavior, the market sizes in the two periods, n_0 and n_2, are endogenously determined by consumers' choices. Hence, we assume that all of the n consumers arrive at the beginning of the first period and decide simultaneously on whether to make their search and purchasing decisions in the first period or postpone their decisions to the beginning of the second period. Assume that consumers discount second-period utility at a rate of δ. Furthermore, assume $\delta \leq \frac{1}{2}$.

SL.1. Social Learning Through Sales Ranking

Consider a consumer with search cost s. His or her expected surplus without decision postponement (i.e., in period 1) is

$$U_1(s) = \max[0, p_h u_h + p_l \max(u_l, p_h u_h + (1 - p_h)u_l - s) - s]$$

Under **ranking information**, his or her expected surplus by postponing his or her decisions to period 2 is

$$U_2^r(s; \pi_1^r, \pi_2^r) = \delta \max[0, \pi_1^r u_h + (1 - \pi_1^r) \max(u_l, \pi_2^r u_h + (1 - \pi_2^r)u_l - s) - s]$$

where the posteriors π_1^r and π_2^r are endogenously determined by all the consumers' choice on whether or not to postpone decisions. By the analysis in the base model, for given n_1 and m, $\pi_1^r \geq p_h \geq \pi_2^r$.

Given π_1^r and π_2^r,

$$U_1(s) - U_2^r(s;\pi_1^r,\pi_2^r)$$
$$= \begin{cases} (p_h u_h + (1-p_h)(p_h u_h + (1-p_h)u_l - s) - s) \\ -\delta(\pi_1^r u_h + (1-\pi_1^r)(\pi_2^r u_h + (1-\pi_2^r)u_l - s) - s), & \text{if } s \le \pi_2^r(u_h - u_l) \\ (p_h u_h + (1-p_h)(p_h u_h + (1-p_h)u_l - s) - s) - \delta(\pi_1^r u_h + (1-\pi_1^r)u_l - s), & \text{if } \pi_2^r(u_h-u_l) < s \le p_h(u_h-u_l) \\ (p_h u_h + (1-p_h)u_l - s) - \delta(\pi_1^r u_h + (1-\pi_1^r)u_l - s), & \text{if } p_h(u_h-u_l) < s \le p_h u_h + (1-p_h)u_l \\ 0 - \delta(\pi_1^r u_h + (1-\pi_1^r)u_l - s) < 0, & \text{if } p_h u_h + (1-p_h)u_l < s \le \pi_1^r u_h + (1-\pi_1^r)u_l \\ 0 & \text{if } s > \pi_1^r u_h + (1-\pi_1^r)u_l \end{cases}$$

$$\frac{d[U_1(s) - U_2^r(s;\pi_1^r,\pi_2^r)]}{ds}$$
$$= \begin{cases} -p_l - 1 - \delta(-(1-\pi_1^r) - 1) = -1 + \delta + \delta(1-\pi_1^r) - p_l < 0, & \text{if } s \le \pi_2^r(u_h - u_l) \\ -p_l - 1 - \delta(-1) = -p_l - 1 + \delta < 0, & \text{if } \pi_2^r(u_h-u_l) < s \le p_h(u_h-u_l) \\ -1 - \delta(-1) < 0, & \text{if } p_h(u_h-u_l) < s \le p_h u_h + (1-p_h)u_l \\ 0 - \delta(-1) > 0, & \text{if } p_h u_h + (1-p_h)u_l < s \le \pi_1^r u_h + (1-\pi_1^r)u_l \\ 0 & \text{if } s > \pi_1^r u_h + (1-\pi_1^r)u_l \end{cases}$$

Thus, $\frac{d[U_1(s) - U_2^r(s;\pi_1^r,\pi_2^r)]}{ds} < 0$ for $s \le p_h u_h + (1-p_h)u_l$. Also note that $U_1(s) - U_2^r(s;\pi_1^r,\pi_2^r) < 0$ if $p_h u_h + (1-p_h)u_l < s \le \pi_1^r u_h + (1-\pi_1^r)u_l$. Hence, for given π_1^r and π_2^r, there exists a unique $\hat{s} \in [0, p_h u_h + (1-p_h)u_l]$ such that $U_1(s) > U_2^r(s;\pi_1^r,\pi_2^r)$ when $s < \hat{s}$ and $U_1(s) \le U_2^r(s;\pi_1^r,\pi_2^r)$ when $s \ge \hat{s}$. That is, consumers with lower search costs are less likely to postpone decisions, as they expect a higher utility from the decisions and thus, if they wait, they would suffer a greater loss of utility due to time discounting. This structural result is similar to the one in Yu et al. (2016), where consumers with higher valuations are less inclined to wait for reviews.

Now, we shall endogenize π_1^r and π_2^r. By the analysis of the consumer behavior for given π_1^r and π_2^r, we will focus on a threshold-type equilibrium: there exists a threshold $\hat{s}^r$, such that consumers with search cost s less than $\hat{s}^r$ make their searching and purchasing decisions in period 1 and those with search cost s greater than or equal to $\hat{s}^r$ postpone their decisions to period 2. In particular, $\hat{s}^r \in [0, p_h u_h + (1-p_h)u_l]$, implying that all the consumers making decisions in period 1 will make a purchase. To explicitly recognize the dependence of π_1^r and π_2^r on $\hat{s}^r$, we now denote them as $\pi_1^r(\hat{s}^r)$ and $\pi_2^r(\hat{s}^r)$. As follows, we define the variables as functions of the threshold search cost s, for the sake of generality. Specifically, let $n_0(s) = n_1(s) = \lfloor nF(s) \rfloor$, $m(s) = \lfloor nF(\min(s, p_h(u_h-u_l))) \rfloor$, and for $x \in [0, n_1]$,

$$g_s(x;s) := Binomial(x, n_1(s), 1/2),$$
$$g_a(x;s) := Binomial(x - m(s), n_1(s) - m(s), 1/2) \text{ if } x \ge m(s), \text{ and } 0 \text{ if } x < m(s),$$

where $Binomial(x,y,p)$ is the probability that among y independent trials, x of them succeed, where the probability of success is p. Let $G_s(x;s)$ and $G_a(x;s)$ be the cumulative distribution functions corresponding to $g_s(x;s)$ and $g_a(x;s)$, respectively. Let $\bar{G}_s(x;s) := 1 - G_s(x;s)$ and $\bar{G}_a(x;s) := 1 - G_a(x;s)$.

When $n_1(s)$ is odd,

$$\pi_1^r(s) = p_h^2 + 2p_h p_l(1 - G_a(\frac{n_1(s)}{2};s));$$
$$\pi_2^r(s) = \frac{p_h G_a(\frac{n_1(s)}{2};s)}{p_h G_a(\frac{n_1(s)}{2};s) + p_l \bar{G}_s(\frac{n_1(s)}{2};s)}$$

When $n_1(s)$ is even,

$$\pi_1^r(s) = p_h^2 + 2p_hp_l(1 - G_a(\frac{n_1(s)}{2}; s) + g_a(\frac{n_1(s)}{2}; s)/2);$$
$$\pi_2^r(s) = \frac{p_h(G_a(\frac{n_1(s)}{2}; s) - g_a(\frac{n_1(s)}{2}; s)/2)}{p_h(G_a(\frac{n_1(s)}{2}; s) - g_a(\frac{n_1(s)}{2}; s)/2) + p_l\bar{G}_s(\frac{n_1(s)}{2}; s)}$$

With posteriors endogenously determined by consumers' strategic waiting behavior, we now define an equilibrium among consumers. First, we define a threshold function for given $\pi_1^r(s)$ and $\pi_2^r(s)$:

$$\mathcal{S}_r(s) = \min\{s' \in [0, p_hu_h + (1-p_h)u_l] : U_1(s') \leq U_2^r(s'; \pi_1^r(s), \pi_2^r(s))\} \tag{S.37}$$

Note that $U_1(s') - U_2^r(s'; \pi_1^r(s), \pi_2^r(s))$ is continuous in s'. It is negative at $s = p_hu_h + (1-p_h)u_l$. Hence, the set in S.37 is well defined.

Now, an equilibrium among consumers is sustained if there exists a value $\hat{s}^r \in [0, p_hu_h + (1-p_h)u_l]$ such that

$$\hat{s}^r = \mathcal{S}_r(\hat{s}^r)$$

If there are multiple equilibria, we pick the one with the smallest $\hat{s}^r$.

The expected total sales of two periods as a function of the marginal consumer's search cost s and that in equilibrium are denoted by

$$\xi_r(s) = nF(\pi_1^r(s)u_h + (1-\pi_1^r(s))u_l), \text{ and}$$
$$\xi_r^* = nF(\pi_1^r(\hat{s}^r)u_h + (1-\pi_1^r(\hat{s}^r))u_l),$$

respectively.

Lemma S.22. *Let $\underline{s} := \min\{s : nF(s) \geq 1\}$. If $p_h(u_h - u_l) \leq \underline{s}$, $\hat{s}^r = p_hu_h + p_lu_l$; otherwise, $\hat{s}^r \geq \underline{s}$ if an equilibrium is sustained.*

The critical number, $\underline{s}$, defined in Lemma S.22 is the threshold of marginal consumer's search cost below which no consumer performs the first search in the first period, i.e., $n_0(s) = n_1(s) = 0$. Notice that when $s \leq \underline{s}$, no sales information is available and, thus, social learning does not occur, i.e., $\pi_1^r(s) = \pi_2^r(s) = p_h$.

By Lemma S.22, when $p_h(u_h - u_l) > \underline{s}$, it suffices to consider $s \in [\underline{s}, p_hu_h + p_lu_l]$ in our search for the equilibrium threshold $\hat{s}^r$.

Proposition S.22. *If $p_h(u_h - u_l) > \underline{s}$, both $\pi_1^r(s)$ and $\xi_r(s)$ are non-increasing in $s \in [\underline{s}, p_hu_h + p_lu_l]$.*

The intuition behind Proposition S.22 is as follows: when s is small, $m = n_1$, i.e., everyone who buys in the first period is informed. As s increases, as m is bounded by $nF(p_h(u_h - u_l))$, the proportion of informed buyers among all the first-period buyers becomes smaller. That is, the bestseller information is increasingly noisy and thus $\pi_1^r(s)$ decreases, leading to the second-period sales shrinking in s due to a reduction in both the market size and the proportion of consumers who conduct a first search in the second period. Since the total sales in the two periods is determined by $\pi_1^r(s)$, the total sales declines as s increases, as illustrated in Figure S.2.

Initially, as the early sales volume increases from zero, the first-period purchases start from the consumers with the lowest search cost. Before the early sales volume reaches a certain threshold (i.e., the number of consumers who are willing to do a second search under the prior belief), the informativeness of the bestseller information remains unchanged, as all the early buyers search twice and make "informed" purchases. Nevertheless, as the early sales volume continues to increase, the number of informed buyers remains unchanged (as it is a fixed portion of the total consumer pool) while that of uninformed buyers keeps rising. The more consumers buying early, the more noisy the sales information is, and the less consumers are willing to wait. This is opposite to review information, under which waiting incentive is reinforced by a higher early sales as the informativeness of review information increases in review volume.

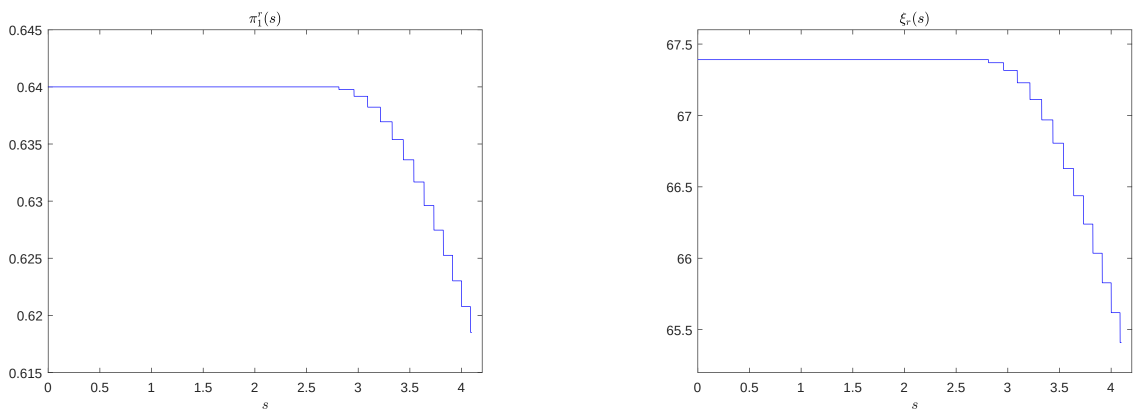

Figure S.2 $\pi_1^r(s)$ **and** $\xi_r(s)$ **under ranking information:** $F(x) = (\alpha + (1-\alpha)\Phi((x-\mu)/\sigma))\mathbb{I}(x \geq 0)$ **with** $\mu = 4.5$, $\sigma = 1.5$ **and** $\alpha = 0.08$, $n = 100$, $\delta = 0.3$, $u_h = 6.5$, $u_l = 2.5$, $p_h = 0.4$.

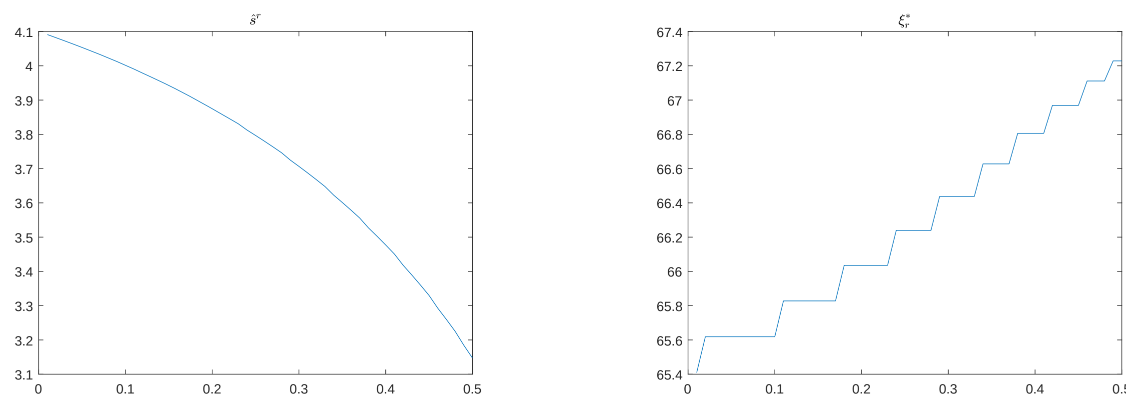

Figure S.3 $\hat{s}^r$ **and** ξ_r^* **under ranking information:** $F(x) = (\alpha + (1-\alpha)\Phi((x-\mu)/\sigma))\mathbb{I}(x \geq 0)$ **with** $\mu = 4.5$, $\sigma = 1.5$ **and** $\alpha = 0.08$, $n = 100$, $u_h = 6.5$, $u_l = 2.5$, $p_h = 0.4$.

Now, when the early sales volume is endogenous, as δ increases, the volume of early buyers decreases and, yet, a higher portion of them are informed. Thus, sales ranking becomes more informative and the total sales of the two periods increases as it is determined by the first-search belief in the second period (Figure S.3). This implies that, under social learning through bestseller information, consumers' strategic waiting can be advantageous to the platform. This is because, when consumers are strategic, those with lowest search cost self-select to purchase early and make informed purchasing decision after searching both products. Hence, the more patient the consumers are (i.e., a higher δ), the higher the proportion of informed purchases in first-period sales, and the more informative the sales ranking. This, thus, promotes the second-period sales and benefits the platform.

In the meanwhile, since $\pi_1^r(s) \geq p_h$ for any given s, $\pi_1^r(\hat{s}^r) \geq p_h$. Thus, same as in the base model, ranking information increases total expected sales for the platform.

SL.2. Social Learning Through Sales Volume

We follow a similar approach to formulate the consumers' problem under **volume information**. Under volume information, the focal consumer's expected surplus by postponing his or her decisions to period 2 is

$$U_2^v(s;\pi_1^v(\cdot),\pi_2^v(\cdot)) = E_x[\delta \max[0, \pi_1^v(x)u_h + (1-\pi_1^v(x))\max(u_l, \pi_2^v(x)u_h + (1-\pi_2^v(x))u_l - s) - s]]$$

where the posteriors $\pi_1^v(\cdot)$ and $\pi_2^v(\cdot)$ are endogenously determined by all the consumers' choice on whether or not to postpone decisions. By the analysis in the base model, for given n_1 and m, $\pi_2^v(x) \leq p_h$ but $\pi_1^v(x)$ may not always be greater than p_h.

First, for given $\pi_1^v(\cdot)$ and $\pi_2^v(\cdot)$, we have

$$\begin{aligned}
&U_1(s) - U_2^v(s;\pi_1^v(\cdot),\pi_2^v(\cdot)) \\
&= \begin{cases} (p_h u_h + (1-p_h)(p_h u_h + (1-p_h)u_l - s) - s) \\ -E_x[\delta \max[0, \pi_1^v(x)u_h + (1-\pi_1^v(x))\max(u_l, \pi_2^v(x)u_h + (1-\pi_2^v(x))u_l - s) - s]], & \text{if } s \leq p_h(u_h - u_l) \\ (p_h u_h + (1-p_h)u_l - s) \\ -E_x[\delta \max[0, \pi_1^v(x)u_h + (1-\pi_1^v(x))\max(u_l, \pi_2^v(x)u_h + (1-\pi_2^v(x))u_l - s) - s]], & \text{if } p_h(u_h - u_l) < s \leq p_h u_h + (1-p_h)u_l \\ -E_x[\delta \max[0, \pi_1^v(x)u_h + (1-\pi_1^v(x))\max(u_l, \pi_2^v(x)u_h + (1-\pi_2^v(x))u_l - s) - s]], & \text{if } s > p_h u_h + (1-p_h)u_l \end{cases} \\
&\frac{d[U_1(s) - U_2^v(s;\pi_1^v(\cdot),\pi_2^v(\cdot))]}{ds} \\
&= \begin{cases} -p_l - 1 + \delta E_x[(2-\pi_1^v(x))1_{s \leq \pi_2^v(x)(u_h - u_l)} + 1_{\pi_2^v(x)(u_h-u_l) < s \leq \pi_1^v(x)u_h + (1-\pi_1^v(x))u_l}], & \text{if } s \leq p_h(u_h - u_l) \\ -1 + \delta E_x[(2-\pi_1^v(x))1_{s \leq \pi_2^v(x)(u_h - u_l)} + 1_{\pi_2^v(x)(u_h-u_l) < s \leq \pi_1^v(x)u_h + (1-\pi_1^v(x))u_l}], & \text{if } p_h(u_h - u_l) < s \leq p_h u_h + (1-p_h)u_l \\ \delta E_x[(2-\pi_1^v(x))1_{s \leq \pi_2^v(x)(u_h - u_l)} + 1_{\pi_2^v(x)(u_h-u_l) < s \leq \pi_1^v(x)u_h + (1-\pi_1^v(x))u_l}] & \text{if } s > p_h u_h + (1-p_h)u_l \end{cases}
\end{aligned}$$

Since $E_x[(2-\pi_1^v(x))1_{s \leq \pi_2^v(x)(u_h - u_l)} + 1_{\pi_2^v(x)(u_h-u_l) < s \leq \pi_1^v(x)u_h + (1-\pi_1^v(x))u_l}] \leq 2$ and $\delta \leq \frac{1}{2}$, we have: $\frac{d[U_1(s) - U_2^v(s;\pi_1^v(\cdot),\pi_2^v(\cdot))]}{ds} \leq 0$ if $s \leq p_h u_h + (1-p_h)u_l$ and $U_1(s) - U_2^v(s;\pi_1^v(\cdot),\pi_2^v(\cdot)) \leq 0$ if $s > p_h u_h + (1-p_h)u_l$. Thus, consumers with lower search costs are less likely to postpone decisions.

Now we endogenize the posterior beliefs and again focus on a threshold-type equilibrium: there exists a threshold $\hat{s}^v$, such that consumers with search cost s less than $\hat{s}^v$ make their searching and purchasing decisions in period 1 and those with search cost s greater than or equal to $\hat{s}^v$ postpone their decisions to period 2. In particular, $\hat{s}^v \in [0, p_h u_h + (1-p_h)u_l]$, implying that all the consumers making decisions in period 1 will make a purchase. To explicitly recognize the dependence of $\pi_1^v(\cdot)$ and $\pi_2^v(\cdot)$ on $\hat{s}^v$, we now denote them as $\pi_1^v(\cdot;\hat{s}^v)$ and $\pi_2^v(\cdot;\hat{s}^v)$. Similar to those for the ranking information, we denote the variables as functions of s, and refer to s as the threshold or marginal search cost. Specifically, under the definitions of $n_1(s)$, $m(s)$, $g_s(x;s)$, $g_a(x;s)$, $G_a(x;s)$, $G_s(x;s)$, $\bar{G}_a(x;s)$, and $\bar{G}_s(x;s)$ (see the analysis of ranking information), we have: for $x \in [\frac{n_1(s)}{2}, n_1(s)]$,

$$\begin{aligned}
\pi_1^v(x;s) &= \frac{p_h^2 g_s(x;s) + p_h p_l g_a(x;s)}{p_h^2 g_s(x;s) + p_h p_l g_a(x;s) + p_h p_l g_a(n_1(s)-x) + p_l^2 g_s(x;s)}; \\
\pi_2^v(x;s) &= \frac{p_h g_a(n_1(s)-x;s)}{p_h g_a(n_1(s)-x;s) + p_l g_s(x;s)}
\end{aligned}$$

With posteriors endogenously determined by consumers' strategic waiting behavior, we now define an equilibrium among consumers. First, we define a threshold function for given $\pi_1^v(\cdot;s)$ and $\pi_2^v(\cdot;s)$:

$$\mathcal{S}_v(s) = \min\{s' \in [0, p_h u_h + (1-p_h)u_l] : U_1(s') \le U_2^v(s'; \pi_1^v(\cdot;s), \pi_2^v(\cdot;s))\}$$

Note: $U_1(s') - U_2^v(s'; \pi_1^v(\cdot;s), \pi_2^v(\cdot;s))$ is continuous in s'. It is negative at $s = p_h u_h + (1-p_h)u_l$. Hence, this set is well defined.

Now, an equilibrium among consumers is sustained if there exists a value $\hat{s}^v \in [0, p_h u_h + (1-p_h)u_l]$ such that

$$\hat{s}^v = \mathcal{S}_v(\hat{s}^v)$$

If there are multiple equilibria, we pick the one with the smallest $\hat{s}^v$.

The expected total sales of two periods as a function of the marginal consumer's search cost s and that in equilibrium are denoted by

$$\xi_v(s) = nE_x[F(\pi_1^v(x;s)u_h + (1-\pi_1^v(x;s))u_l)], \text{and}$$
$$\xi_v^* = nE_x[F(\pi_1^v(x;\hat{s}^v)u_h + (1-\pi_1^v(x;\hat{s}^v))u_l)],$$

respectively.

We first note that, different from the result under ranking information that $\xi_r(s)$ is non-increasing in s, the expected total sales under volume information, $\xi_v(s)$, may not be monotonic in s (see right panel of Figure S.4). This is driven by the fact that, compared to sales ranking, sale volume provides additional information about bestseller utility, which leads to extra variability in the posterior belief, $\pi^v(X;s)$, such that the posterior, despite of its mean declining in s (see left panel of Figure S.4), may not be first-order stochastically decreasing in s. Consequently, how $\xi_v(s)$ varies in s is determined by both the distribution of $\pi^v(X;s)$ and the search-cost distribution $F(\cdot)$.

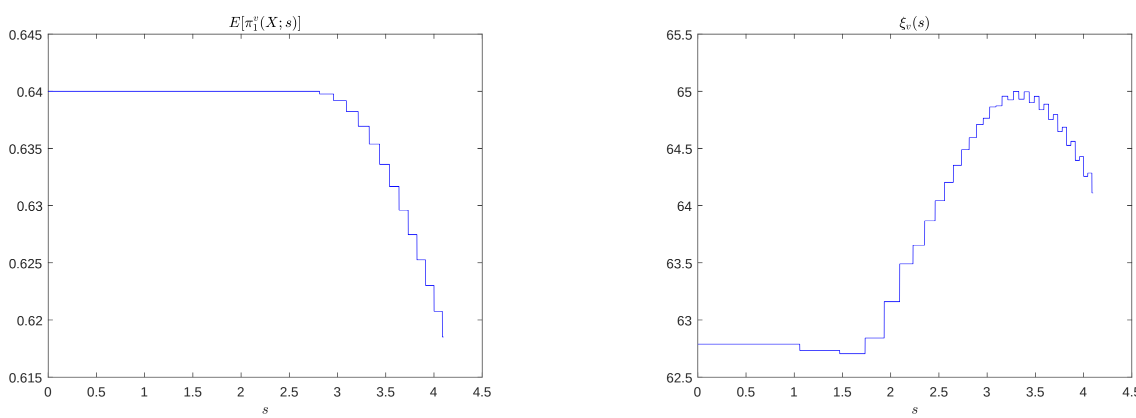

Figure S.4 $E[\pi_1^v(X;s)]$ **and** $\xi_v(s)$ **under volume information:** $F(x) = (\alpha + (1-\alpha)\Phi((x-\mu)/\sigma))\mathbb{I}(x \ge 0)$ **with** $\mu = 4.5$, $\sigma = 1.5$ **and** $\alpha = 0.08$, $n = 100$, $\delta = 0.3$, $u_h = 6.5$, $u_l = 2.5$, $p_h = 0.4$.

To elaborate and exemplify the effects of increasing s on the posterior $\pi^v(X;s)$, consider the special case of $s \le p_h(u_h - u_l)$. For s in this range, by the definition of $m(s)$ and $n_1(s)$ (recall: $m(s) = \lfloor nF(\min(s, p_h(u_h - u_l)))\rfloor$

and $n_1(s) = \lfloor nF(s) \rfloor$), we have $m(s) = n_1(s)$, implying that all the early buyers have performed two searches. In such a case, under ranking information, $\pi_1^r(s)$ equals to $p_h^2 + 2p_h p_l$ because, conditional on different product utilities, all the consumers purchase the high-type product and thus the bestseller is of high type for sure, i.e., $G_a(\frac{n_1(s)}{2}; s) = 0$. Note that this posterior belief is independent of the common value of $m(s)$ and $n_1(s)$, and holds as long as $s \le p_h(u_h - u_l)$.

Now, given $s \le p_h(u_h - u_l)$, consider the distribution of the posterior under volume information, $\pi_1^v(x; s)$. We find that the posterior follows a binary distribution and may become more "variable" when s increases in the range. To see the rationale, first note that, for any $x < n_1(s)$, $\pi_1^v(x; s) = p_h^2/(p_h^2 + p_l^2)$ because $m(s) = n_1(s)$ and the bestseller's sales lower than $m(s)$ implies that the product utilities are identical to each other (as otherwise, the bestseller's sales is at least $m(s)$). Hence, $\pi_1^v(x; s)$ is binary and equals to either $p_h^2/(p_h^2 + p_l^2)$ or $\pi_1^v(n_1(s); s)$.

To see the effect of increasing s on $\pi_1^v(x; s)$, first note that the mean of $\pi_1^v(x; s)$ equals to $\pi_1^r(s)$, which is always $p_h^2 + 2p_h p_l$ when $s \le p_h(u_h - u_l)$, as discussed above. On the other hand, $\pi_1^v(n_1(s); s)$ increases as a higher s leads to a higher common value of $m(s)$ and $n_1(s)$. This is because: conditional on different product utilities, the probability of the bestseller's sales equal to $m(s)$ is one; conditional on identical product utilities, all the early buyers randomly choose between the two products and the probability that all of them pick the same product becomes smaller as $n_1(s)$ increases. Hence, as $n_1(s)$ increases, upon observing all the early buyers choosing a same product, consumers believe that the product utilities are more likely to differ than to coincide. Furthermore, since all the early buyers search twice (as $m(s) = n_1(s)$), conditional on different product utilities, the bestseller is of high type for sure. Thus, as the common value of $m_1(s)$ and $n_1(s)$ increases, upon observing the early buyers' unanimous choice of a product, consumers' belief about the bestseller being a high type increases. Also note that, as $n_1(s)$ increases, the event $x = n_1(s)$ is less likely to occur, as noted earlier. Hence, as s increases, the common value of $m(s)$ and $n_1(s)$ increases and the posterior distribution $\pi_1^v(x; s)$, while preserving its mean, assigns a higher value for $\pi_1^v(n_1(s); s)$ and yet a lower probability for taking this value. It can be shown that this change leads to a higher variance of $\pi_1^v(x; s)$.

The impact of such a change in the posterior distribution on the total expected sales $\xi_v(s)$ is ambiguous: even though $\xi_v(s) = nE_x[F(\pi_1^v(x; s)u_h + (1 - \pi_1^v(x; s))u_l)]$ is an increasing function of π_1^v, how it varies in response to a higher variability of $\pi_1^v(x; s)$ depends on the search-cost distribution $F(\cdot)$. Two detailed examples are presented in Table S.10, whereby $\xi_v(s)$ (weakly) decreases in s in the first example ($\mu = 4.5, \sigma = 1.5$) and (weakly) increases in s in the second example ($\mu = 7, \sigma = 3$).

Table S.10 Impact of s on posterior $\pi_1^v(x; s)$ and $\xi_v(s)$, where x denotes the bestseller's sales volume: $p_h = 0.4, F(x) = (\alpha + (1 - \alpha)\Phi((x - \mu)/\sigma))\mathbb{I}(x \ge 0)$ with $\alpha = 0.08$, $u_h = 6.5$, $u_l = 2.5$

			$\mu = 4.5, \sigma = 1.5$			$\mu = 7, \sigma = 3$		
s	$\pi_1^r(s)$	$\pi_1^v(x; s)$ when $x < n_1(s)$	$\pi_1^v(n_1(s); s)$	$\Pr(x = n_1(s))$	$\xi_v(s)$	$\pi_1^v(n_1(s); s)$	$\Pr(x = n_1(s))$	$\xi_v(s)$
0.3	0.64	0.3077	0.9942	0.4841	62.7890	0.9971	0.4820	33.7425
0.6	0.64	0.3077	0.9942	0.4841	62.7890	0.9971	0.4820	33.7425
0.9	0.64	0.3077	0.9942	0.4841	62.7890	0.9971	0.4820	33.7425
1.2	0.64	0.3077	0.9971	0.4820	62.7334	0.9985	0.4810	33.7488
1.5	0.64	0.3077	0.9985	0.4810	62.7055	0.9993	0.4805	33.7519

Now we endogenize the marginal search cost s and examine the equilibrium under volume information. We observe that, similar to that under ranking information, more consumers postpone their purchases to the second period as they become more patient, i.e., $\hat{s}^v$ decreases in δ (as illustrated in the left panel of Figure S.5). On the other hand, due to the aforementioned effects of increasing s on $\xi_v(s)$, the total expected sales in equilibrium, ξ_v^*, may not be monotonic in δ (see Figure S.5, right panel). Nevertheless, consistent with the finding under ranking information, ξ_v^* may increase in δ, implying that strategic consumer waiting can benefit the platform under social learning through bestseller information.

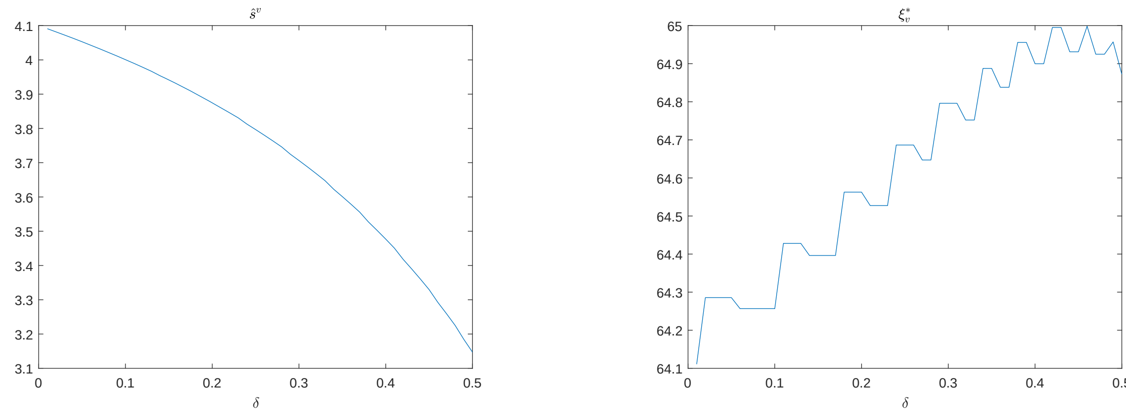

Figure S.5 $\hat{s}^v$ **and** ξ_v^* **under volume information:** $F(x) = (\alpha + (1-\alpha)\Phi((x-\mu)/\sigma))\mathbb{I}(x \geq 0)$ **with** $\mu = 4.5$**,** $\sigma = 1.5$ **and** $\alpha = 0.08$**,** $n = 100$**,** $u_h = 6.5$**,** $u_l = 2.5$**,** $p_h = 0.4$**.**

To examine the impact of sales information granularity, we numerically compare the equilibria under ranking and volume information. We observe that, compared to ranking information, volume information induces more consumers to postpone purchases, i.e., $\hat{s}_r > \hat{s}_v$, as illustrated in the left panels of Figure S.6. This is because consumers expect a higher surplus in the second period under sales information of a higher granularity. It echoes well with the finding in the base model that consumers benefit more from the sales information as the information becomes finer. This result also implies that the platform can provide bestseller information of a lower granularity to dampen consumers' incentives to wait for bestseller information (if the platform finds it desirable to dampen the incentives). In addition, similar to that in the base model, ranking information may or may not lead to higher total sales than volume information, as exemplified in the right panels of Figure S.6.

We also note that several other key results of the base model remain robust. For example, for given s, it can be shown that the posterior under volume information, $\pi_1^v(x;s)$, is a mean-preserving spread of that under ranking information, i.e., $\pi_1^r(s)$. Likewise, the reinforcement-by-homogeneity effect continues to be valid for given s and, due to this effect, no information can outperform volume information in terms of total expected sales.

SL.3. Appendix

Proof of Lemma S.22 Note that, for $s \leq \underline{s}$, $n_1(s) = 0$, implying $\pi_1^r(s) = \pi_2^r(s) = p_h$. Thus, consider three cases:

• if $p_h u_h + p_l u_l \leq \underline{s}$, fixing any $s \leq p_h u_h + p_l u_l$, $U_1(s') > U_2^r(s'; \pi_1^r(s), \pi_2^r(s))$ for all $s' < p_h u_h + p_l u_l$ and $U_1(s') = U_2^r(s'; \pi_1^r(s), \pi_2^r(s))$ for $s' = p_h u_h + p_l u_l$. Thus, $\hat{s}^r = p_h u_h + p_l u_l$.

• If $p_h u_h + p_l u_l > \underline{s} \geq p_h(u_h - u_l)$, fixing any $s \leq \underline{s}$, $U_1(s') > U_2^r(s'; \pi_1^r(s), \pi_2^r(s))$ for all $s' < \underline{s}$. Hence, $\hat{s}^r \geq \underline{s}$. Furthermore, fixing any $\underline{s} < s \leq p_h u_h + p_l u_l$, $n_1(s) > 0$ and yet $m(s) = 0$ since $p_h(u_h - u_l) < \underline{s}$. Thus, $U_1(s') > U_2^r(s'; \pi_1^r(s), \pi_2^r(s))$ for all $s' < p_h u_h + p_l u_l$ and and $U_1(s') = U_2^r(s'; \pi_1^r(s), \pi_2^r(s))$ for $s' = p_h u_h + p_l u_l$. Thus, $\hat{s}^r =$

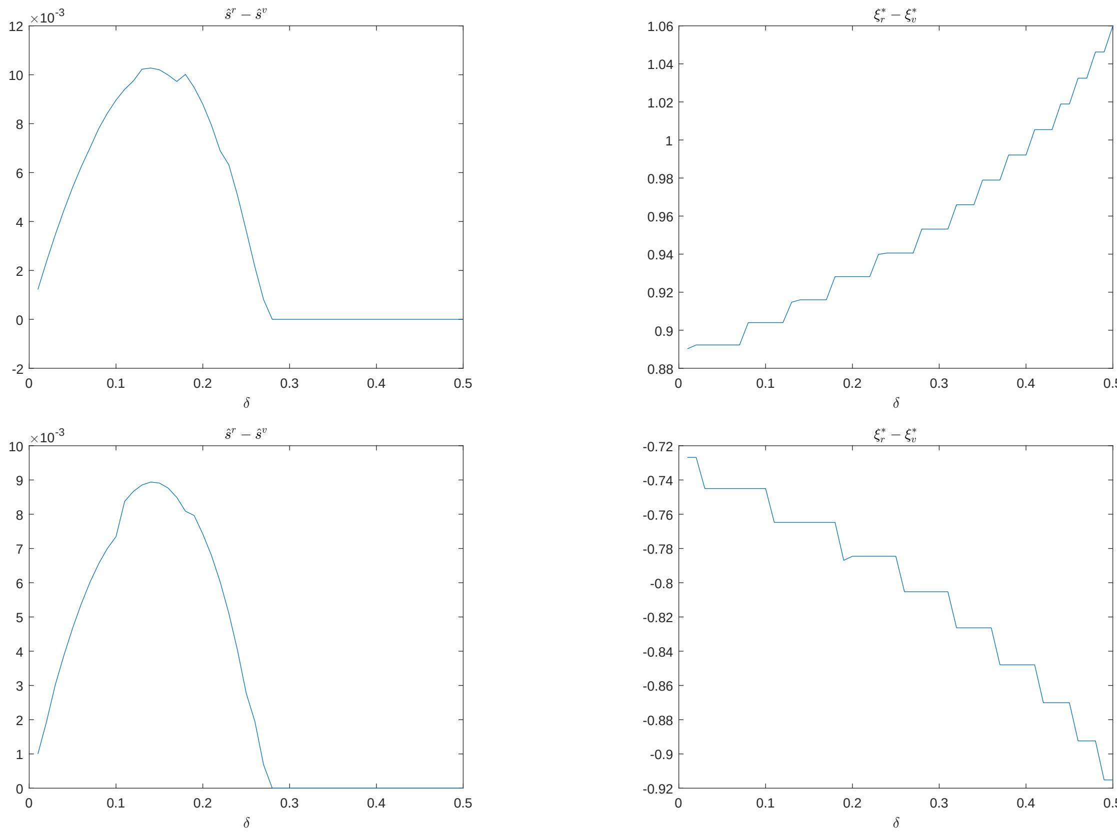

Figure S.6 **Comparison between ranking and volume in equilibrium:** $n = 100$, $u_h = 6.5$, $u_l = 2.5$, $p_h = 0.4$, $F(x) = (\alpha + (1-\alpha)\Phi((x-\mu)/\sigma))\mathbb{I}(x \geq 0)$ **with** $\alpha = 0.08$**, and the top panels correspond to** $\mu = 4.5$ **and** $\sigma = 2.5$**, and the bottom panels correspond to** $\mu = 6$ **and** $\sigma = 3$**.**

$p_h u_h + p_l u_l$.

- If $\underline{s} < p_h(u_h - u_l)$, fixing any $s \leq \underline{s}$, $U_1(s') > U_2^r(s'; \pi_1^r(s), \pi_2^r(s))$ for all $s' < \underline{s}$. Hence, $\hat{s}^r \geq \underline{s}$. □

Proof of Proposition S.22 Given $\underline{s} < p_h(u_h - u_l)$, we consider two cases, (i) $\underline{s} \leq s \leq p_h(u_h - u_l)$ and (ii) $p_h(u_h - u_l) < s \leq p_h u_h + p_l u_l$, to prove the proposition. Within this proof we omit the argument s in functions, whenever no confusion arises.

(i) $\underline{s} \leq s \leq p_h(u_h - u_l)$: In this case, $m(s) = n_1(s) \geq 1$ and all the consumers who perform the first search in the first period are willing to perform the second search if the first search reveals a low type. Therefore in this case consumers always purchase a high type whenever there is one. Therefore $g_a(n_1) = 1$ and $g_a(x) = 0$ for all $x < n_1$. Hence, when n_1 is odd, $\pi_1^r = p_h^2 + 2p_h p_l(1 - G_a(n_1/2))$ and $G_a(n_1/2) = 0$, implying $\pi_1^r = p_h^2 + 2p_h p_l$. Similarly, when n_1 is even, $\pi_1^r = p_h^2 + 2p_h p_l(1 - G_a(n_1/2) + g_a(n_1/2)/2)$ (recall that when the two products have equal sales, each product is ranked first with equal probability) and $G_a(n_1/2) + g_a(n_1/2)/2 = 0$, implying $\pi_1^r = p_h^2 + 2p_h p_l$. Thus, π_1^r is independent of s for $\underline{s} \leq s \leq p_h(u_h - u_l)$.

(ii) $p_h(u_h - u_l) < s \leq p_h u_h + p_l u_l$: In this case, $m(s) = \lfloor nF(p_h(u_h - u_l))\rfloor$ and $n_1(s) = \lfloor nF(s)\rfloor$. When s increases, $m(s)$ remains unchanged while $n_1(s)$ increases. Therefore, in this case $\pi_1^r(s)$ depends on s only through $n_1(s)$ and, thus, it suffices to consider π_1^r as a function of n_1 (within this proof we shall re-write it as $\pi_1^r(n_1)$) and show that it decreases in n_1. We prove this by considering the change in π_1^r when n_1 increases from t to $t+1$.

To explicitly recognize the dependence of the functions $\bar{G}_a(\cdot)$ and $g_a(\cdot)$ on n_1, within this proof we write them as $\bar{G}_a(\cdot|n_1)$ and $g_a(\cdot|n_1)$. Recall $g_a(i|n_1) = \frac{1}{2^{n_1-m}}\frac{(n_1-m)!}{(i-m)!(n_1-i)!}$, for $i = m, ..., n_1$. Consider two subcases:
(ii-a) t is even: $\pi_1^r(t) = p_h^2 + 2p_hp_l(1 - G_a(\frac{t}{2}|t) + g_a(\frac{t}{2}|t)/2)$ and $\pi_1^r(t+1) = p_h^2 + 2p_hp_l(1 - G_a(\frac{t+1}{2}|t+1))$. By definition,

$$
\begin{aligned}
\bar{G}_a((t+1)/2|t+1) =& \sum_{i=t/2+1}^{t+1} g_a(i|t+1) \\
=& \sum_{i=t/2+1}^{t+1} \frac{1}{2^{t+1-m}}\frac{(t+1-m)!}{(i-m)!(t+1-i)!} \\
=& \sum_{i=t/2+1}^{t} \frac{1}{2^{t+1-m}}\Big(\frac{(t-m)!}{(i-1-m)!(t-i+1)!} + \frac{(t-m)!}{(i-m)!(t-i)!}\Big) + \frac{1}{2^{t+1-m}} \\
=& \frac{1}{2^{t+1-m}}\Big(\frac{(t-m)!}{(t/2-m)!(t/2)!} + \frac{(t-m)!}{(t/2+1-m)!(t/2-1)!}\Big) \\
&+ \frac{1}{2^{t+1-m}}\Big(\frac{(t-m)!}{(t/2+1-m)!(t/2-1)!} + \frac{(t-m)!}{(t/2+2-m)!(t/2-2)!}\Big) + ... \\
&+ \frac{1}{2^{t+1-m}}(t-m+1) + \frac{1}{2^{t+1-m}} \\
=& \frac{1}{2^{t+1-m}}\frac{(t-m)!}{(t/2-m)!(t/2)!} + \frac{1}{2^{t-m}}\sum_{i=t/2+1}^{t-1}\frac{(t-m)!}{(i-m)!(t-i)!} + 2\cdot\frac{1}{2^{t+1-m}} \\
=& \frac{1}{2^{t-m}}\frac{(t-m)!}{(t/2-m)!(t/2)!}\cdot\frac{1}{2} + \frac{1}{2^{t-m}}\sum_{i=t/2+1}^{t}\frac{(t-m)!}{(i-m)!(t-i)!} \\
=& g_a(t/2|t)/2 + \sum_{i=t/2+1}^{t} g_a(i|t) \\
=& g_a(t/2|t)/2 + \bar{G}_a(t/2|t)
\end{aligned}
$$

where we use the identity:

$$
\frac{t!}{(i-1)!(t-i+1)!} + \frac{t!}{i!(t-i)!} = \frac{(t+1)!}{i!(t+1-i)!}
$$

for $i = 1, 2, ..., t$. Hence, $\pi_1^r(t) = \pi_1^r(t+1)$.
(ii-b) t is odd: $\pi_1^r(t) = p_h^2 + 2p_hp_l(1 - G_a(\frac{t}{2}|t))$ and $\pi_1^r(t+1) = p_h^2 + 2p_hp_l(1 - G_a(\frac{t+1}{2}|t+1) + g_a(\frac{t+1}{2}|t+1)/2)$. By definition,

$$
\begin{aligned}
& g_a((t+1)/2|t+1)/2 + \bar{G}_a((t+1)/2|t+1) \\
=& g_a((t+1)/2|t+1)/2 + \sum_{i=(t+1)/2+1}^{t+1} g_a(i|t+1) \\
=& \frac{1}{2^{t+1-m}}\frac{(t+1-m)!}{((t+1)/2-m)!((t+1)/2)!}\cdot\frac{1}{2} + \sum_{i=(t+1)/2+1}^{t+1}\frac{1}{2^{t+1-m}}\frac{(t+1-m)!}{(i-m)!(t+1-i)!} \\
=& \frac{1}{2^{t+1-m}}\frac{(t+1-m)!}{((t+1)/2-m)!((t+1)/2)!}\cdot\frac{1}{2} + \sum_{i=(t+1)/2+1}^{t}\frac{1}{2^{t+1-m}}\Big(\frac{(t-m)!}{(i-1-m)!(t-i+1)!} + \frac{(t-m)!}{(i-m)!(t-i)!}\Big) + \frac{1}{2^{t+1-m}} \\
=& \frac{1}{2^{t+1-m}}\frac{(t+1-m)!}{((t+1)/2-m)!((t+1)/2)!}\cdot\frac{1}{2} + \frac{1}{2^{t+1-m}}\Big(\frac{(t-m)!}{((t+1)/2-m)!((t-1)/2)!} + \frac{(t-m)!}{((t+1)/2+1-m)!((t-1)/2-1)!}\Big) \\
&+ \frac{1}{2^{t+1-m}}\Big(\frac{(t-m)!}{((t+1)/2+1-m)!((t-1)/2-1)!} + \frac{(t-m)!}{((t+1)/2+2-m)!((t-1)/2-2)!}\Big) + ... + \frac{1}{2^{t+1-m}}(t-m+1) + \frac{1}{2^{t+1-m}}
\end{aligned}
$$

$$
\begin{aligned}
=&\frac{1}{2^{t+1-m}}\frac{(t+1-m)!}{((t+1)/2-m)!((t+1)/2)!}\cdot\frac{1}{2}+\frac{1}{2^{t+1-m}}\frac{(t-m)!}{((t+1)/2-m)!((t-1)/2)!}\\
&+\sum_{i=(t+1)/2+1}^{t-1}\frac{1}{2^{t-m}}\frac{(t-m)!}{(i-m)!(t-i)!}+2\cdot\frac{1}{2^{t+1-m}}\\
=&\frac{1}{2^{t+1-m}}\frac{(t+1-m)!}{((t+1)/2-m)!((t+1)/2)!}\cdot\frac{1}{2}+\frac{1}{2^{t+1-m}}\frac{(t-m)!}{((t+1)/2-m)!((t-1)/2)!}+\sum_{i=(t+1)/2+1}^{t}\frac{1}{2^{t-m}}\frac{(t-m)!}{(i-m)!(t-i)!}\\
=&\frac{1}{2^{t+1-m}}\frac{(t+1-m)!}{((t+1)/2-m)!((t+1)/2)!}\cdot\frac{1}{2}+\frac{1}{2^{t+1-m}}\frac{(t-m)!}{((t+1)/2-m)!((t-1)/2)!}\\
&+\bar{G}_a(t/2|t)-\frac{1}{2^{t-m}}\frac{(t-m)!}{((t+1)/2-m)!((t-1)/2)!}\\
=&\bar{G}_a(t/2|t)+\frac{1}{2^{t+1-m}}\frac{(t+1-m)!}{((t+1)/2-m)!((t+1)/2)!}\cdot\frac{1}{2}-\frac{1}{2^{t+1-m}}\frac{(t-m)!}{((t+1)/2-m)!((t-1)/2)!}\\
=&\bar{G}_a(t/2|t)+\frac{1}{2^{t+1-m}}[\frac{(t-m)!}{((t-1)/2-m)!((t+1)/2)!}+\frac{(t-m)!}{((t+1)/2-m)!((t-1)/2)!}]\cdot\frac{1}{2}\\
&-\frac{1}{2^{t+1-m}}\frac{(t-m)!}{((t+1)/2-m)!((t-1)/2)!}\\
=&\bar{G}_a(t/2|t)+\frac{1}{2^{t+1-m}}[\frac{(t-m)!}{((t-1)/2-m)!((t+1)/2)!}-\frac{(t-m)!}{((t+1)/2-m)!((t-1)/2)!}]\cdot\frac{1}{2}\\
\le&\bar{G}_a(t/2|t)
\end{aligned}
$$

where the last inequality follows from the fact that $\frac{(t-m)!}{((t-1)/2-m)!((t+1)/2)!}-\frac{(t-m)!}{((t+1)/2-m)!((t-1)/2)!}\le 0$. To see this fact, notice that $\frac{t!}{i!(t-i)!}$ increases in i for $i\le t/2$ and $(t-1)/2-m\le(t+1)/2-m\le(t-m)/2$. Hence, $\pi_1^r(t)\ge\pi_1^r(t+1)$. Thus, it implies that both $\pi_1^r(s)$ and $\xi_r(s)$ are non-increasing in s. □

SM. Personalized Information Provision

In the base model we assume that sales information provision is public. In particular, all the consumers access to the same type of information if the platform offers it. This is in line with platforms' current practices as noted in the introduction section. Nevertheless, web-analytics techniques have allowed platforms to gain a better understanding of individual consumers' searching and purchasing behavior, and may soon create opportunities for personalized information provision. In this extension we explore the situation when the platform is aware of each late consumer's search cost (e.g., by analyzing a consumer's product search and browsing history on the platform) and can customize the type of sales information provision (ranking or volume) accordingly.

Recall that we show in Proposition 11 that the optimal personalized provision strategy is characterized by a simple cutoff on a consumer's search cost. The proposition is stated and explained in the main body. It is replicated below for readers' convenience.

Proposition 11 *(Personalized information provision)*
It is optimal for the platform to provide ranking (volume) information to a consumer if her search cost, s, is less than or equal to (greater than) $\pi_1^r u_h+(1-\pi_1^r)u_l$.

In terms of welfare implication of the personalization strategy, the platform never gets worse off by customizing its information provision, which represents a higher level of operational flexibility. Nevertheless, the following proposition reveals that information personalization may hurt the consumers.

Proposition S.23. *(Impact of personalized information provision on consumer welfare)* *If ranking (resp. volume) information is optimal to the platform in the base model, aggregate consumer welfare is higher (resp. lower) due to information personalization.*

This result is implied by our previous analysis of consumer surplus. As we showed in §5.2, consumer surplus increases in the information level. If the optimal information level in absence of personalization is ranking (resp. volume), since customized information provision may result in more (resp. less) information offered to some consumers, aggregate consumer surplus is enhanced (resp. reduced) by information personalization. Therefore, a platform may, in its own interest, individualize its information provision, which may be detrimental to consumers.

SM.1. Appendix

Proof of Proposition 11 The first part of the proposition follows from the fact that, for any consumer, ranking information always increases the chance of purchase in the second period compared to no information. For the second part of the proposition, notice that consumers with search cost no greater than $\pi_1^r u_h + (1-\pi_1^r)u_l$ purchase for sure under ranking information and may not purchase under volume information, and thus it is optimal for the platform to offer them ranking information. For consumers with search cost greater than $\pi_1^r u_h + (1-\pi_1^r)u_l$, they do not purchase under ranking information but might purchase under volume information if the sales difference between the two products is significant. Thus, it is optimal to offer them volume information. □

Proof of Proposition S.23 This proposition follows from the fact that volume information results in higher consumer surplus than ranking information. □

SN. Consumer Heterogeneity in Reservation Utility

In the base model we have focused on consumer heterogeneity in search cost. Search cost is an important determinant of consumers' purchasing decisions as most consumers search before making a purchase. Anderson (2022) note that a majority (81%) of shoppers perform product research online before purchasing. Good search and navigation features are considered as a key factor of online shopping experience by nearly 50% of consumers (Brophy 2023). Moreover, search can be very costly to consumers in practice: as estimated by Koulayev (2014), the median of search cost is around \$10 per page of search results, and can be as high as \$30 per page for a subset of consumers. In line with these estimations, a Millward-Brown study in 2014 finds that 70% of Amazon users never browse beyond the first page of search results (Derakhshan et al. 2022). Thus, search cost plays a critical role in determining the set of products that consumers examine before purchasing.

We now incorporate additional consumer heterogeneity to enrich the model and results. Recall that in the base model we assume that consumers are homogeneous in terms of their reservation utility (i.e., the utility of the no-purchase alternative). In particular, the reservation utility, denoted by u_0, is lower than the low value of a product. In this extension we relax these assumptions and consider the setting where the reservation utility is heterogeneous across consumers and may be higher than the low product value. Specifically, consumers are privately informed about their own reservation utility, which follows a general distribution on $[\underline{u}_0, \bar{u}_0]$, with probability density function $h(\cdot)$ and cumulative distribution function $H(\cdot)$. That is, similar to the assumption on search-cost distribution, among any m randomly-selected consumers, the number of consumers whose reservation utility is less than x equals to $mH(x)$. Here we assume $\underline{u}_0 < u_l < u_h < \bar{u}_0$, implying that the reservation utility is higher than the low product value for some consumers. Note that in this extension consumers are heterogenous in both search cost and reservation utility. Under this two-dimensional consumer heterogeneity, we start our analysis in §SN.1 from the same setting as in the base model, where search is required for purchase. Subsequently, we relax this assumption in §SN.2 to examine the scenario where consumers are allowed to purchase without search. All other settings remain the same as in the base model.

SN.1. Search Required for Purchase

We start from the case that a consumer needs to search a product before purchasing it. This is aligned with the assumption in the base model and allows us to disentangle the effects of (heterogenous) reservation utility by comparing the findings in this subsection with those in the base model.

SN.1.1. First Period We start the analysis from the first period. The following lemma characterizes the optimal search and purchasing strategies for first-period consumers.

LEMMA S.23. *In the first period, the optimal search and purchasing strategies of a consumer with reservation utility u_0 and search cost s are as follows:*

(i) If $u_0 \geq u_h$: the consumer neither searches nor purchases.

(ii) If $u_l \leq u_0 < u_h$: the consumer performs the first search if $s \leq p_h(u_h - u_0)$ and leaves without search or purchase if $s > p_h(u_h - u_0)$. If the first search reveals a high-value product, the consumer purchases the searched product. If, however, the first search reveals a low-value product, the consumer performs the second search. Subsequently, the consumer makes a purchase (and buys a high-value product) if the second search reveals a high-type product and leaves without purchase otherwise. In particular, the consumer with $u_l \leq u_0 < u_h$ never purchases a low-value product.

(iii) If $u_0 < u_l$: the consumer performs the first search if $s \leq p_h u_h + p_l u_l - u_0$ and leaves without search or purchase if $s > p_h u_h + p_l u_l - u_0$. If the first search reveals a high-value product, the consumer purchases the searched product. If, however, the first search reveals a low-value product, the consumer performs the second search if $s \leq p_h(u_h - u_l)$ and buys the searched product otherwise. The consumer purchases a low-value product (after search) if the second search also reveals a low-type product and buys the high-value product otherwise.

Lemma S.23 shows how a first-period consumer's optimal search and purchasing strategies vary as her search cost and reservation utility fall in different ranges. In particular, those consumers with reservation utility higher than u_l would never purchase a low-value product and, thus, some of these consumers search and leave without purchase if both products are found to be of low value, as in case (ii) of Lemma S.23. This is in contrast to the result in the base model, whereby all of the first-period consumers purchase after search under the assumptions of homogenous u_0 and $u_0 < u_l$. As we shall elaborate below, this change in some consumers' purchasing strategy has important implication for the sales distribution in the first period.

Based on Lemma S.23, we now derive the first-period sales distribution. Let

$$k_1 := \lfloor n_0 \cdot \Pr[u_l \leq u_0 < u_h, s \leq p_h(u_h - u_0)] \rfloor,$$

$$k_2 := \lfloor n_0 \cdot \Pr[u_0 < u_l, s \leq p_h u_h + p_l u_l - u_0] \rfloor,$$

$$m_1 := \lfloor n_0 \cdot \Pr[u_0 < u_l, s \leq p_h(u_h - u_l)] \rfloor,$$

where, as in the base model, n_0 denotes the number of consumers in the first period. Define

$$g_s^h(x) := Binomial(x, k_1 + k_2, 1/2),$$
$$g_s^l(x) := Binomial(x, k_2, 1/2),$$
$$g_a(x) := Binomial(x - m_1 - k_1, k_2 - m_1, 1/2) \text{ if } x \geq m_1 + k_1, \text{ and } 0 \text{ if } x < m_1 + k_1,$$

Let $G_s^h(\cdot)$, $G_s^l(\cdot)$, and $G_a(\cdot)$ be the cumulative distribution function of $g_s^h(\cdot)$, $g_s^l(\cdot)$, and $g_a(\cdot)$, respectively. Define $\bar{G}_s^h(x) = 1 - G_s^h(x)$, $\bar{G}_s^l(x) = 1 - G_s^l(x)$, and $\bar{G}_a(x) = 1 - G_a(x)$. Here we slightly abuse the notation $G_a(x)$ for expositional convenience (note that its definition is different from and yet very similar to that in the base model). Lemma S.24 characterizes the sales distribution in the first period.

Lemma S.24. *(i) The total sales in the first period is $k_1 + k_2$ if at least one of the products is of high value, and is k_2 if neither product is of high value.*

(ii) If both products are of high value, the sales of either product follows distribution $G_s^h(x)$.

(iii) If both products are of low value, the sales of either product follows distribution $G_s^l(x)$.

(iv) If the product values are different, the sales of the high-value (resp. low-value) product follows distribution $G_a(x)$ (resp. $\bar{G}_a(k_1 + k_2 - x)$).

Lemma S.24 shows that, unlike in the base model where the total sales in the first period is independent of the product values, here the total sales in the first period is determined by (and thus also reveals) whether at least one of the products is of high value. This is driven by the fact that consumers with $u_l \leq u_0 < u_h$ only purchase a high-value product. Specifically, if there is at least one high-value product, all of the consumers who perform a search (including those with $u_l \leq u_0 < u_h$) purchase a product; otherwise, consumers with $u_l \leq u_0 < u_h$ do not purchase any product even if they have searched some product(s) and thus the first-period total sales is lower. Since the total sales is now informative about the product values, the learning process under volume information is qualitatively different from that in the base model, as we detail next.

SN.1.2. Second Period

Next we consider the optimal search and purchasing strategies for consumers in the second period where different levels of first-period sales information are released by the platform.

Sales Ranking Information

We first consider sales ranking information. As in the base model, we define π_1^r as the belief that the bestseller product is of high value and π_{-1}^r as the belief that the product with lower sales ranking is of high value. Let π_2^r be the belief that the product of lower sales ranking is of high value when the bestseller product has been revealed to be of low value. Since the two products are ex ante homogeneous, we assume without loss of generality that product 1 has higher sales ranking throughout this extension. Let X_1, X_2 be the first-period sales of product 1 and product 2, respectively. Denote x_1, x_2 as the sales realization of X_1 and X_2, respectively.

By similar analysis to that in the base model, when both k_2 and $k_1 + k_2$ are odd,

$$\begin{aligned}
\pi_1^r &= \Pr[u_1 = u_h | X_1 \geq X_2] \\
&= \frac{\Pr[u_1 = u_2 = u_h, X_1 \geq \frac{k_1+k_2}{2}] + \Pr[u_1 = u_h, u_2 = u_l, X_1 \geq \frac{k_1+k_2}{2}]}{\begin{pmatrix} \Pr[u_1 = u_2 = u_h, X_1 \geq \frac{k_1+k_2}{2}] + \Pr[u_1 = u_h, u_2 = u_l, X_1 \geq \frac{k_1+k_2}{2}] \\ +\Pr[u_1 = u_l, u_2 = u_h, X_1 \geq \frac{k_1+k_2}{2}] + \Pr[u_1 = u_2 = u_l, X_1 \geq \frac{k_2}{2}] \end{pmatrix}} \\
&= \frac{p_h^2 \bar{G}_s^h(\frac{k_1+k_2}{2}) + p_h p_l \bar{G}_a(\frac{k_1+k_2}{2})}{p_h^2 \bar{G}_s^h(\frac{k_1+k_2}{2}) + p_h p_l \bar{G}_a(\frac{k_1+k_2}{2}) + p_h p_l G_a(\frac{k_1+k_2}{2}) + p_l^2 \bar{G}_s^l(\frac{k_2}{2})} \\
&= p_h^2 + 2 p_h p_l (1 - G_a(\tfrac{k_1+k_2}{2})),
\end{aligned}$$

$$
\begin{aligned}
\pi_{-1}^r &= \Pr[u_2 = u_h | X_1 \geq X_2] \\
&= \frac{\Pr[u_1 = u_2 = u_h, X_1 \geq \frac{k_1+k_2}{2}] + \Pr[u_1 = u_l, u_2 = u_h, X_1 \geq \frac{k_1+k_2}{2}]}{\left(\begin{array}{c}\Pr[u_1 = u_2 = u_h, X_1 \geq \frac{k_1+k_2}{2}] + \Pr[u_1 = u_h, u_2 = u_l, X_1 \geq \frac{k_1+k_2}{2}] \\ +\Pr[u_1 = u_l, u_2 = u_h, X_1 \geq \frac{k_1+k_2}{2}] + \Pr[u_1 = u_2 = u_l, X_1 \geq \frac{k_2}{2}]\end{array}\right)} \\
&= \frac{p_h^2 \bar{G}_s^h(\frac{k_1+k_2}{2}) + p_h p_l G_a(\frac{k_1+k_2}{2})}{p_h^2 \bar{G}_s^h(\frac{k_1+k_2}{2}) + p_h p_l \bar{G}_a(\frac{k_1+k_2}{2}) + p_h p_l G_a(\frac{k_1+k_2}{2}) + p_l^2 \bar{G}_s^l(\frac{k_2}{2})} \\
&= p_h^2 + 2 p_h p_l G_a(\tfrac{k_1+k_2}{2}),
\end{aligned}
$$

and

$$
\begin{aligned}
\pi_2^r &= \Pr[u_2 = u_h | X_1 \geq X_2, u_1 = u_l] \\
&= \frac{\Pr[u_1 = u_l, u_2 = u_h, X_1 \geq \frac{k_1+k_2}{2}]}{\Pr[u_1 = u_l, u_2 = u_h, X_1 \geq \frac{k_1+k_2}{2}] + \Pr[u_1 = u_2 = u_l, X_1 \geq \frac{k_2}{2}]} \\
&= \frac{p_h G_a(\frac{k_1+k_2}{2})}{p_h G_a(\frac{k_1+k_2}{2}) + p_l \bar{G}_s(\frac{k_2}{2})} \\
&= \frac{p_h G_a(\frac{k_1+k_2}{2})}{p_h G_a(\frac{k_1+k_2}{2}) + p_l/2}
\end{aligned}
$$

The case where either k_2 or $k_1 + k_2$ is even can be derived similarly and is omitted here. As in the base model, we have the following lemma.

LEMMA S.25. *The posterior beliefs satisfy:* $\pi_1^r \geq p_h \geq \pi_{-1}^r \geq \pi_2^r$.

Lemma S.25 shows that, despite the additional complexity in sales distribution due to consumer heterogeneity in reservation utility, the ordering of the posterior beliefs under ranking information is sustained as in the base model. It allows for a full characterization of the optimal search and purchasing strategies for the second-period consumers under ranking information, as in Proposition S.24 below.

PROPOSITION S.24. *When first-period sales ranking information is released, the optimal search and purchasing strategies of a second-period consumer with reservation utility* u_0 *and search cost* s *are as follows:*

(i) If $u_0 \geq u_h$*: the consumer neither searches nor purchases.*

(ii) If $u_l \leq u_0 < u_h$*: the consumer searches the bestseller product if* $s \leq \pi_1^r(u_h - u_0)$ *and leaves without search or purchase otherwise. If the bestseller is of high value, the consumer purchases the bestseller. If, however, the bestseller is of low value, she performs the second search if* $s \leq \pi_2^r(u_h - u_0)$ *and leaves without further search or purchase otherwise. If the second search reveals a high-value product, the consumer purchases the high-value product; otherwise, she leaves without purchase.*

(iii) If $u_0 < u_l$*: the consumer searches the bestseller product if* $s \leq \pi_1^r u_h + (1 - \pi_1^r) u_l - u_0$ *and leaves without search or purchase otherwise. If the bestseller is of high value, the consumer purchases the bestseller. If, however, the bestseller is of low value, she performs the second search if* $s \leq \pi_2^r(u_h - u_l)$ *and purchases the bestseller otherwise. If the second search reveals a high-value product, the consumer purchases the high-value product; otherwise, she randomly purchases a product.*

Proposition S.24 proves that, same as in the base model, it is optimal for the second-period consumers to search the bestseller product first when sales ranking information is released. The intuition is also the same as in the base model: a high-value product is more likely to be the bestseller product compared to a low-value product. This confirms our finding in the base model that sales ranking information strengthens the belief that the bestseller product is of high-value and converts random search into directional search.

Sales Volume Information

We now proceed to the case where first-period sales volume information is released by the platform. In this case, by Lemma S.24 a second-period consumer is able to learn from the total sales whether there is at least one high-value product and update her belief about the product values, as detailed below.

Given first-period sales volume x_1, x_2 with $x_1 \geq x_2$, let $\pi_1^v(x_1,x_2)$ and $\pi_{-1}^v(x_1,x_2)$ be the posterior beliefs that product 1 and product 2 are of high value, respectively. Let $\pi_2^v(x_1,x_2)$ denote the belief of product 2 being of high value when product 1 is revealed to be of low value. Consider the following two cases:

- If $x_1+x_2=k_2$, by Lemma S.24 neither product is of high value and thus

$$\pi_1^v(x_1,x_2)=\pi_{-1}^v(x_1,x_2)=\pi_2^v(x_1,x_2)=0$$

- If $x_1+x_2=k_1+k_2$, by Lemma S.24 there is at least one high-value product and thus

$$\begin{aligned}\pi_1^v(x_1,x_2) &= \frac{\Pr[u_1=u_2=u_h, X_1=x_1, X_2=x_2]+\Pr[u_1=u_h, u_2=u_l, X_1=x_1, X_2=x_2]}{\begin{pmatrix}\Pr[u_1=u_2=u_h, X_1=x_1, X_2=x_2]+\Pr[u_1=u_h, u_2=u_l, X_1=x_1, X_2=x_2]\\ +\Pr[u_1=u_l, u_2=u_h, X_1=x_1, X_2=x_2]+\Pr[u_1=u_2=u_l, X_1=x_1, X_2=x_2]\end{pmatrix}}\\ &= \frac{p_h^2 g_s^h(x_1)+p_h p_l g_a(x_1)}{p_h^2 g_s^h(x_1)+p_h p_l g_a(x_1)+p_h p_l g_a(x_2)+0}\\ &= \frac{p_h g_s^h(x_1)+p_l g_a(x_1)}{p_h g_s^h(x_1)+p_l g_a(x_1)+p_l g_a(x_2)},\end{aligned}$$

$$\begin{aligned}\pi_{-1}^v(x_1,x_2) &= \frac{\Pr[u_1=u_2=u_h, X_1=x_1, X_2=x_2]+\Pr[u_1=u_l, u_2=u_h, X_1=x_1, X_2=x_2]}{\begin{pmatrix}\Pr[u_1=u_2=u_h, X_1=x_1, X_2=x_2]+\Pr[u_1=u_h, u_2=u_l, X_1=x_1, X_2=x_2]\\ +\Pr[u_1=u_l, u_2=u_h, X_1=x_1, X_2=x_2]+\Pr[u_1=u_2=u_l, X_1=x_1, X_2=x_2]\end{pmatrix}}\\ &= \frac{p_h^2 g_s^h(x_1)+p_h p_l g_a(x_2)}{p_h^2 g_s^h(x_1)+p_h p_l g_a(x_1)+p_h p_l g_a(x_2)+0}\\ &= \frac{p_h g_s^h(x_1)+p_l g_a(x_2)}{p_h g_s^h(x_1)+p_l g_a(x_1)+p_l g_a(x_2)},\end{aligned}$$

and

$$\begin{aligned}\pi_2^v(x_1,x_2) &= \frac{\Pr[u_1=u_l, u_2=u_h, X_1=x, X_2=x_2]}{\Pr[u_1=u_l, u_2=u_h, X_1=x_1, X_2=x_2]+\Pr[u_1=u_2=u_l, X_1=x_1, X_2=x_2]}\\ &= \frac{p_h p_l g_a(x_1)}{p_h p_l g_a(x_1)+0}\\ &= 1\end{aligned}$$

Notice that here $\pi_2^v(x_1,x_2)=1$ when $x_1+x_2=k_1+k_2$. That is, in this case the unsearched product is of high type for sure. This is because a total sales volume of k_1+k_2 indicates that a high-value product is present and the probability $\pi_2^v(\cdot,\cdot)$ is conditional on product 1 being revealed in the first search to be of low value.

We note that a few important properties of the posterior beliefs shown in the base model are preserved in this extended model. First, Lemma S.26 proves that, consistent with our findings in the base model, the bestseller product is more likely to be of high value than the other product. It implies that, in this extended model and under volume information, it is optimal to search the bestseller first if a consumer would like to search.

Lemma S.26. *We have* $\pi_1^v(x_1,x_2) \geq \pi_{-1}^v(x_1,x_2)$, Furthermore, the properties of mean-preserving spread and reinforcement-by-homogeneity effect remain valid, as below.

Mean-preserving spread The following proposition shows that the mean-preserving-spread result still hold.

PROPOSITION S.25. *The belief that the bestseller product is of high value with sales volume information is a mean-preserving spread of the belief that the bestseller product is of high value with sales ranking information.*

Reinforcement-by-homogeneity: In the base model we show that sales volume information gives rise to the reinforcement-by-homogeneity effect, which may render the belief that the bestseller product is of high value lower than the initial belief. The underlying reason is that sales volume information, in contrast to sales ranking information, allows the second-period consumers to update their belief about the number of high-value products. In this extension, this effect is even stronger as the second-period consumers are able to fully deduce from sales volume information about whether there is at least a high-value product. When consumers deduce that neither product is of high value (i.e., when $x_1 + x_2 = k_2$), their posterior belief is zero, clearly lower than the initial belief.

Building on the findings so far, Proposition S.26 characterizes the optimal search and purchasing strategies for the second-period consumers when sales volume information is released by the platform.

PROPOSITION S.26. *Upon observing the first-period sales volume x_1 and x_2, the optimal search and purchasing strategies of a second-period consumer with reservation utility u_0 and search cost s are as follows:*

(i) $x_1 + x_2 = k_2$:

- *If $u_0 \geq u_l$, the consumer neither searches nor purchases.*
- *If $u_0 < u_l$, the consumer randomly chooses a product to search and purchase if $s \leq u_l - u_0$ and leaves without search or purchase otherwise.*

(ii) $x_1 + x_2 = k_1 + k_2$:

- *If $u_0 \geq u_h$, the consumer neither searches nor purchases.*
- *If $u_l \leq u_0 < u_h$: the consumer searches the bestseller product first if $s \leq \frac{u_h - u_0}{2 - \pi_1^v(x_1, x_2)}$ and leaves without search or purchase otherwise. If the bestseller is of high value, the consumer purchases the bestseller. If, however, the bestseller is revealed to be of low value, the consumer performs the second search and purchases the searched product (as it is of high type for sure) if $s \leq u_h - u_0$ and leaves without search or purchase otherwise.*
- *If $u_0 < u_l$: there exists a threshold $\underline{s}_1$ such that the consumer searches the bestseller product if $s \leq \underline{s}_1$ and leaves without search or purchase otherwise. If the bestseller is of high value, the consumer purchases the bestseller. If, however, the bestseller is revealed to be of low value, the consumer performs the second search and purchases the searched product (as it is of high type for sure) if $s \leq u_h - u_l$ and buys the bestseller otherwise. Specifically, $\underline{s}_1 = \frac{u_h - u_0}{2 - \pi_1^v(x_1, x_2)}$ when $u_l - (1 - \pi_1^v(x_1, x_2))(u_h - u_l) \leq u_0 < u_l$ and $\underline{s}_1 = \pi_1^v(x_1, x_2)u_h + (1 - \pi_1^v(x_1, x_2))u_l - u_0$ when $u_0 < u_l - (1 - \pi_1^v(x_1, x_2))(u_h - u_l)$.*

Proposition S.26 suggests that, compared to that in the base model, sales volume becomes more informative of the product values in this extended model, as it allows the second-period consumers to perfectly infer about whether both product values are low. In particular, if the sales volume implies that neither product is of high value, consumers' uncertainty about the product values is fully resolved. Such a perfect-learning outcome is never possible in the base model. As aforementioned, the difference in results is driven by the heterogeneity in consumers' reservation utility: in particular, consumers with $u_l \le u_0 < u_h$ never purchase a low-value product. Sales volume reveals these consumers' purchasing decision and thus is indicative of the product values.

Numerical Illustration

Table S.11 exemplifies the expected second-period sales under no sales information, sales ranking information, and sales volume information. The reservation utility is assumed to be uniformly distributed in $[\frac{u_h+u_l}{2} - \theta, \frac{u_h+u_l}{2} + \theta]$ and the search cost distribution is $F(x) = (\alpha + (1-\alpha)\Phi((x-\mu)/\sigma))\mathbb{I}(x \ge 0)$, where $\Phi(\cdot)$ is the cumulative distribution function for the standard normal distribution and $\mathbb{I}(\cdot)$ is the indicator function. The parameter values are as in the table and its caption.

The results in Table S.11 confirm that bestseller information can improve total product sales and either ranking or volume can be the preferred form of the information to the platform. This is consistent with the findings in the base model.

Table S.11 Impact of p_h on total expected sales in the second period (search required for purchase):
$\mu = 4.5, \sigma = 1.5, n_2 = 100, u_h = 10, u_l = 6, \alpha = 0.08$

	$\theta = 6$				$\theta = 6.5$			
p_h	*Ranking*	*Volume*	*No Info.*	*Opt. Info.*	*Ranking*	*Volume*	*No Info.*	*Opt. Info.*
0.3	14.46	14.18	11.02	Ranking	16.02	14.85	13.02	Ranking
0.4	17.83	17.94	13.28	Volume	19.28	18.62	15.28	Ranking
0.5	21.18	21.88	16.25	Volume	23.17	22.99	18.50	Ranking
0.6	25.26	25.28	19.52	Volume	25.46	26.40	21.52	Volume
0.7	27.58	28.06	21.73	Volume	28.57	29.19	24.73	Volume
0.8	29.77	30.29	25.84	Volume	30.82	31.47	26.88	Volume

SN.2. Optional Search for Purchase

So far we have assumed that search is required for purchase. In this subsection we relax the assumption and consider the setting where consumers are allowed to purchase a product without searching it.

SN.2.1. Equilibrium Analysis

We first analyze the consumers' optimal strategies and start from the first period.

First Period

Lemma S.27 characterizes the optimal search and purchasing strategies for consumers in the first period.

LEMMA S.27. *In the first period, the optimal search and purchasing strategies of a first-period consumer with reservation utility u_0 and search cost s are as follows:*

(i) If $u_0 \ge u_h$, the consumer neither searches nor purchases.

(ii) If $p_h u_h + p_l u_l \le u_0 < u_h$, the consumer performs the first search if $s \le p_h(u_h - u_0)$ and leaves without search or purchase otherwise. If the first search reveals a high-value product, the consumer purchases the searched product. If, however, the first search reveals a low-value product, the consumer performs the second search if $s \le p_h(u_h - u_0)$ and leaves without search or purchase otherwise. If the second search reveals a high-value product, the consumer purchases the high-value product; otherwise, she leaves without purchase.

(iii) If $u_l \le u_0 < p_h u_h + p_l u_l$, the consumer randomly purchases a product without search if $s > p_h p_l(u_h - u_l)$ and, otherwise, performs the first search and purchases the searched product if it is of high value. If the first search reveals a low-value product, the consumer purchases the unsearched product without searching it if $s \ge p_l(u_0 - u_l)$ and performs the second search otherwise. If the second search again reveals a low-value product, the consumer leaves without purchase.

(iv) If $u_0 < u_l$, the consumer randomly purchases a product without search if $s > p_h p_l(u_h - u_l)$ and, otherwise, performs the first search and purchases the searched product if it is of high value. If the first search reveals a low-value product, the consumer purchases the unsearched product without performing the second search.

Lemma S.27 shows that the first-period consumers' optimal strategies become more diverse when the requirement of search for purchase is lifted. Specifically, as in cases (iii) and (iv) of Lemma S.27, consumers with both low reservation utility ($u_0 < p_h u_h + p_l u_l$) and high search cost ($s > p_h p_l(u_h - u_l)$) randomly purchase a product without any search and, after the first search reveals a low-value product, consumers with medium search cost ($\max(p_l(u_0 - u_l), 0) \le s \le p_h p_l(u_h - u_l)$) purchase the unsearched product without searching it. An interesting implication of these strategies is that, in contrast with the result in §SN.1 that consumers with $u_0 > u_l$ never purchase a low-value product, with the option of purchase without search some consumers with $u_0 > u_l$ may end up buying a low-value product even if there is a high-value product. Nevertheless, some other consumers with $u_0 > u_l$ search at least once and make a purchase if and only if a high-value product is present. This leads to a finding as in §SN.1 that the total sales volume is informative of the product values, as elaborated below.

Define

$$\begin{aligned}
\kappa_1 &:= \lfloor n_0 \Pr(p_h u_h + p_l u_l \le u_0 < u_h, s \le p_h(u_h - u_0)) \rfloor \\
\kappa_{21} &:= \lfloor n_0 \Pr(u_l \le u_0 < p_h u_h + p_l u_l, s > p_h p_l(u_h - u_l)) \rfloor \\
\kappa_{22} &:= \lfloor n_0 \Pr(u_l \le u_0 < p_h u_h + p_l u_l, p_l(u_0 - u_l) < s \le p_h p_l(u_h - u_l)) \rfloor \\
\kappa_{23} &:= \lfloor n_0 \Pr(u_l \le u_0 < p_h u_h + p_l u_l, s \le p_l(u_0 - u_l)) \rfloor \\
\kappa_{31} &:= \lfloor n_0 \Pr(u_0 \le u_l, s > p_h p_l(u_h - u_l)) \rfloor \\
\kappa_{32} &:= \lfloor n_0 \Pr(u_0 \le u_l, s \le p_h p_l(u_h - u_l)) \rfloor
\end{aligned}$$

and

$t_s^h(x) := Binomial(x, \kappa_{21} + \kappa_{31} + \kappa_{22} + \kappa_{32} + \kappa_1 + \kappa_{23}, 1/2)$,
$t_s^l(x) := Binomial(x, \kappa_{21} + \kappa_{31} + \kappa_{22} + \kappa_{32}, 1/2)$,
$t_a(x) := Binomial(x - (\kappa_{22} + \kappa_{32} + \kappa_1 + \kappa_{23}), \kappa_{21} + \kappa_{31}, 1/2)$ if $x \ge \kappa_{22} + \kappa_{32} + \kappa_1 + \kappa_{23}$, and 0 if $x < \kappa_{22} + \kappa_{32} + \kappa_1 + \kappa_{23}$.

Let $T_s^h(\cdot)$, $T_s^l(\cdot)$, and $T_a(\cdot)$ be the cumulative distribution function of $t_s^h(\cdot)$, $t_s^l(\cdot)$, and $t_a(\cdot)$, respectively. Define $\bar{T}_s^h(x) = 1 - T_s^h(x)$, $\bar{T}_s^l(x) = 1 - T_s^l(x)$, and $\bar{T}_a(x) = 1 - T_a(x)$.

Lemma S.28 characterizes the sales distribution in the first period.

Lemma S.28. *(i) The total sales in the first period is $\kappa_{21} + \kappa_{31} + \kappa_{22} + \kappa_{32} + \kappa_1 + \kappa_{23}$ if there is at least one high-value product and is $\kappa_{21} + \kappa_{31} + \kappa_{22} + \kappa_{32}$ if there is no high-value product.*

(ii) If both products are of high value, the sales of either product follows distribution $T_s^h(x)$.

(iii) If both products are of low value, the sales of either product follows distribution $T_s^l(x)$.

(iv) If the two product's value are different, the sales of the high-value (resp. low-value) product follows distribution $T_a(x)$ (resp. $\bar{T}_a(\kappa_{21}+\kappa_{31}+\kappa_{22}+\kappa_{32}+\kappa_1+\kappa_{23}-x)$).

Lemma S.28 shows that sales distribution in the first period is similar to that in §SN.1 where search is required for purchase. In particular, the total sales is determined by and thus also implies whether there is at least one high-value product. This is because consumers in the cases corresponding to κ_1 and κ_{23} make a purchase if and only if one or both of the products are of high value.

By comparing the first-period sales distributions with or without the requirement of search for purchase, we note that lifting the requirement raises the total sales in the first period. The sales increase is due to some consumers with high search cost either purchasing a product without searching at all or conducting a first search and purchasing the unsearched product without searching it. On the other hand, the number of first-period consumers who make "informed" purchases (i.e., always purchase a high-value product if the product values differ from each other) may be lower when search becomes optional for purchase. For example, when $u_0 < u_l$, if search is required for purchase, consumers with $s \le p_h(u_h-u_l)$ are willing to conduct two searches and purchase a high-value product if there is any; while under optional search, some of these consumers (specifically, those with relatively higher search cost, $p_hp_l(u_h-u_l) < s \le p_h(u_h-u_l)$) randomly purchase a product without searching at all. Hence, in this case, fewer consumers in this case ($u_0 < u_l$) make informed purchases when search is optional for purchase. The number of informed purchases, however, may increase for other consumer cases (e.g., $u_l \le u_0 < p_hu_h+p_lu_l$), as some of the consumers in this case may skip the second search and directly purchase the unsearched product if the first search reveals a low-value product. Thus, the overall impact of search requirement on the number of informed purchases is ambiguous.

Second Period

Next we consider the optimal search and purchasing strategies for the consumers in the second period where different levels of first-period sales information are released by the platform.

Sales Ranking Information

We start from ranking information. Define $\tilde{\pi}_1^r$ as the belief that the bestseller product is of high value and $\tilde{\pi}_{-1}^r$ as the belief that the product with lower sales ranking is of high value. Let $\tilde{\pi}_2^r$ be the belief that the product of lower sales ranking is of high value when the bestseller product is revealed to be of low value. For ease of exposition, define $\zeta_1 := \kappa_{21}+\kappa_{31}+\kappa_{22}+\kappa_{32}+\kappa_1+\kappa_{23}$ and $\zeta_2 := \kappa_{21}+\kappa_{31}+\kappa_{22}+\kappa_{32}$. When both ζ_1 and ζ_2 are odd, we have

$$\begin{aligned}
\tilde{\pi}_1^r &= \Pr[u_1=u_h|X_1\ge X_2]\\
&= \frac{\Pr[u_1=u_2=u_h, X_1\ge\frac{\zeta_1}{2}]+\Pr[u_1=u_h,u_2=u_l,X_1\ge\frac{\zeta_1}{2}]}{\left(\begin{array}{l}\Pr[u_1=u_2=u_h, X_1\ge\frac{\zeta_1}{2}]+\Pr[u_1=u_h,u_2=u_l,X_1\ge\frac{\zeta_1}{2}]\\ +\Pr[u_1=u_l,u_2=u_h,X_1\ge\frac{\zeta_1}{2}]+\Pr[u_1=u_2=u_l,X_1\ge\frac{\zeta_2}{2}]\end{array}\right)}\\
&= \frac{p_h^2\bar{T}_s^h(\frac{\zeta_1}{2})+p_hp_l\bar{T}_a(\frac{\zeta_1}{2})}{p_h^2\bar{T}_s^h(\frac{\zeta_1}{2})+p_hp_l\bar{T}_a(\frac{\zeta_1}{2})+p_hp_lT_a(\frac{\zeta_1}{2})+p_l^2\bar{T}_s^l(\frac{\zeta_2}{2})}\\
&= p_h^2+2p_hp_l(1-T_a(\tfrac{\zeta_1}{2})),
\end{aligned}$$

$$\tilde{\pi}_{-1}^r = \Pr[u_2=u_h|X_1\ge X_2]$$

$$
\begin{aligned}
&= \frac{\Pr[u_1=u_2=u_h, X_1 \geq \frac{\zeta_1}{2}] + \Pr[u_1=u_l, u_2=u_h, X_1 \geq \frac{\zeta_1}{2}]}{\begin{pmatrix} \Pr[u_1=u_2=u_h, X_1 \geq \frac{\zeta_1}{2}] + \Pr[u_1=u_h, u_2=u_l, X_1 \geq \frac{\zeta_1}{2}] \\ +\Pr[u_1=u_l, u_2=u_h, X_1 \geq \frac{\zeta_1}{2}] + \Pr[u_1=u_2=u_l, X_1 \geq \frac{\zeta_2}{2}] \end{pmatrix}} \\
&= \frac{p_h^2 \bar{T}_s^h(\frac{\zeta_1}{2}) + p_h p_l T_a(\frac{\zeta_1}{2})}{p_h^2 \bar{T}_s^h(\frac{\zeta_1}{2}) + p_h p_l \bar{T}_a(\frac{\zeta_1}{2}) + p_h p_l T_a(\frac{\zeta_1}{2}) + p_l^2 \bar{T}_s^l(\frac{\zeta_2}{2})} \\
&= p_h^2 + 2 p_h p_l T_a(\tfrac{\zeta_1}{2}),
\end{aligned}
$$

and

$$
\begin{aligned}
\tilde{\pi}_2^r &= \Pr[u_2=u_h | X_1 \geq X_2, u_1=u_l] \\
&= \frac{\Pr[u_1=u_l, u_2=u_h, X_1 \geq \frac{\zeta_1}{2}]}{\Pr[u_1=u_l, u_2=u_h, X_1 \geq \frac{\zeta_1}{2}] + \Pr[u_1=u_2=u_l, X_1 \geq \frac{\zeta_2}{2}]} \\
&= \frac{p_h T_a(\frac{\zeta_1}{2})}{p_h T_a(\frac{\zeta_1}{2}) + p_l \bar{T}_s(\frac{\zeta_2}{2})} \\
&= \frac{p_h T_a(\frac{\zeta_1}{2})}{p_h T_a(\frac{\zeta_1}{2}) + p_l/2}
\end{aligned}
$$

The case where either ζ_1 or ζ_2 is even can be derived similarly and is omitted here. The following lemma shows that the belief ordering under ranking information remains the same as that in §SN.1 and in the base model.

LEMMA S.29. *We have $\tilde{\pi}_1^r \geq p_h \geq \tilde{\pi}_{-1}^r \geq \tilde{\pi}_2^r$.*

Lemma S.29 allows for a full characterization of the optimal search and purchasing strategies for the second-period consumers under ranking information. Proposition S.27 follows.

PROPOSITION S.27. *When first-period sales ranking information is released, the optimal search and purchasing strategies of a second-period consumer with reservation utility u_0 and search cost s are as follows:*

(i) If $u_0 \geq u_h$: the consumer neither searches nor purchases.

(ii) If $\tilde{\pi}_1^r u_h + (1-\tilde{\pi}_1^r) u_l \leq u_0 < u_h$, the consumer searches the bestseller if $s \leq \tilde{\pi}_1^r (u_h - u_0)$ and leaves without search or purchase otherwise. If the bestseller is of high value, the consumer purchases the bestseller. If, however, the bestseller is of low value, she performs the second search if $s \leq \tilde{\pi}_2^r (u_h - u_0)$ and leaves without further search or purchase otherwise. If the second search reveals a high-value product, the consumer purchases the high-value product; otherwise, she leaves without purchase.

(iii) If $\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r) u_l \leq u_0 < \tilde{\pi}_1^r u_h + (1-\tilde{\pi}_1^r) u_l$, there exists a threshold $\underline{s}_2$ such that the consumer searches the bestseller if $s \leq \underline{s}_2$ and purchases the bestseller without searching otherwise. If the bestseller is of high value, the consumer purchases the bestseller. If, however, the bestseller is of low value, she performs the second search if $s \leq \tilde{\pi}_2^r (u_h - u_0)$ and leaves without purchase otherwise. If the second search reveals a high-value product, the consumer purchases the high-value product; otherwise, she leaves without purchase. Specifically, $\underline{s}_2 = (1-\tilde{\pi}_1^r)(u_0 - u_l)$ when $\frac{\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_1^r) u_l}{1-\tilde{\pi}_1^r+\tilde{\pi}_2^r} \leq u_0 < \tilde{\pi}_1^r u_h + (1-\tilde{\pi}_1^r) u_l$ and $\underline{s}_2 = (1-\tilde{\pi}_1^r) \frac{\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r) u_0 - u_l}{2-\tilde{\pi}_1^r}$ when $\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r) u_l \leq u_0 < \frac{\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_1^r) u_l}{1-\tilde{\pi}_1^r+\tilde{\pi}_2^r}$.

(iv) If $u_l \leq u_0 < \tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r) u_l$, there exists a threshold $\underline{s}_3$ such that the consumer searches the bestseller if $s \leq \underline{s}_3$ and purchases the bestseller without searching otherwise. If the bestseller is of high value, the consumer purchases the bestseller. If, however, the bestseller is of low value, she performs the second search if $s \leq (1-\tilde{\pi}_2^r)(u_0 - u_l)$ and purchases the unsearched product otherwise. If the second search reveals a high-value product, the consumer purchases the high-value product; otherwise, she leaves without purchase. Specifically, $\underline{s}_3 = (1-$

$\tilde{\pi}_1^r)\tilde{\pi}_2^r(u_h - u_l)$ *when* $\frac{(1-\tilde{\pi}_1^r)\tilde{\pi}_2^r(u_h-u_l)}{1-\tilde{\pi}_2^r} + u_l \leq u_0 < \tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_l$ *and* $\underline{s}_3 = \frac{\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_0 - u_l}{2-\tilde{\pi}_1^r}$ *when* $u_l \leq u_0 < \frac{(1-\tilde{\pi}_1^r)\tilde{\pi}_2^r(u_h-u_l)}{1-\tilde{\pi}_2^r} + u_l$.

(v) If $u_l < u_0$, the consumer searches the bestseller if $s \leq (1-\tilde{\pi}_1^r)\tilde{\pi}_2^r(u_h - u_l)$ and purchases the bestseller otherwise. If the first search reveals a low-value product, the consumer purchases the unsearched product.

Proposition S.27 confirms, once again, that it is optimal for a second-period consumer to first search the bestseller if she ever searches. In addition, the proposition shows that the bestseller is also a consumer's optimal product choice for purchasing without search if the consumer decides to do so. Thus, under this model, sales ranking guides consumers in both their first search and their purchase without any search.

Sales Volume Information

Now we consider sales volume information. Akin to the case of search required for purchase, when sales volume information is released, the second-period consumers are able to fully deduce from the total sales about whether there exists a high-value product. Given observed first-period sales x_1 and x_2 with $x_1 \geq x_2$, let $\tilde{\pi}_1^v(x_1, x_2)$ and $\tilde{\pi}_{-1}^v(x_1, x_2)$ be the beliefs that product 1 and product 2 are of high value, respectively, and let $\tilde{\pi}_2^v(x_1, x_2)$ be the belief that product 2 is of high value when product 1 is revealed to be of low value.

If $x_1 + x_2 = \zeta_2$, by Lemma S.28 there is no high-value product and

$$\tilde{\pi}_1^v(x_1, x_2) = \tilde{\pi}_{-1}^v(x_1, x_2) = \tilde{\pi}_2^v(x_1, x_2) = 0$$

If $x_1 + x_2 = \zeta_1$, by Lemma S.28 there is at least one high-value product,

$$\begin{aligned}
\tilde{\pi}_1^v(x_1, x_2) &= \frac{\Pr[u_1 = u_2 = u_h, X_1 = x_1, X_2 = x_2] + \Pr[u_1 = u_h, u_2 = u_l, X_1 = x_1, X_2 = x_2]}{\begin{pmatrix} \Pr[u_1 = u_2 = u_h, X_1 = x_1, X_2 = x_2] + \Pr[u_1 = u_h, u_2 = u_l, X_1 = x_1, X_2 = x_2] \\ +\Pr[u_1 = u_l, u_2 = u_h, X_1 = x_1, X_2 = x_2] + \Pr[u_1 = u_2 = u_l, X_1 = x_1, X_2 = x_2] \end{pmatrix}} \\
&= \frac{p_h^2 t_s^h(x_1) + p_h p_l t_a(x_1)}{p_h^2 t_s^h(x) + p_h p_l t_a(x_1) + p_h p_l t_a(x_2) + 0} \\
&= \frac{p_h t_s^h(x_1) + p_l t_a(x_1)}{p_h t_s^h(x) + p_l t_a(x_1) + p_l t_a(x_2)},
\end{aligned}$$

and

$$\begin{aligned}
\tilde{\pi}_{-1}^v(x_1, x_2) &= \frac{\Pr[u_1 = u_2 = u_h, X_1 = x_1, X_2 = x_2] + \Pr[u_1 = u_l, u_2 = u_h, X_1 = x_1, X_2 = x_2]}{\begin{pmatrix} \Pr[u_1 = u_2 = u_h, X_1 = x_1, X_2 = x_2] + \Pr[u_1 = u_h, u_2 = u_l, X_1 = x_1, X_2 = x_2] \\ +\Pr[u_1 = u_l, u_2 = u_h, X_1 = x_1, X_2 = x_2] + \Pr[u_1 = u_2 = u_l, X_1 = x_1, X_2 = x_2] \end{pmatrix}} \\
&= \frac{p_h^2 t_s^h(x_1) + p_h p_l t_a(x_2)}{p_h^2 t_s^h(x) + p_h p_l t_a(x_1) + p_h p_l t_a(x_2) + 0} \\
&= \frac{p_h t_s^h(x_1) + p_l t_a(x_2)}{p_h t_s^h(x) + p_l t_a(x_1) + p_l t_a(x_2)},
\end{aligned}$$

and

$$\begin{aligned}
\tilde{\pi}_2^v(x_1, x_2) &= \frac{\Pr[u_1 = u_l, u_2 = u_h, X_1 = x, X_2 = x_2]}{\Pr[u_1 = u_l, u_2 = u_h, X_1 = x_1, X_2 = x_2] + \Pr[u_1 = u_2 = u_l, X_1 = x_1, X_2 = x_2]} \\
&= \frac{p_h p_l t_a(x_1)}{p_h p_l t_a(x_1) + 0} \\
&= 1
\end{aligned}$$

Recall that, if the total sales equals ζ_1, it indicates that there is at least one high-value product. Conditional on product 1 found to be of low value in the first search, product 2 must be of high-value, i.e., $\tilde{\pi}_2^v(x_1, x_2) = 1$ when $x_1 + x_2 = \zeta_1$. This result is in line with its counterpart in §SN.1. Following a similar approach as in §SN.1, we prove an ordering of the posterior beliefs in Lemma S.30. Similarly, the mean-preserving-spread property and the reinforcement-by-homogeneity effect can also be shown and are omitted for brevity.

LEMMA S.30. *We have* $\tilde{\pi}_1^v(x_1,x_2) \geq \tilde{\pi}_{-1}^v(x_1,x_2)$*.*

The following proposition characterizes the second-period consumers' optimal search and purchasing strategies under sales volume information.

PROPOSITION S.28. *Upon observing the first-period sales volume* x_1 *and* x_2*, the optimal search and purchasing strategies of a second-period consumer with reservation utility* u_0 *and search cost* s *are as follows:*

(i) $x_1 + x_2 = \zeta_2$*:*

- *If* $u_0 \geq u_l$*, the consumer neither searches nor purchases.*
- *If* $u_0 < u_l$*, the consumer randomly chooses a product to purchase without searching it.*

(ii) $x_1 + x_2 = \zeta_1$*:*

- *If* $u_0 \geq u_h$*, the consumer neither searches nor purchases.*
- *If* $\tilde{\pi}_1^v(x_1,x_2)u_h + (1-\tilde{\pi}_1^v(x_1,x_2))u_l \leq u_0 < u_h$*, the consumer searches the bestseller product first if* $s \leq u_h - u_0$ *and leaves without search or purchase otherwise. If the bestseller is of high value, the consumer purchases the bestseller. If, however, the bestseller is revealed to be of low value, the consumer purchases the unsearched product without searching it (as it is of high value for sure).*
- *If* $u_0 < \tilde{\pi}_1^v(x_1,x_2)u_h + (1-\tilde{\pi}_1^v(x_1,x_2))u_l$*, the consumer searches the bestseller product first if* $s \leq (1-\tilde{\pi}_1^v(x_1,x_2))(u_h - u_l)$ *and purchases the bestseller directly without search otherwise. If the bestseller is of high value, the consumer who performs the first search purchases the bestseller. If, however, the bestseller is of low value, the consumer purchases the unsearched product without searching it (as it is of high value for sure).*

Similar to the roles of ranking information shown in Proposition S.27, sales volume guides the second-period consumers in both their first search and their purchase without any search (unless it reveals low value in both products, inducing the consumers to randomly purchase without search), as confirmed by Proposition S.28. In addition, Proposition S.28 implies that, as in the case of search required for purchase (§SN.1), heterogenous reservation utility renders sales volume more informative about the product values. This is again reflected in the fact that the second-period consumers are able to perfectly infer from the total sales volume about whether or not there is any high-value product.

While the learning outcome is largely the same with or without the requirement of search for purchase, an interesting difference is that the second search never occurs when search is optional for purchase. This is because the second-period consumers only perform the first search when they infer that there is at least one high-value product. Thus, if the first search reveals a low-value product, the remaining product must be of high value and thus consumers purchase it without the second search.

Numerical Illustration

Under the same assumptions for search-cost and reservation-utility distributions as in §SN.1, Table S.12 illustrates that relaxing the requirement of search for purchase does not alter the key insights of the model. That is, bestseller information can be advantageous to the platform by enhancing the total product sales and sales volume is sometimes, but not always, the platform's preferred form of bestseller information.

Table S.12 Impact of p_h on total expected sales in the second period (search optional for purchase):

$\mu = 4.5, \sigma = 1.5, n_2 = 100, u_h = 6, u_l = 2, \alpha = 0.08$

	$\theta = 1$				$\theta = 1.5$			
p_h	*Ranking*	*Volume*	*No Info.*	*Opt. Info.*	*Ranking*	*Volume*	*No Info.*	*Opt. Info.*
0.3	50.08	51.00	12.57	Volume	47.05	49.89	24.57	Volume
0.4	67.48	63.53	31.48	Ranking	61.44	62.91	38.12	Volume
0.5	85.94	75.00	51.00	Ranking	73.19	73.31	51.00	Volume
0.6	94.04	84.00	68.88	Ranking	82.11	82.34	63.72	Volume
0.7	95.37	91.00	88.37	Ranking	90.29	89.57	76.37	Ranking
0.8	98.68	96.00	98.68	Ranking	96.72	95.85	88.72	Ranking

SN.2.2. Impact of Sales Information on Consumer Search and Welfare

In this subsection we take the perspective of consumers and verify, under the assumptions of optional search for purchase and heterogeneous reservation utility, the effects of sales information on consumers from two aspects: search probabilities and aggregate welfare.

Search Probabilities

We consider ranking and volume information separately and start from the impact of ranking information.

Sales Ranking Information

Lemma S.31 characterizes the effects of ranking information on consumers' search probabilities.

LEMMA S.31. *Consider a second-period consumer with search cost s and reservation utility u_0. Compared to the case where no sales information is provided, ranking information provision leads to the following changes in the consumer's search probabilities:*

- $u_0 \geq u_h$: *both search probabilities remain the same for all s;*
- $\tilde{\pi}_1^r u_h + (1-\tilde{\pi}_1^r)u_l \leq u_0 < u_h$: *first-search probability increases while second-search probability decreases for all s;*
- $p_h u_h + p_l u_h \leq u_0 < \tilde{\pi}_1^r u_h + (1-\tilde{\pi}_1^r)u_l$: *change in first-search probability is ambiguous; second-search probability decreases for all s;*
- $\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_l \leq u_0 < p_h u_h + p_l u_h$: *change in both search probabilities is ambiguous;*
- $\frac{(1-\tilde{\pi}_1^r)\tilde{\pi}_2^r(u_h-u_l)}{1-\tilde{\pi}_2^r} + u_l \leq u_0 < \tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_l$: *first-search probability decreases and second-search probability increases for all s;*
- $u_l \leq u_0 < \frac{(1-\tilde{\pi}_1^r)\tilde{\pi}_2^r(u_h-u_l)}{1-\tilde{\pi}_2^r} + u_l$: *change in first-search probability is ambiguous; second-search probability increases for all s;*
- $u_0 < u_l$: *first-search probability decreases and second-search probability remains the same for all s;*

In the base model we find, under the assumption of search required for purchase, that public learning can be complementary to private learning, as public sales information sometimes promotes private product search. Lemma S.31 shows that the same phenomenon may occur when the requirement of search for purchase is lifted. In particular, as exemplified in the second case of the lemma, the first-search probability may increase since ranking information enhances consumers' confidence about discovering a high-value product through their search (of the bestseller). This is similar to the rationale in the base model. The increase in the second-search probability is, however, driven by the assumption of optional search for purchase, as explained below.

When a consumer's reservation utility satisfies $u_l \le u_0 < \tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_l$ (i.e., the fifth and sixth cases in Lemma S.31), conditional on the first search revealing a low-value product, the expected utility of purchasing the other product (say, product 2) without searching it dominates that of leaving without purchase. Thus, the consumer compares two options: (i) purchasing product 2 without searching it and (ii) searching product 2 and, if it turns out to be of low value, leaving without purchase and obtaining u_0. In addition to the search cost associated with option (ii), a key difference between the two options is that, if product 2 is of low value, the consumer obtains u_l under option (i) and u_0 under option (ii). Since $u_0 \ge u_l$, option (ii) becomes more desirable in relative to option (i) when product 2 is increasingly likely to be of low value. (Intuitively, the less confident a consumer is about a product being of high value, the less inclined he is to purchase it without search.) As in Lemma S.29, the belief that the lower-ranked product is of high value is lower than the prior (i.e., $\tilde{\pi}_2^r \le p_h$), implying that ranking information increases the belief about the lower-ranked product being of low value and thus renders the second search (i.e., option (ii)) more appealing.

In other cases, consumers may become more reluctant to search due to availability of the ranking information. For example, in the last case of Lemma S.31, a consumer with low reservation utility ($u_0 < u_l$) and intermediate search cost (e.g., $\tilde{\pi}_2^r(1-\tilde{\pi}_1^r)(u_h-u_l) < s \le p_h p_l(u_h-u_l)$) conducts the first search in absence of sales information, while purchases without search at all under ranking information. This highlights another potential benefit of social learning to consumers (in addition to enabling better informed purchases): it can alleviate consumers' time and efforts spent in product search. We shall reiterate on this point in the subsequent welfare analysis.

Sales Volume Information

Now we consider the situation when sales volume information is released. Recall that, under heterogeneous reservation utility, sales volume information allows consumers to learn from total sales volume about whether there is (at least) a high-value product. If it is inferred that both products are of low value (i.e., when $x_1+x_2 = \zeta_2$), none of the second-period consumers searches as the product value uncertainty is fully resolved. Thus, in such a case, volume information provision reduces both first-search and second-search probabilities for all the consumers. Lemma S.32 and S.33 below focus on the more interesting case, when volume information reveals that there is at least one high-value product (i.e., $x_1+x_2=\zeta_1$). For this case, recall that the second search never occurs, as discussed after Proposition S.28. Thus, the second-search probability under volume information is (weakly) lower than that in absence of any sales information and that under ranking information.

Lemma S.32. *Consider a second-period consumer with search cost s and reservation utility u_0. Compared to the case where no sales information is provided, volume information provision leads to the following changes in the consumer's search probabilities (assuming $x_1+x_2=\zeta_1$):*

- $u_0 \ge u_h$*: both search probabilities remain the same for all s;*
- $\tilde{\pi}_1^v(\zeta_1,0)u_h + (1-\tilde{\pi}_1^v(\zeta_1,0))u_l \le u_0 < u_h$*: first-search probability increases and second-search probability decreases for all s;*
- $p_h u_h + p_l u_l \le u_0 < \tilde{\pi}_1^v(\zeta_1,0)u_h + (1-\tilde{\pi}_1^v(\zeta_1,0))u_l$*: first-search probability decreases for $s \le p_h(u_h-u_0)$ and increases for $s > p_h(u_h-u_0)$, second-search probability decreases for all s;*
- $u_l \le u_0 < p_h u_h + p_l u_l$*: first-search probability decreases for $s \le p_h p_l(u_h-u_l)$ and increases for $s > p_h p_l(u_h-u_l)$, second-search probability decreases for all s;*

- $u_0 < u_l$: *first-search probability decreases for* $s \leq p_h p_l(u_h - u_l)$ *and increases for* $s > p_h p_l(u_h - u_l)$*, second-search probability remains the same for all* s*.*

Lemma S.33. *Consider a second-period consumer with search cost* s *and reservation utility* u_0*. Compared to the case where first-period sales ranking information is released, volume information provision leads to the following changes in the consumer's search probabilities (assuming* $x_1 + x_2 = \zeta_1$*):*

- $u_0 \geq u_h$: *both search probabilities remain the same for all* s*;*
- $\tilde{\pi}_1^v(\zeta_1, 0)u_h + (1 - \tilde{\pi}_1^v(\zeta_1, 0))u_l \leq u_0 < u_h$: *first-search probability increases and second-search probability decreases for all* s*;*
- $\tilde{\pi}_1^r u_h + (1 - \tilde{\pi}_1^r))u_l \leq u_0 < \tilde{\pi}_1^v(\zeta_1, 0)u_h + (1 - \tilde{\pi}_1^v(\zeta_1, 0))u_l$: *first-search probability decreases for* $s \leq \tilde{\pi}_1^r(u_h - u_0)$ *and increases for* $s > \tilde{\pi}_1^r(u_h - u_0)$*, second-search probability decreases for all* s*;*
- $\tilde{\pi}_2^r u_h + (1 - \tilde{\pi}_2^r))u_l \leq u_0 < \tilde{\pi}_1^r u_h + (1 - \tilde{\pi}_1^r))u_l$: *first-search probability decreases for* $s \leq \underline{s}_2$ *and increases for* $s > \underline{s}_2$*, second-search probability decreases for all* s*;*
- $u_l \leq u_0 < \tilde{\pi}_2^r u_h + (1 - \tilde{\pi}_2^r))u_l$: *first-search probability decreases for* $s \leq \underline{s}_3$ *and increases for* $s > \underline{s}_3$*, second-search probability decreases for all* s*;*
- $u_0 < u_l$: *first-search probability decreases for* $s \leq (1 - \tilde{\pi}_1^r)\tilde{\pi}_2^r(u_h - u_l)$ *and increases for* $s > (1 - \tilde{\pi}_1^r)\tilde{\pi}_2^r(u_h - u_l)$*, second-search probability remains the same for all* s*,*

where $\underline{s}_2$ *and* $\underline{s}_3$ *are as defined in Proposition S.27.*

Lemmas S.32 and S.33 detail the changes in the search probabilities due to volume information provision, in comparison with no sales information and ranking information provision, respectively. Once again, we note that, under (finer) public sales information, consumers may become more willing to conduct costly private search. This is, again, because bestseller information guides consumers to the product with higher expected value. Thus, the comparative analysis of the search probabilities confirms that public sales information may promote private product search, even when search is no longer required for purchase. Note that, as search becomes optional for purchase, the only incentive for consumers to search is to acquire product information. Thus, the finding that sales information sometimes boosts consumer search under optional search strengthens the result regarding complementarity between public learning and private learning.

In the meanwhile, similar to the results under ranking information, we note that some consumers may lower their search probabilities when volume information is provided. For example, in the last case of Lemma S.32, when the sales realization is such that $1 - \tilde{\pi}_1^v(x_1, x_2) < p_h p_l$, a consumer with low reservation utility $u_0 < u_l$ and intermediate search cost (e.g., $(1 - \tilde{\pi}_1^v(x_1, x_2))(u_h - u_l) < s \leq p_h p_l(u_h - u_l)$) performs the first search when no sales information is provided but purchases without search at all with sales volume information. In addition, as aforementioned, none of the second-period consumers performs the second search under volume information. This, again, suggests that social learning can be advantageous to consumers by lowering the search cost incurred before purchase, as we discuss in the following welfare analysis.

Aggregate Welfare

To evaluate the effects of bestseller information on aggregate consumer welfare, as well consumers' benefits from public learning and private learning, we numerically compare aggregate consumer surplus in four settings:

(i) no search and no (public) learning, labeled as "$NL\&NS$", where product search is infeasible (equivalently, search cost is infinite for all the consumers) and no sales information is provided;

(ii) search without learning, labeled as "$NL\&S$", where product search is feasible (with heterogenous search costs) and yet no sales information is provided;

(iii) learning without search, labeled as "$L\&NS$", where product search is infeasible and yet sales information is provided (ranking and volume information are considered separately);

(iv) learning with search, labeled as "$L\&S$", where product search is feasible (with heterogenous search costs) and sales information is provided (ranking and volume information are considered separately)

The welfare gains due to public or private learning are defined as the differences in the aggregate consumer surplus of two different settings. Specifically, $+S, NL$ and $+S, L$ represent the welfare gains due to search in the absence and presence of social learning, respectively, and $+L, NS$ and $+L, S$ represent the welfare gains due to public sales information when product search is infeasible and feasible, respectively. The assumptions on reservation-utility distribution and search-cost distribution (if search is feasible) are as in §SN.1 and the parameter values are as given in the captions of Tables S.13 and S.14.

Table S.13 Aggregate consumer welfare: $\mu = 4.5$, $\sigma = 1.5$, $u_h = 6$, $u_l = 2$, $\alpha = 0.08$, $p_h = 0.6$

Ranking Information								
θ	$NL\&NS$	$NL\&S$	$L\&NS$	$L\&S$	$+S,NL$	$+S,L$	$+L,NS$	$+L,S$
0.5	4.4050	4.4829	5.1100	5.1281	0.0779	0.0181	0.7050	0.6451
1	4.4900	4.5377	5.1680	5.1855	0.0477	0.0175	0.6780	0.6478
1.5	4.6017	4.6361	5.1619	5.1874	0.0344	0.0255	0.5602	0.5513
2	4.7200	4.7518	5.2782	5.2981	0.0318	0.0199	0.5582	0.5463
Volume Information								
θ	$NL\&NS$	$NL\&S$	$L\&NS$	$L\&S$	$+S,NL$	$+S,L$	$+L,NS$	$+L,S$
0.5	4.4050	4.4829	5.4300	5.4414	0.0779	0.0114	1.0250	0.9585
1	4.4900	4.5377	5.4880	5.4983	0.0477	0.0103	0.9980	0.9606
1.5	4.6017	4.6361	5.4612	5.4720	0.0344	0.0108	0.8595	0.8359
2	4.7200	4.7518	5.5260	5.5356	0.0318	0.0096	0.8060	0.7837

Table S.14 Aggregate consumer welfare: $\mu = 4.5$, $\sigma = 1.5$, $u_h = 6$, $u_l = 2$, $\alpha = 0.08$, $p_h = 0.15$

Ranking Information								
θ	$NL\&NS$	$NL\&S$	$L\&NS$	$L\&S$	$+S,NL$	$+S,L$	$+L,NS$	$+L,S$
0.5	4.0000	4.0185	4.0000	4.0245	0.0185	0.0245	0	0.0060
1	4.0000	4.0187	4.0030	4.0277	0.0187	0.0247	0.0030	0.0090
1.5	4.0017	4.0207	4.0620	4.0854	0.0190	0.0234	0.0604	0.0647
2	4.0450	4.0639	4.1522	4.1738	0.0189	0.0216	0.1072	0.1099
Volume Information								
θ	$NL\&NS$	$NL\&S$	$L\&NS$	$L\&S$	$+S,NL$	$+S,L$	$+L,NS$	$+L,S$
0.5	4.0000	4.0185	4.5550	4.5574	0.0185	0.0024	0.555	0.5388
1	4.0000	4.0187	4.5550	4.5574	0.0187	0.0024	0.555	0.5387
1.5	4.0017	4.0207	4.5550	4.5574	0.0190	0.0024	0.5533	0.5367
2	4.0450	4.0639	4.5489	4.5514	0.0189	0.0025	0.5039	0.4874

As observed from Tables S.13 and S.14, product search and sales information, either in isolation or combined with each other, improve consumer welfare. This demonstrates consumers' gain from both private learning and public learning. In particular, we note that the welfare gains from ranking information (i.e., $+L, NS$ and $+L, S$)

are lower than their counterparts from volume information, with or without product search. This resonates with the finding in the base model that the value of bestseller information to consumers increases as the information becomes finer.

In the meanwhile, the welfare gain from private learning can be reduced by the opportunity to learn from public sales information (i.e., $+S,L < +S,NL$). This is in line with some cases in Lemmas S.31 and S.32 where some consumers leverage social learning and save on search costs. In some other cases, e.g., ranking information in Table S.14, the result is opposite and the welfare gain from private learning is higher due to social learning (i.e., $+S,L > +S,NL$). This corresponds to the discussion immediately following Lemma S.31, where sales information may promote private search for consumers with relatively high reservation utilities, by either enhancing their confidence about discovering a high-value product through search or lowering the relative value of their option to purchase without search.

SN.3. Appendix

Proof of Lemma S.23 For the first case, as the utility of the no-purchase alternative is higher than that of a high-value product, the consumer does not search or purchase.

For the second case, as the utility of the no-purchase alternative is higher than that of a low-value product, the consumer never purchases a low-value product. Given that the first search reveals a low-value product, the expected utility of second search is $p_h u_h + p_l u_0 - s$ and the utility of not performing the second search is u_0 (as $u_0 \geq u_l$). The consumer is willing to perform the second search if and only if

$$p_h u_h + p_l u_0 - s \geq u_0$$

which is $s \leq p_h(u_h - u_0)$. The expected utility of the first search is $p_h u_h + p_l \max[p_h u_h + p_l u_0 - s, u_0] - s$ and the consumer performs the first search if and only if

$$p_h u_h + p_l \max[p_h u_h + p_l u_0 - s, u_0] - s \geq u_0$$

which is again $s \leq p_h(u_h - u_0)$. To see this, notice that $p_h u_h + p_l u_0 \leq p_h u_h + p_l \max[p_h u_h + p_l u_0 - s, u_0]$ so the consumer must perform the first search when $s \leq p_h(u_h - u_0)$. Conversely, when $s > p_h(u_h - u_0)$, the consumer does not perform the second search, $p_h u_h + p_l u_0 = p_h u_h + p_l \max[p_h u_h + p_l u_0 - s, u_0]$ and the consumer does not perform the first search.

For the third case, the consumer is willing to purchase a low-value product as the utility of the no-purchase alternative is lower than that of a low-value product. Given that the first search reveals a low-value product, the expected utility of second search is $p_h u_h + p_l u_l - s$ and the utility of not performing the second search is u_l (as $u_0 < u_l$). The consumer is willing to perform the second search if and only if

$$p_h u_h + p_l u_l - s \geq u_l$$

which is $s \leq p_h(u_h - u_l)$. The expected utility of the first search is $p_h u_h + p_l \max[p_h u_h + p_l u_l - s, u_l]$ and the consumer performs the first search if and only if

$$p_h u_h + p_l \max[p_h u_h + p_l u_l - s, u_l] - s \geq u_0$$

which is $s \leq p_h u_h + p_l u_l - u_0$. To see this, notice that $p_h u_h + p_l u_l \leq p_h u_h + p_l \max[p_h u_h + p_l u_l - s, u_l]$ so the consumer with $s \leq p_h u_h + p_l u_l - u_0$ must perform the first search. Conversely, when $s > p_h u_h + p_l u_l - u_0$, the consumer does not perform the second search as $p_h(u_h - u_l) < p_h u_h + p_l u_l - u_0$ (as $u_0 < u_l$). So $p_h u_h + p_l u_l = p_h u_h + p_l \max[p_h u_h + p_l u_l - s, u_l]$ and the consumer does not perform the first search. □

Proof of Lemma S.24 By definition, k_1 is the number of consumers who search in the second case and k_2 is the number of consumers who search in the third case. As consumers in the second case only purchase when there is at least one high-value product and consumers in the third case always purchase after search, total sales equals to $k_1 + k_2$ if there is at least one high-value product and equals to k_2 if there is no high-value product.

When both products are of high value, the two products are symmetric and total sales is $k_1 + k_2$. So the sales distribution for either product follows distribution $G_s^h(x)$. Similarly, when both products are of low value, the two products are symmetric and total sales is k_2 and the sales distribution for either product follows distribution $G_s^l(x)$.

When the two products are of different values, the k_1 consumers in the second case and the m_1 consumers in the third case must purchase the high-value product as they are willing to perform the second search when the first search reveals a low-value product. For the $k_2 - m_1$ consumers in the third case, they do not perform the second search and randomly pick a product to search and purchase. So the sales of the high-value product follows distribution $G_a(x)$ and the sales of the low value product follows distribution $\bar{G}_a(k_1 + k_2 - x)$. □

Proof of Lemma S.25 The proof is same to the base model by replacing n_1 with $k_1 + k_2$. □

Proof of Proposition S.24 For the first case, as the utility of the no-purchase alternative is higher than that of a high-value product, they leave without search or purchase.

For the second case, the expected utility of performing the second search is $\pi_2^r u_h + (1 - \pi_2^r) u_0 - s$ and utility of leaving without purchase is u_0. So the consumer performs the second search if and only if $\pi_2^r u_h + (1 - \pi_2^r) u_0 - s \geq u_0$, which is $s \leq \pi_2^r (u_h - u_0)$. The expected utility of first search is

$$\pi_1^r u_h + (1 - \pi_1^r) \max[u_0, \pi_2^r u_h + (1 - \pi_2^r) u_0 - s] - s$$

while the utility of not performing the first search is u_0. The consumer performs the first search if and only if $s \leq \pi_1^r (u_h - u_0)$. To see this, notice that $\pi_1^r u_h + (1 - \pi_1^r) u_0 \leq \pi_1^r u_h + (1 - \pi_1^r) \max[u_0, \pi_2^r u_h + (1 - \pi_2^r) u_0 - s]$ so the consumer must perform the first search if $s \leq \pi_1^r (u_h - u_0)$. Conversely, when $s > \pi_1^r (u_h - u_0)$, the consumer does not perform the second search. So $\pi_1^r u_h + (1 - \pi_1^r) u_0 = \pi_1^r u_h + (1 - \pi_1^r) \max[u_0, \pi_2^r u_h + (1 - \pi_2^r) u_0 - s]$ and the consumer does not perform the first search. As the consumer has higher utility for the no-purchase alternative than the low-value product, she never purchases a low-value product.

For the third case, the expected utility of performing the second search is $\pi_2^r u_h + (1 - \pi_2^r) u_l - s$ and the utility of purchasing the low-value product is u_l. The consumer performs the second search if and only if $\pi_2^r u_h + (1 - \pi_2^r) u_l - s \geq u_l$, which is $s \leq \pi_2^r (u_h - u_l)$. The expected utility of first search is

$$\pi_1^r u_h + (1 - \pi_1^r) \max[u_l, \pi_2^r u_h + (1 - \pi_2^r) u_l - s] - s$$

while the utility of not performing the first search is u_0. The consumer performs the first search if and only if $s \leq \pi_1^r u_h + (1 - \pi_1^r) u_l - u_0$. To see this, notice that $\pi_1^r u_h + (1 - \pi_1^r) u_l \leq \pi_1^r u_h + (1 - \pi_1^r) \max[u_l, \pi_2^r u_h + (1 - \pi_2^r) u_l - s]$ so the consumer must perform the first search when $s \leq \pi_1^r u_h + (1 - \pi_1^r) u_l - u_0$. Conversely, when $s > \pi_1^r u_h +$

$(1-\pi_1^r)u_l - u_0$, the consumer does not perform the second search as $\pi_1^r u_h + (1-\pi_1^r)u_l - u_0 > \pi_2^r(u_h - u_l)$ (as $u_l > u_0$ and $\pi_1^r \geq \pi_2^r$). So $\pi_1^r u_h + (1-\pi_1^r)u_l = \pi_1^r u_h + (1-\pi_1^r)\max[u_l, \pi_2^r u_h + (1-\pi_2^r)u_l - s]$ and the consumer does not perform the first search. As the consumer in this case prefer a low-value product over the no-purchase alternative, she always purchases a product after search. □

Proof of Proposition S.25 Denote $P(x_1, x_2)$ as the probability that the sales realization is $X_1 = x_1, X_2 = x_2$. Notice that $\pi_1^v(x_1, x_2) = 0$ when $x_1 + x_2 = k_2$ and it suffices to consider the case $x_1 + x_2 = k_1 + k_2$. Assuming $k_1 + k_2$ is odd, we have

$$\begin{aligned}
\mathbb{E}[\pi_1^v(x_1, x_2)] &= \frac{\sum_{x_1+x_2=k_1+k_2, x_1 \geq x_2} P(x_1, x_2)\pi_1^v(x_1, x_2)}{\Pr(x_1 \geq x_2 | x_1 + x_2 = k_1 + k_2)} \\
&= 2 \sum_{x_1+x_2=k_1+k_2, x_1\geq x_2} P(x_1, x_2) \frac{p_h g_s^h(x_1) + p_l g_a(x_1)}{p_h g_s^h(x) + p_l g_a(x_1) + p_l g_a(x_2)} \\
&= 2 \sum_{x_1+x_2=k_1+k_2, x_1\geq x_2} (p_h^2 g_s^h(x_1) + p_h p_l g_a(x_1) + p_h p_l g_a(x_2)) \frac{p_h g_s^h(x_1) + p_l g_a(x_1)}{p_h g_s^h(x) + p_l g_a(x_1) + p_l g_a(x_2)} \\
&= 2 \sum_{x_1+x_2=k_1+k_2, x_1\geq x_2} (p_h^2 g_s^h(x_1) + p_h p_l g_a(x_1)) \\
&= 2p_h^2 \bar{G}_s^h(\frac{k_1+k_2}{2}) + 2p_h p_l \bar{G}_a(\frac{k_1+k_2}{2}) \\
&= p_h^2 + 2p_h p_l \bar{G}_a(\frac{k_1+k_2}{2}) = \pi_1^r
\end{aligned}$$

The second equality follows from $\Pr(x_1 \geq x_2 | x_1 + x_2 = k_1 + k_2) = 1/2$ as the two products are ex ante homogeneous. The third equality follows from the identity that $P(x_1, x_2) = p_h^2 g_s^h(x_1) + p_h p_l g_a(x_1) + p_h p_l g_a(x_2)$. The case where $k_1 + k_2$ is even can be proved similarly and is omitted here. □

Proof of Lemma S.26 It is equivalent to show $g_a(x_1) \geq g_a(x_2)$ for $x_1 \geq x_2, x_1 + x_2 = k_1 + k_2$, which is the same as the proof in the base model by replacing n_1 with $k_1 + k_2$. □

Proof of Proposition S.26 When $x_1 + x_2 = k_2$, both products are of low value. So only consumers with $u_0 \leq u_l$ and $s \leq u_l - u_0$ search and make purchase.

Next we consider the case where $x_1 + x_2 = k_1 + k_2$. For the consumer with $u_0 \geq u_h$, she does not search or purchase. For the consumer with $u_l \leq u_0 < u_h$, if the first search reveals a low-value product, then the unsearched product must be of high value. So she performs the second search if and only if $u_h - s \geq u_0$, which is $s \leq u_h - u_0$. The expected utility of the first search is

$$\pi_1^v(x_1, x_2)u_h + (1 - \pi_1^v(x_1, x_2))\max[u_0, u_h - s] - s$$

and the consumer performs the first search if and only if $\pi_1^v(x_1, x_2)u_h + (1-\pi_1^v(x_1, x_2))\max[u_0, u_h - s] - s \geq u_0$, which is $s \leq \frac{u_h - u_0}{2-\pi_1^v(x_1,x_2)}$. To see this, notice that $\pi_1^v(x_1, x_2)u_h + (1 - \pi_1^v(x_1, x_2))(u_h - s) \leq \pi_1^v(x_1, x_2)u_h + (1 - \pi_1^v(x_1, x_2))\max[u_0, u_h - s]$ so the consumer must perform the first search when $s \leq \frac{u_h - u_0}{2-\pi_1^v(x_1,x_2)}$. Conversely, when $s > \frac{u_h - u_0}{2-\pi_1^v(x_1,x_2)}$, $\pi_1^v(x_1, x_2)u_h + (1-\pi_1^v(x_1, x_2))(u_h - s) - s < u_0$ and $\pi_1^v(x_1, x_2)u_h + (1-\pi_1^v(x_1, x_2))u_0 - s < u_0$ (as $\frac{u_h - u_0}{2-\pi_1^v(x_1,x_2)} \geq \pi_1^v(x_1, x_2)(u_h - u_0)$), and the consumer does not perform the first search.

For the consumer with $u_l - (1 - \pi_1^v(x_1, x_2))(u_h - u_l) \leq u_0 < u_l$, she performs the second search if and only if $u_h - s \geq u_l$ (as $u_l > u_0$), which is $s \leq u_h - u_l$. The expected utility of the first search is

$$\pi_1^v(x_1, x_2)u_h + (1 - \pi_1^v(x_1, x_2))\max[u_l, u_h - s] - s$$

and the consumer performs the first search if and only if $\pi_1^v(x_1,x_2)u_h + (1-\pi_1^v(x_1,x_2))\max[u_l, u_h - s] - s \geq u_0$, which is $s \leq \frac{u_h - u_0}{2-\pi_1^v(x_1,x_2)}$.

For the consumer with $u_0 < u_l - (1-\pi_1^v(x_1,x_2))(u_h - u_l)$, she performs the second search if and only if $u_h - s \geq u_l$ (as $u_l > u_0$), which is $s \leq u_h - u_l$. The expected utility of the first search is

$$\pi_1^v(x_1,x_2)u_h + (1-\pi_1^v(x_1,x_2))\max[u_l, u_h - s] - s$$

and the consumer performs the first search if and only if $\pi_1^v(x_1,x_2)u_h + (1-\pi_1^v(x_1,x_2))\max[u_l, u_h - s] - s \geq u_0$, which is $s \leq \pi_1^v(x_1,x_2)u_h + (1-\pi_1^v(x_1,x_2))\max[u_l, u_h - s] - u_0$. This is equivalent to

$$s \leq \max[\pi_1^v(x_1,x_2)u_h + (1-\pi_1^v(x_1,x_2))u_l - u_0, \frac{u_h - u_0}{2-\pi_1^v(x_1,x_2)}].$$

So $\underline{s}_1 = \frac{u_h - u_0}{2-\pi_1^v(x_1,x_2)}$ when $u_l - (1-\pi_1^v(x_1,x_2))(u_h - u_l) \leq u_0 < u_l$ and $\underline{s}_1 = \pi_1^v(x_1,x_2)u_h + (1-\pi_1^v(x_1,x_2))u_l - u_0$ when $u_0 < u_l - (1-\pi_1^v(x_1,x_2))(u_h - u_l)$. □

Proof of Lemma S.27 For the first case ($u_0 \geq u_h$), the consumer neither searches nor purchases as the utility of the no-purchase alternative is higher than the utility of a high-value product.

For the second case ($p_h u_h + p_l u_l \leq u_0 < u_h$), as the utility of the no-purchase alternative is higher than the expected utility of an unsearched product, the consumer does not purchase a product without search. For the second search, the consumer performs the second search if and only if

$$p_h u_h + p_l u_0 - s \geq u_0$$

which is $s \leq p_h(u_h - u_0)$. The consumer is willing to perform the first search if and only if

$$p_h u_h + p_l \max[p_h u_h + p_l u_0 - s, u_0] - s \geq u_0$$

which is again $s \leq p_h(u_h - u_0)$ (see Lemma S.23). The consumer in this case does not purchase a low-value product as $u_0 > u_l$.

For the third case ($u_l \leq u_0 < p_h u_h + p_l u_l$), as $u_0 > u_l$, the consumer does not purchase a product that is revealed to be of low value. For the second search, the consumer considers the second search when the first search reveals a low-value product. The utility of purchasing the unsearched product is $p_h u_h + p_l u_l$, the utility of choosing the no-purchase alternative is u_0, and the utility of searching the unsearched product is $p_h u_h + p_l u_0 - s$ (note that the consumer chooses the no-purchase alternative when the second search again reveals a low-value product). As $u_0 < p_h u_h + p_l u_l$, the consumer performs the second search if and only if $p_h u_h + p_l u_l \leq p_h u_h + p_l u_0 - s$, which is $s < p_l(u_0 - u_l)$, and purchases the unseareched product directly otherwise.

Then consider the first search. As the two products are ex-ante homogeneous, the expected utility of purchasing a product without search is $p_h u_h + p_l u_l$, the utility of choosing the no-purchase alternative is u_0, and the utility of searching a product is $p_h u_h + p_l \max(p_h u_h + p_l u_l, p_h u_h + p_l u_0 - s) - s$. Since $u_0 < p_h u_h + p_l u_l$, the consumer performs the first search if and only if $p_h u_h + p_l \max(p_h u_h + p_l u_l, p_h u_h + p_l u_0 - s) - s > p_h u_h + p_l u_l$, which is $s < p_h p_l(u_h - u_l)$. The sufficiency of the condition follows from $p_h u_h + p_l \max(p_h u_h + p_l u_l, p_h u_h + p_l u_0 - s) - s \geq p_h u_h + p_l(p_h u_h + p_l u_l) - s > p_h u_h + p_l u_l$. To see the necessity, consider the consumer with search cost $s > p_h p_l(u_h - u_l)$. Notice that $p_h p_l(u_h - u_l) - p_l(u_0 - u_l) = p_l(p_h u_h + p_l u_l - u_0) > 0$ as $u_0 < p_h u_h + p_l u_l$, thus $s > p_l(u_0 - u_l)$. Therefore, if the first search reveals a low value product, $p_h u_h + p_l u_0 - s < p_h u_h + p_l u_l$ and the consumer does not perform the second search and purchases the unsearched product directly. Hence, her expected utility of first search is $p_h u_h + p_l \max(u_0, p_h u_h + p_l u_l, p_h u_h + p_l u_0 - s) - s = p_h u_h + p_l(p_h u_h + p_l u_l) - s < p_h u_h + p_l u_l$.

To summarize, in the third case, the consumer with search cost $s > p_h p_l(u_h - u_l)$ does not perform the first search. As the two products are symmetric, she purchases either product with equal probability. For the consumer with search cost $p_l(u_0 - u_l) < s < p_h p_l(u_h - u_l)$, she performs the first search but not the second search. When the first search reveals a high-value product, she purchases it; if it reveals a low-value product, as $p_h u_h + p_l u_l > u_0$, she purchases the other product without searching it. For the consumer with search cost $s < p_l(u_0 - u_l)$ and $u_l \le u_0 < p_h u_h + p_l u_l$, she performs the first search and the second search if the first search reveals a low value product. Thus, the consumer never purchases a product without searching it. If both products are of low value, she leaves without purchase as $u_0 > u_l$.

For the fourth case ($u_0 < u_l$): The consumer is willing to purchase a low-value product. If the first search reveals a low-value product, the expected utility of performing the second search is $p_h u_h + p_l u_l - s$ and the expected utility of purchasing the second product without search is $p_h u_h + p_l u_l$. So the consumer purchases the unsearched product directly without performing the second search. For the first search, the expected utility of purchasing a product directly is $p_h u_h + p_l u_l$ and the expected utility of performing the first search is $p_h u_h + p_l(p_h u_h + p_l u_l) - s$. The consumer chooses to search if and only if $p_h u_h + p_l(p_h u_h + p_l u_l) - s \ge p_h u_h + p_l u_l$, which is $s \le p_h p_l(u_h - u_l)$. When search cost is higher than $p_h p_l(u_h - u_l)$, the consumer randomly chooses a product to purchase as $u_0 < u_l$. □

Proof of Lemma S.28: From Lemma S.27, $\kappa_1 + \kappa_{23}$ consumers only make purchase when there is a high-value product, $\kappa_{21} + \kappa_{31}$ consumers randomly purchase a product, and $\kappa_{22} + \kappa_{32}$ consumers makes one search and purchase the searched product if it is of high value and purchase the other product if it is of low value. Hence, the total sales in the first period is $\kappa_{21} + \kappa_{31} + \kappa_{22} + \kappa_{32} + \kappa_1 + \kappa_{23}$ if there is at least one high-value product and $\kappa_{21} + \kappa_{31} + \kappa_{22} + \kappa_{32}$ if both products are of low value and (i) is proved. For (ii) and (iii), it suffices to notice that the two products are symmetric and consumers purchase each product with equal probability. For (iv), notice that the $\kappa_{22} + \kappa_{32}$ consumers purchase a high-value product for sure whenever there is one and $\kappa_1 + \kappa_{23}$ consumers only make purchase when there is a high-value product and purchase a high-value product when there is one. So the $\kappa_1 + \kappa_{23} + \kappa_{22} + \kappa_{32}$ consumers always purchase the high-value product while the remaining consumers randomly pick a product to purchase. □

Proof of Lemma S.29 The proof is the same as the base model by replacing n_1 with ζ_1. □

Proof of Proposition S.27 For the first case with $u_0 \le u_h$, the consumer does not search or purchase as $u_0 > u_h$. For the second case with $\tilde{\pi}_1^r u_h + (1 - \tilde{\pi}_1^r) u_l \le u_0 < u_h$, the consumer does not purchase a product without search and only purchases high-value product as $\tilde{\pi}_1^r u_h + (1 - \tilde{\pi}_1^r) u_l \le u_0$. The expected utility of performing the second search when the first search reveals a low-value product is $\tilde{\pi}_2^r u_h + (1 - \tilde{\pi}_2^r) u_0 - s$ and the utility of not performing the second search is u_0. The consumer in this case performs the second search if and only if $u_0 \le \tilde{\pi}_2^r u_h + (1 - \tilde{\pi}_2^r) u_0 - s$, which is $s \le \tilde{\pi}_2^r (u_h - u_0)$. The expected utility of first search is $\tilde{\pi}_1^r u_h + (1 - \tilde{\pi}_1^r) \max[u_0, \tilde{\pi}_2^r u_h + (1 - \tilde{\pi}_2^r) u_0 - s] - s$ and the consumer performs the first search if and only if $\tilde{\pi}_1^r u_h + (1 - \tilde{\pi}_1^r) \max[u_0, \tilde{\pi}_2^r u_h + (1 - \tilde{\pi}_2^r) u_0 - s] - s \ge u_0$, which is $s \le \tilde{\pi}_1^r (u_h - u_0)$ (see the proof of Proposition S.24).

For the third case with $\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_l \le u_0 < \tilde{\pi}_1^r u_h + (1-\tilde{\pi}_1^r)u_l$, the consumer is willing to purchase the bestseller product directly but not the lower sales product. The expected utility of performing the second search is $\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_0 - s$ and the utility of not performing the second search is u_0 given that the first search reveals a low-value product. The consumer in this case performs the second search if and only if $\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_0 - s \ge u_0$, which is $s \le \tilde{\pi}_2^r(u_h - u_0)$. For the consumer whose first search reveals a low-value product and is unwilling to perform the second search, she leaves without purchase. The expected utility of the first search is $\tilde{\pi}_1^r u_h + (1-\tilde{\pi}_1^r)\max[u_0, \tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_0 - s] - s$ and the expected utility of purchasing product 1 directly is $\tilde{\pi}_1^r u_h + (1-\tilde{\pi}_1^r)u_l$. The consumer performs the first search if and only if $\tilde{\pi}_1^r u_h + (1-\tilde{\pi}_1^r)\max[u_0, \tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_0 - s] - s \ge \tilde{\pi}_1^r u_h + (1-\tilde{\pi}_1^r)u_l$, which is

$$s \le \max[(1-\tilde{\pi}_1^r)(u_0 - u_l), (1-\tilde{\pi}_1^r)\frac{\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_0 - u_l}{2-\tilde{\pi}_1^r}].$$

So $\underline{s}_2 = (1-\tilde{\pi}_1^r)(u_0 - u_l)$ when $\frac{\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_1^r)u_l}{1-\tilde{\pi}_1^r+\tilde{\pi}_2^r} \le u_0 < \tilde{\pi}_1^r u_h + (1-\tilde{\pi}_1^r)u_l$ and $\underline{s}_2 = (1-\tilde{\pi}_1^r)\frac{\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_0 - u_l}{2-\tilde{\pi}_1^r}$ when $\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_l \le u_0 < \frac{\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_1^r)u_l}{1-\tilde{\pi}_1^r+\tilde{\pi}_2^r}$.

For the fourth case with $u_l \le u_0 < \tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_l$, the consumer is willing to purchase both products without search. The expected utility of performing the second search is $\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_0 - s$ and the utility of purchasing the unsearched product directly is $\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_l$ when the first search reveals a low-value product. The consumer in this case performs the second search if and only if $\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_l \le \tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_0 - s$, which is $s \le (1-\tilde{\pi}_2^r)(u_0 - u_l)$. When the consumer has higher search cost, she purchases the unsearched product directly. The expected utility of the first search is $\tilde{\pi}_1^r u_h + (1-\tilde{\pi}_1^r)\max[\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_l, \tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_0 - s] - s$ and the expected utility of purchasing the bestseller directly is $\tilde{\pi}_1^r u_h + (1-\tilde{\pi}_1^r)u_l$. The consumer performs the first search if and only if $\tilde{\pi}_1^r u_h + (1-\tilde{\pi}_1^r)\max[\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_l, \tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_0 - s] - s \ge \tilde{\pi}_1^r u_h + (1-\tilde{\pi}_1^r)u_l$, which is equivalent to

$$s \le \max[(1-\tilde{\pi}_1^r)\tilde{\pi}_2^r(u_h - u_l), \frac{\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_0 - u_l}{2-\tilde{\pi}_1^r}].$$

So $\underline{s}_3 = (1-\tilde{\pi}_1^r)\tilde{\pi}_2^r(u_h - u_l)$ when $\frac{(1-\tilde{\pi}_1^r)\tilde{\pi}_2^r(u_h-u_l)}{1-\tilde{\pi}_2^r} + u_l \le u_0 < \tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_l$ and $\underline{s}_3 = \frac{\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_0 - u_l}{2-\tilde{\pi}_1^r}$ when $u_l \le u_0 < \frac{(1-\tilde{\pi}_1^r)\tilde{\pi}_2^r(u_h-u_l)}{1-\tilde{\pi}_2^r} + u_l$.

For the last case with $u_0 < u_l$, the consumer always purchases and never performs the second search as $u_0 < u_l$. If the first search reveals a low-value product, the consumer purchases the lower sales product directly as $\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_l \ge u_l$. The expected utility of searching the bestseller is $\tilde{\pi}_1^r u_h + (1-\tilde{\pi}_1^r)(\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r)u_l) - s$ and the expected utility of purchasing the bestseller directly is $\tilde{\pi}_1^r u_h + (1-\tilde{\pi}_1^r)u_l$. The consumer performs the first search if and only if $s \le (1-\tilde{\pi}_1^r)\tilde{\pi}_2^r(u_h - u_l)$. The consumer with higher search cost purchases the bestseller directly. □

Proof of Lemma S.30 Same as base model by replacing n_1 with ζ_1. □

Proof of Proposition S.28 When $x_1 + x_2 = \zeta_2$, both products are of low value. The consumer with $u_0 \ge u_h$ leaves without purchase while the consumer with $u_0 < u_l$ randomly chooses a product to purchase directly.

Next we consider the case where $x_1 + x_2 = \zeta_1$. Notice that as $\tilde{\pi}_2^v(x_1, x_2) = 1$, the consumer does not perform the second search. If the first search reveals a low-value product, the consumer purchases the unsearched product.

For the first case, the consumer does not search nor purchase. For the second case, the expected utility of searching the bestseller is $u_h - s$ (as one of the two products must be of high value and the consumer can purchase without search) and the utility of not searching is u_0. The consumer performs the first search if and only if $u_0 \leq u_h - s$, which is $s \leq u_h - u_0$. The consumer leaves without search or purchase when search cost is higher. For the third case, as $u_0 < \tilde{\pi}_1^v(x_1, x_2)u_h + (1 - \tilde{\pi}_1^v(x_1, x_2))u_l$. the utility of not searching is $\tilde{\pi}_1^v(x_1, x_2)u_h + (1 - \tilde{\pi}_1^v(x_1, x_2))u_l$ and the utility of searching the bestseller is $u_h - s$. The consumer performs the first search if and only if $\tilde{\pi}_1^v(x_1, x_2)u_h + (1 - \tilde{\pi}_1^v(x_1, x_2))u_l \leq u_h - s$, which is $s \leq (1 - \tilde{\pi}_1^v(x_1, x_2))(u_h - u_l)$. The consumer purchases the bestseller directly when search cost is higher. □

Proof of Lemma S.31 We consider different cases as discussed below.

- $u_0 \geq u_h$: In this case, the consumer does not search and both search probabilities remain the same.
- $\tilde{\pi}_1^r u_h + (1 - \tilde{\pi}_1^r)u_l \leq u_0 < u_h$: In this case, without sales information the consumer performs the first search and the second search if and only if $s \leq p_h(u_h - u_0)$. When sales ranking information is released, the consumer performs the first search if and only if $s \leq \tilde{\pi}_1^r(u_h - u_0)$ and the second search if and only if $s \leq \tilde{\pi}_2^r(u_h - u_0)$. As $\tilde{\pi}_2^r \leq p_h \leq \tilde{\pi}_1^r$, first-search probability increases and second-search probability decreases.
- $p_h u_h + p_l u_h \leq u_0 < \tilde{\pi}_1^r u_h + (1 - \tilde{\pi}_1^r)u_l$: In this case, without sales information the consumer performs the first search and the second search if and only if $s \leq p_h(u_h - u_0)$. When sales ranking information is released, the consumer performs the first search if and only if $s \leq (1 - \tilde{\pi}_1^r)(u_0 - u_l)$ or $s \leq (1 - \tilde{\pi}_1^r)\frac{\tilde{\pi}_2^r u_h + (1 - \tilde{\pi}_2^r)u_0 - u_l}{2 - \tilde{\pi}_1^r}$ and the second search if and only if $s \leq \tilde{\pi}_2^r(u_h - u_0)$. As $\tilde{\pi}_2^r \leq p_h$, second-search probability decreases. Change of first-search probability is ambiguous.
- $\tilde{\pi}_2^r u_h + (1 - \tilde{\pi}_2^r)u_l \leq u_0 < p_h u_h + p_l u_h$: In this case, without sales information the consumer performs the first search if and only if $s \leq p_h p_l(u_h - u_l)$ and performs the second search if and only if $s < p_l(u_0 - u_l)$. When sales ranking information is released, the consumer performs the first search if and only if $s \leq (1 - \tilde{\pi}_1^r)(u_0 - u_l)$ or $s \leq (1 - \tilde{\pi}_1^r)\frac{\tilde{\pi}_2^r u_h + (1 - \tilde{\pi}_2^r)u_0 - u_l}{2 - \tilde{\pi}_1^r}$ and the second search if and only if $s \leq \tilde{\pi}_2^r(u_h - u_0)$. Change is ambiguous here.
- $\frac{(1 - \tilde{\pi}_1^r)\tilde{\pi}_2^r(u_h - u_l)}{1 - \tilde{\pi}_2^r} + u_l \leq u_0 < \tilde{\pi}_2^r u_h + (1 - \tilde{\pi}_2^r)u_l$: In this case, without sales information the consumer performs the first search if and only if $s \leq p_h p_l(u_h - u_l)$ and performs the second search if and only if $s < p_l(u_0 - u_l)$. When sales ranking information is released, the consumer performs the first search if and only if $s \leq (1 - \tilde{\pi}_1^r)\tilde{\pi}_2^r(u_h - u_l)$ and the second search if and only if $s \leq (1 - \tilde{\pi}_2^r)(u_0 - u_l)$. As $\tilde{\pi}_2^r \leq p_h \leq \tilde{\pi}_1^r$, second-search probability increases and first-search probability decreases.
- $u_l \leq u_0 < \frac{(1 - \tilde{\pi}_1^r)\tilde{\pi}_2^r(u_h - u_l)}{1 - \tilde{\pi}_2^r} + u_l$: second-search probability increases as the previous case while the change for first-search probability is ambiguous.
- $u_0 < u_l$: In this case, without sales information the consumer performs the first search if and only if $s \leq p_h p_l(u_h - u_l)$ and never performs the second search. When sales ranking information is released, the consumer performs the first search when $s \leq (1 - \tilde{\pi}_1^r)\tilde{\pi}_2^r(u_h - u_l)$ and never performs the second search. So first-search probability decreases and second-search probability remains the same. □

Proof of Lemma S.32 We consider the following cases. Recall that no consumer performs the second search when sales volume information is released.

- $u_0 \geq u_h$: the consumer does not search and both search probabilities remain the same.
- $\tilde{\pi}_1^v(\zeta_1, 0)u_h + (1 - \tilde{\pi}_1^v(\zeta_1, 0))u_l \leq u_0 < u_h$: Without sales information, the consumer performs the first and the second search if and only if $s \leq p_h(u_h - u_0)$. When sales volume information is released, the consumer performs the first search if and only if $s \leq u_h - u_0$. So first-search probability increases and second-search probability decreases.

- $p_h u_h + p_l u_l \leq u_0 < \tilde{\pi}_1^v(n_1,0)u_h + (1-\tilde{\pi}_1^v(n_1,0))u_l$: Without sales information, the consumer performs the first and second search if and only if $s \leq p_h(u_h - u_0)$. With sales volume information, the consumer performs the first search if and only if $s \leq u_h - u_0$ or $s \leq (1-\tilde{\pi}_1^v(x_1,x_2))(u_h - u_l)$ depending on the value of x_1, x_2. So first-search probability decreases for $s \leq p_h(u_h - u_0)$, increases for $s > p_h(u_h - u_0)$, and second search probability decreases.
- $u_l \leq u_0 < p_h u_h + p_l u_l$: Without sales information, the consumer performs the first if and only if $s \leq p_h p_l(u_h - u_l)$ and performs the second search if and only if $s \leq p_l(u_0 - u_l)$. With sales volume information, the consumer performs the first search if and only if $s \leq u_h - u_0$ or $s \leq (1-\tilde{\pi}_1^v(x_1,x_2))(u_h - u_l)$ depending on the value of x_1, x_2. So first-search probability decreases for $s \leq p_h p_l(u_h - u_l)$, increases for $s > p_h p_l(u_h - u_l)$, and second search probability decreases.
- $u_0 < u_l$: Without sales information, the consumer never performs the second search and performs the first search if and only if $s \leq p_h p_l(u_h - u_l)$. With sales volume information, the consumer performs the first search if and only if $s \leq (1-\tilde{\pi}_1^v(x_1,x_2))(u_h - u_l)$. So first-search probability decreases for $s \leq p_h p_l(u_h - u_l)$, increases for $s > p_h p_l(u_h - u_l)$, and second search probability remain the same. □

Proof of Lemma S.33 We consider the following cases. Again notice that no consumer performs the second search when sales volume information is released.

- $u_0 \geq u_h$: the consumer does not search and both search probabilities remain the same.
- $\tilde{\pi}_1^v(\zeta_1,0)u_h + (1-\tilde{\pi}_1^v(\zeta_1,0))u_l \leq u_0 < u_h$: When sales ranking information is released, the consumer performs the first search if and only if $s \leq \tilde{\pi}_1^r(u_h - u_0)$ and the second search if and only if $s \leq \tilde{\pi}_2^r(u_h - u_0)$. When sales volume information is released, the consumer performs the first search if and only if $s \leq u_h - u_0$. So first-search probability increases and second-search probability decreases.
- $\tilde{\pi}_1^r u_h + (1-\tilde{\pi}_1^r))u_l \leq u_0 < \tilde{\pi}_1^v(\zeta_1,0)u_h + (1-\tilde{\pi}_1^v(\zeta_1,0))u_l$: With sales ranking information, the consumer performs the first search if and only if $s \leq \tilde{\pi}_1^r(u_h - u_0)$ and the second search if and only if $s \leq \tilde{\pi}_2^r(u_h - u_0)$. With sales volume information, the consumer performs the first search if and only if $s \leq u_h - u_0$ or $s \leq (1-\tilde{\pi}_1^v(x_1,x_2))(u_h - u_l)$ depending on the value of x_1, x_2. So first-search probability decreases for $s \leq \tilde{\pi}_1^r(u_h - u_0)$, increases for $s > \tilde{\pi}_1^r(u_h - u_0)$, and second-search probability decreases.
- $\tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r))u_l \leq u_0 < \tilde{\pi}_1^r u_h + (1-\tilde{\pi}_1^r))u_l$: With sales ranking information, the consumer performs the first search if and only if $s \leq \underline{s}_2$ and the second search if and only if $s \leq \tilde{\pi}_2^r(u_h - u_0)$. With sales volume information, the consumer performs the first search if and only if $s \leq u_h - u_0$ or $s \leq (1-\tilde{\pi}_1^v(x_1,x_2))(u_h - u_l)$ depending on the value of x_1, x_2. So first-search probability decreases for $s \leq \underline{s}_2$, increases for $s > \underline{s}_2$, and second-search probability decreases.
- $u_l \leq u_0 < \tilde{\pi}_2^r u_h + (1-\tilde{\pi}_2^r))u_l$: With sales ranking information, the consumer performs the first search if and only if $s \leq \underline{s}_3$ and the second search if and only if $s \leq \tilde{\pi}_2^r(u_h - u_0)$. With sales volume information, the consumer performs the first search if and only if $s \leq u_h - u_0$ or $s \leq (1-\tilde{\pi}_1^v(x_1,x_2))(u_h - u_l)$ depending on the value of x_1, x_2. So first-search probability decreases for $s \leq \underline{s}_3$, increases for $s > \underline{s}_3$, and second search probability decreases.
- $u_0 < u_l$: With sales ranking information, the consumer never performs the second search and performs the first search if and only if $s \leq (1-\tilde{\pi}_1^r)\tilde{\pi}_2^r(u_h - u_l)$. With sales volume information, the consumer performs the first search if and only if $s \leq (1-\tilde{\pi}_1^v(x_1,x_2))(u_h - u_l)$. So first-search probability decreases for $s \leq (1-\tilde{\pi}_1^r)\tilde{\pi}_2^r(u_h - u_l)$, increases for $s > (1-\tilde{\pi}_1^r)\tilde{\pi}_2^r(u_h - u_l)$, and second-search probability remain the same. □

SO. Consumer Heterogeneity in Valuation

In the base model we assume that consumers share a common valuation for a product. In practice, idiosyncratic factors (e.g., personal tastes) can lead to heterogeneity in consumers' product valuation. In this extension we generalize the base model by incorporating consumer heterogeneity in product valuation. Specifically, let τ be a consumer's individual valuation for the products. Assume that, for a consumer with individual valuation τ, consuming a high-value (resp. low-value) product results in a total (gross) utility of $u_h+\tau$ (resp. $u_l+\tau$) and the reservation utility is u_0, independent of the individual valuation. Such an additive utility model is widely adopted in the operations-management literature (see, for example, Yu et al. 2015 and 2016). We assume that τ is privately known by each consumer and follows a given distribution $\Gamma(\cdot)$.

In what follows, we consider two different settings: in the first setting the individual valuation is known by a consumer upfront, i.e., before she searches or purchases a product; and in the second setting the individual valuation is not known until after a consumer purchases a product.

Heterogeneous Valuation Privately Known Before Search or Purchase

We first consider the setting where a consumer is aware of her individual valuation before she searches or purchases. Thus, she takes account of the individual valuation in her search and purchasing decisions. We note that, under this setting, the model with *heterogenous* valuation and *homogenous* reservation utility is equivalent to a model with *homogenous* valuation and *heterogenous* reservation utility. Our analysis is detailed as follows.

As consumers choose between the reservation utility and consumption of either product, the case where a high-value product is of utility $u_h+\tau$, a low-value product is of utility $u_l+\tau$, and the reservation utility is u_0 is equivalent to the case where a high-value product is of utility u_h, a low-value product is of utility u_l, and the reservation utility is $u_0-\tau$. More precisely, when a consumer decides whether or not to search/purchase a product, it is the difference of utilities between different options that matters, not the absolute utilities. For instance, a consumer who discovers low-value products only chooses a low-value product instead of the no-purchase alternative when $u_l+\tau\geq u_0$, which is equivalent to $u_l\geq u_0-\tau$. Hence, heterogeneity in consumer valuation (with homogenous reservation utility) is equivalent to heterogeneity in reservation utility (with homogenous consumer valuation). Thus, the analysis and results in §SN apply to this setting of heterogeneous valuation.

Heterogeneous Valuation Privately Known After Purchase

We next consider the second setting where the value of τ is known by a consumer only after the consumer has purchased a product (e.g., experience goods). Denote $\mathbb{E}[\tau]$ as the expected value of τ. Same as the base model, we assume $u_0<u_l$.

We start by considering the optimal search and purchasing strategy for a first-period consumer. Following backward induction, we first consider the second search. The consumer considers the second search when the first search reveals a low-value product. The expected utility of skipping the second search and purchasing the low-value product is $\mathbb{E}[u_l+\tau]$ and that of performing the second search is $\mathbb{E}[p_h(u_h+\tau)+p_l(u_l+\tau)]-s=p_hu_h+p_lu_l+\mathbb{E}[\tau]-s$. Hence a first-period consumer performs the second search if the first search reveals a low-value product and her search cost satisfies $u_l+\mathbb{E}[\tau]\leq p_hu_h+p_lu_l+\mathbb{E}[\tau]-s$, which is $s\leq p_h(u_h-u_l)$. Note that the condition for the second search is the same as in the base model. Therefore, the individual valuation τ has no impact on the second search for a first-period consumer.

Next consider the first search. The expected utility of the first search is $\mathbb{E}[p_h(u_h+\tau)+p_l\max[u_l+\tau,p_h(u_h+\tau)+p_l(u_l+\tau)-s]-s]=p_hu_h+p_l\max[u_l,p_hu_h+p_lu_l-s]+\mathbb{E}[\tau]-s$. As the utility of the no-purchase alternative is u_0, the first-period consumer performs the first search when $u_0\le p_hu_h+p_lu_l+\mathbb{E}[\tau]-s$ (the analysis is the same as that in the base model), which is $s\le p_hu_h+p_lu_l+\mathbb{E}[\tau]-u_0$. Denote $\tilde{u}_h=u_h+\mathbb{E}[\tau]$ and $\tilde{u}_l=u_l+\mathbb{E}[\tau]$. Then the first-period consumer's optimal search and purchasing strategy is the same as the case where a high-value product has utility $\tilde{u}_h$ and a low-value product has utility $\tilde{u}_l$ as $\tilde{u}_h-\tilde{u}_l=u_h-u_l$ and $p_h\tilde{u}_h+p_l\tilde{u}_l=p_hu_h+p_lu_l+\mathbb{E}[\tau]$.

Similarly, let π_1^t and π_2^t be the beliefs that the bestseller is of high value and the lower sales product is of high value when the bestseller is revealed to be of low value, respectively, when sales information type t is released, where $t\in\{r,v\}$ represents either sales ranking information or sales volume information. The same argument shows that consumers perform the second search when $s\le\pi_2^t(u_h-u_l)$ (and the first search reveals a low-value product) and the first search when $s\le\pi_1^tu_h+(1-\pi_1^t)u_l+\mathbb{E}[\tau]$. This is again the same as the case where a high-value product has utility $\tilde{u}_h$ and a low-value product has utility $\tilde{u}_l$. Therefore, this extension can be reduced to the base model by redefining the utility of products as $\tilde{u}_h=u_h+\mathbb{E}[\tau]$ and $\tilde{u}_l=u_l+\mathbb{E}[\tau]$ and our analysis in the base model applies.

SP. Real-Time Sales Information Provision

In the base model we assume that sales information is publicized after an endogenously-determined number of consumers arrive. It allows us to focus on the impact of information about aggregate, instead of individual, purchases. It also enables a separate analysis of the exploration and exploitation stages of social learning, an approach not uncommon in the prior studies of social learning (e.g., Yu et al., 2016, Papanastasiou and Savva 2017). This is also in line with the prevalent practice: platforms usually release bestseller information periodically (e.g., Amazon.com updates sales rankings hourly[21]).

What if there is no delay in sales information provision? In particular, what if sales information is publicized and updated in real time? What is consumers' optimal searching and purchasing strategy, and how is it influenced by the bestseller information (and its granularity)? Do consumers get better off by more frequent information provision (i.e., compared to periodic information release as in the base model, does real-time information provision improve consumer surplus)? Likewise, does the platform prefer more frequent information provision (i.e., is the expected total sales higher under real-time provision than that in the base model)? In this extension we address these questions by considering the case where sales information is released in real time and comparing the results with those in the base model.

Specifically, we consider the setting in which consumers arrive sequentially and each consumer knows the number of arrivals prior to him or her. In line with the prevailing practice in online marketplaces, we assume that a consumer cannot observe the actions (i.e., search and purchase) taken by his or her predecessors. Instead, the consumer can observe aggregate information (either sales ranking or sales volume) about the purchases made by the predecessors. The information is publicized and updated in real time. Again, in line with practice, we assume that the information is "memoryless" in the sense that a consumer observes a snapshot of the bestseller information upon arrival and cannot observe its full history. All other settings are the same as in the base model.

[21] References: `kdp.amazon.com/en_US/help/topic/G201648140`, `www.amazon.co.uk/gp/help/customer/display.html?nodeId=GGGMZK378RQPATDJ`.

SP.1. Learning under Real-Time Sales Information Provision

SP.1.1. An Example of Three Consumers We start from an example with a total of three consumers. As in the base model, since the two products are ex ante homogeneous, hereafter whenever sales ranking or volume information is available, we assume without loss of generality that product 1 has (weakly) higher sales than product 2. Let $\pi_1^v(j,x_1,x_2)$ and $\pi_{-1}^v(j,x_1,x_2)$ (with $x_1 \geq x_2$) be the beliefs that product 1 and product 2 are of high value when product 1 and product 2 have sales x_1 and x_2 among the first $j-1$ consumers, respectively. Similarly, let $\pi_1^r(j)$ and $\pi_2^r(j)$ be the beliefs that product 1 and product 2 are of high value, respectively, when product 1 has higher sales ranking among the first $j-1$ consumers, respectively. Here and thereafter, we shall use the term "among the first $j-1$ consumers" to denote a time point that is after the first $j-1$ consumers arrive and before the jth consumer arrives. Let $\pi_2^v(j,x_1,x_2)$ (with $x_1 \geq x_2$) be the belief that product 2 is of high value when product 1 and product 2 have sales x_1 and x_2, respectively, among the first $j-1$ consumers and product 1 is revealed to be of low value. Let $\pi_2^r(j)$ be the belief that product 2 is of high value when product 1 is revealed to be of low value and has higher sales ranking among the first $j-1$ consumers.

The three consumers' search and purchasing strategies are analyzed as follows.

• **Consumer** 1: For the consumer who arrives first, her beliefs for the two products to be of high value are the prior beliefs. The first consumer performs the first search if and only if $s \leq p_h u_h + p_l u_l$ and the second search if and only if $s \leq p_h(u_h - u_l)$ and the first search reveals a low-value product.

• **Consumer** 2: For the second arrival, first consider the situation that she observes real-time sales volume. The sales realization she observes is either both products having zero sales, or one product having one unit of sales while the other product having zero sales. For the first case the second consumer learns that the first consumer does not search as otherwise one of the products would have one unit of sales. Therefore she knows that the first consumer has search cost higher than $p_h u_h + p_l u_l$. As the first consumer does not search and therefore does not have private information regarding the products' values, the second consumer's beliefs in this case remain the same as the prior beliefs. Hence when neither product has sales, consumer 2 performs the first search if and only if $s \leq p_h u_h + p_l u_l$ and performs the second search if and only if $s \leq p_h(u_h - u_l)$ and the first search reveals a low-value product.

For the second case where only one product has positive sales, we have the following lemma.

Lemma S.34. *$\pi_1^v(2,1,0) \geq p_h \geq \pi_{-1}^v(2,1,0)$ and $\pi_2^v(2,1,0) \leq p_h$.*

Lemma S.34 indicates that, in this case, product 1 is believed to be more likely to be of high value than product 2 and, thus, should be searched first. In this case the second consumer performs the first search if and only if $s \leq \pi_1^v(2,1,0)u_h + (1-\pi_1^v(2,1,0))u_l$. If the first search reveals a low-value product, consumer 2 performs the second search if and only if $s \leq \pi_2^v(2,1,0)(u_h - u_l)$.

When only sales ranking information is revealed to the second consumer, we have the following lemma.

Lemma S.35. *$\pi_1^r(2) \geq p_h \geq \pi_{-1}^r(2)$ and $\pi_2^r(2) \leq p_h$.*

Lemma S.35 suggests that, under ranking information and for consumer 2, product 1 (the bestseller product) is more likely to be of high value and should be searched first.

• **Consumer** 3: For the third consumer, first consider the situation that sales volume is revealed in real time.. There are four possible sales realizations that the third consumer may observe: $(x_1,x_2)=(0,0),(x_1,x_2)=(1,0),(x_1,x_2)=(1,1),(x_1,x_2)=(2,0)$. Consider each of these cases as below:

Case $(x_1,x_2)=(0,0)$: In this case neither consumer 1 nor consumer 2 makes purchase and consumer 3 learns that both consumer 1 and consumer 2 have search cost higher than $p_h u_h + p_l u_l$. As neither of the first two consumers searches, their no-purchase actions do not imply any information regarding the product values. Hence the consumer 3's beliefs remain the same as the prior beliefs.

Case $(x_1,x_2)=(1,0)$: In this case one of the first two consumers makes purchase. As a consumer's no-purchase implies no information regarding the product values, the consumer who does not purchase does not make impact on learning and it suffices to consider the consumer who makes purchase. If the first consumer makes purchase, her beliefs at time of her own purchase are the prior beliefs. If the second consumer makes purchase, her beliefs at time of her own purchase are again the prior beliefs, by our analysis in the first case of consumer 2 as the first consumer does not purchase. Hence, the analysis for this case is the same as the second case for consumer 2 and we have the following lemma.

Lemma S.36. *$\pi_i^v(3,1,0)=\pi_i^v(2,1,0)$, for $i=1,-1,2$.*

Case $(x_1,x_2)=(1,1)$: In this case both of the first two consumers make purchase. The two products have the same sales and consumer 3 randomly picks a product to search first. According to our analysis in the second case for consumer 2, consumer 2 must first search the product that is purchased by consumer 1 if any. As consumer 2 purchases the other product, it follows that the product purchased by consumer 1 is of low value and consumer 2 performs the second search and purchases the other product. We have the following lemma.

Lemma S.37. *$\pi_1^v(3,1,1)=\pi_{-1}^v(3,1,1)<p_h$ and $\pi_2^v(3,1,1)\le p_h$.*

In Lemma S.37, the posterior belief $\pi_1^v(3,1,1)$ is lower than p_h, because consumer 3, upon observing sales $(1,1)$, infers that at least one of the products is of low value. This result is similar to its counterpart in the base model and shows that consumers' belief about either product being of high value can be lower than the prior belief when the two products' sales are known to be close to each other. As we shall elaborate in §SP.1.2, however, the driving forces of this result are much more complicated than in the base model since multiple rounds of deduction are involved. In particular, when sales information is released in real time, a consumer needs to consider the belief updating process of all of her predecessors, rendering inference much more complex.

Case $(x_1,x_2)=(2,0)$: In this case the first two consumers purchase the same product. Lemma S.38 follows.

Lemma S.38. *$\pi_1^v(3,2,0)\ge p_h$.*

In this case consumer 3's belief that the bestseller product is of high value increases compared to the prior belief. This is aligned with the result in the base model that unbalanced sales between the two products increases the belief about the bestseller being of high value. The analysis, however, is more complex, for the aforementioned reasons.

When sales ranking is revealed in real time, Lemma S.39 characterizes consumer 3's posterior belief.

Lemma S.39. *$\pi_1^r(3)\ge p_h$ and $\pi_{-1}^r(3)\le p_h$.*

Thus, like the base model, when sales ranking information is released, the bestseller product is more likely to be of high value than the other product and should be searched first.

To summarize, for the case of three consumers, we confirm that, similar to the periodic sales information in the base model, real-time ranking information increases consumers' willingness to search the bestseller product, while real-time volume information may reduce it. Furthermore, we note that the belief updating process for the third consumer becomes more involved as she needs to rationalize the learning process of all the consumers arriving prior to her and also takes account of all of the possible sales realizations if the sales volume is unobservable. Thus, analytical complexity grows rapidly in the number of consumers. Nevertheless, in the next subsection we show that several key results in the base model continue to hold under real-time information provision.

SP.1.2. General Results Now we proceed to consider the general case with any number of consumers. We show that both the mean preserving spread property (i.e., the belief under volume information is a mean preserving spread of that under ranking information) and the reinforcement-by-homogeneity effect, which could lead to lower expected sales when sales volume information is released, continue to hold when the sales volume information is updated in real time.

Mean-preserving spread

Recall that $\pi_1^r(j)$ is consumer j's belief that product 1 is of high value when product 1 has higher sales ranking than product 2 among the first $j-1$ consumers and $\pi_1^v(j, x_1, x_2)$ (resp. $\pi_{-1}^v(j, x_1, x_2)$) is consumer j's belief that product 1 (resp. product 2) is of high value when the sales of the two products among the first $j-1$ consumers are x_1 and x_2, respectively (with $x_1 \geq x_2$).

The following proposition shows that the mean-preserving spread property continues to hold.

Proposition S.29. *We have* $\mathbb{E}_{x_1,x_2}[\pi_1^v(j, x_1, x_2)] = \pi_1^r(j)$.

Reinforcement-by-homogeneity

In the base model, we show that under volume information, a consumer's belief for either product being of high value can be lower than the prior belief. The following proposition shows that volume information may also reduce belief when it is released in real time.

Proposition S.30. *There exist problem instances where* $\pi_1^v(j, x_1, x_2) \leq p_h$ *and* $\pi_{-1}^v(j, x_1, x_2) \leq p_h$.

Proposition S.30 is proved by the three-consumer example in §SP.1.1, where we showed $\pi_1^v(3, 1, 1) \leq p_h$ (ref. Lemma S.37). The result is mainly driven by two forces, as we now elaborate. The first force is unique to real-time information provision. As volume information is released in real time, a consumer needs to consider the belief updating and the corresponding optimal strategies of all of the arrivals prior to her. Consequently, consumer 3's observation of one unit sales for both products indicates that the product purchased by consumer 1 must be of low value as otherwise consumer 2 would have purchased the same product as consumer 1 since it is optimal for consumer 2 to first search the product purchased by consumer 1. Hence, consumer 3 deduces that both products cannot be of high value, which reduces her belief that the bestseller product is of high value. In the base model, such an effect does not exist as the first-period sales distribution conditional on both products being of high value is identical to that conditional both products being of low value and, consequently, a second-period consumer cannot rule out the former case solely by the observed first-period sales.

The second driving force is similar to the reinforcement-by-homogeneity effect identified in the base model. To disentangle the second force from the first one, we consider the probability that a randomly selected product is of high value without sales information conditional on at least of the products being of low value (recall: upon observing the sales realization (1,1), consumer 3 believes that both products cannot be of high value). Denote this probability as q_2. We have

$$q_2 = \frac{p_h p_l}{2p_h p_l + p_l^2} = \frac{p_h}{2p_h + p_l}$$

On the other hand, by the proof of Lemma S.37,

$$\pi_1^v(3,1,1) = \frac{p_h}{2p_h + p_l \frac{F(p_h u_h + p_l u_l)}{F(p_h u_h + p_l u_l) - F(p_h(u_h - u_l))}}$$

As $\frac{F(p_h u_h + p_l u_l)}{F(p_h u_h + p_l u_l) - F(p_h(u_h - u_l))} \geq 1$, $\pi_1^v(1,1,3) \leq q_2$, which suggests that consumer 3's belief about the bestseller product being of high value is further driven down from the prior belief (conditional on at least one of the products being of low value). That is, given that at least one of the products is of low value, compared to the case without volume information, when both products are revealed to have one unit of sales consumer 3 is more inclined to believe that both products are of low value rather than to believe that the two products' values differ from each other. The intuition is that the probability of the first two consumers purchasing different products is higher when both product values are low than when the products are of different values (to be explained in the next paragraph). Thus, the observation of equal product sales increases consumer 3's belief about equal product values (conditional on at least one of the values being low), which, in this case, lowers the consumer's belief about a randomly-chosen product (or equivalently in this case, the bestseller) being of high value.

To see why "the probability of the first two consumers purchasing different products is higher when both product values are low than when the products are of different values", consider two cases: (i) both products are of low value and (ii) the products are of different values. In case (i), both products have one unit of sales if and only if (i-a) consumer 2 performs the second search and (i-b) consumer 2 purchases the product examined in her second search. In contrast, in case (ii), both products have one unit of sales if and only if (ii-a) consumer 1 only searches once, (ii-b) consumer 1 searches the low-value product, and (ii-c) consumer 2 performs the second search. Note that the probability of consumer 2 purchasing the product examined in the second search in case (i) (i.e. event (i-b)) is the same as the probability of consumer 1 first searching the low-value product in case (ii) (i.e. event (ii-b)) and, specifically, both probabilities equal 1/2. Also note that event (i-a) is identical to event (ii-c). Thus, the probability that both products have one unit of sales is lower under case (ii) than under case (i) as it requires consumer 1 not performing the second search (i.e., the event (ii-a)).

SP.2. Impact on Product Sales

Now we investigate the impact of real-time sales information on the expected total sales. We first show that, similar to volume information provided periodically (as in the base model), volume information released in real time could also lead to lower expected total sales and thus hurt the platform. Proposition S.31 follows.

Proposition S.31. *There exist problem instances such that the expected total sales under real-time volume information is lower than that under no information.*

The rationale for Proposition S.31 is that real-time sales volume information allows consumers to update beliefs about the number of high-value products. In particular, as we discussed in the three-consumer example, when the products have equal sales, consumer 3 deduces that at least one of the products is of low value. This reduces her belief about either product being of high value and also lowers the probability of her making a purchase.

Furthermore, by comparing the expected total sales under real-time information and that in the base model, we note in Proposition S.32 that more frequent information provision may be undesirable to the platform.

PROPOSITION S.32. *There exist problem instances such that the expected total sales under real-time volume information is lower than that under volume information in the base model.*

Proposition S.32 is due to the fact that real-time volume information allows consumers to learn more about the product values than volume information released after a certain number of consumers arrive. For example, as in the three-consumer example, consumer 3 deduces that at most one product is of high value after observing equal sales. This kind of inference is never possible in the base model, because two high-value products and two low-value products lead to the same first-period sales distribution and hence the second-period consumers are never able to rule out, based on realized sales volume, the possibility that both products are of high value. The key driver of the difference is multi-round inference under real-time volume information. As noted in the proposition, however, consumers' better knowledge about the product values may backfire and reduce the platform's revenue.

SP.3. Impact on Welfare

By numerically compare the consumer welfare under real-time information provision and that in the base model, we note that real-time information provision may improve consumer welfare from the level in the base model because of consumers' better informed search and purchasing decisions. Details of the numerical results and the corresponding discussion are presented at the end of §SH.

SP.4. Appendix

Define $\Pr_j(y)$ as the probability that event y occurs among the first $j-1$ consumers. This notation will be used in some of the proofs below.

Proof of Lemma S.34 We have

$$\begin{aligned}
\pi_1^v(2,1,0) =& \Pr_2(u_1=u_h|x_1=1,x_2=0)\\
=& \frac{\Pr_2(u_1=u_h,x_1=1,x_2=0)}{\Pr_2(x_1=1,x_2=0)}\\
=& \frac{\Pr_2(u_1=u_h,u_2=u_h,x_1=1,x_2=0)+\Pr_2(u_1=u_h,u_2=u_l,x_1=1,x_2=0)}{\begin{pmatrix}\Pr_2(u_1=u_h,u_2=u_h,x_1=1,x_2=0)+\Pr_2(u_1=u_h,u_2=u_l,x_1=1,x_2=0)\\ +\Pr_2(u_1=u_l,u_2=u_h,x_1=1,x_2=0)+\Pr_2(u_1=u_l,u_2=u_l,x_1=1,x_2=0)\end{pmatrix}}\\
=& \frac{p_h^2F(p_hu_h+p_lu_l)/2+p_hp_l(F(p_h(u_h-u_l))+\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2})}{\begin{pmatrix}p_h^2F(p_hu_h+p_lu_l)/2+p_hp_l(F(p_h(u_h-u_l))+\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2})\\ +p_hp_l\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2}+p_l^2F(p_hu_h+p_lu_l)/2\end{pmatrix}}\\
=& \frac{p_h^2F(p_hu_h+p_lu_l)+2p_hp_l(F(p_h(u_h-u_l))+\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2})}{F(p_hu_h+p_lu_l)}
\end{aligned}$$

$$
\begin{aligned}
&= p_h(p_h + 2p_l \frac{F(p_h(u_h-u_l)) + \frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2}}{F(p_hu_h+p_lu_l)}) \\
&\geq p_h(p_h + 2p_l \frac{\frac{F(p_hu_h+p_lu_l)}{2}}{F(p_hu_h+p_lu_l)}) \\
&= p_h(p_h+p_l) = p_h
\end{aligned}
$$

For the fourth equality, notice that $\Pr_2(u_1=u_h,u_2=u_h,x_1=1,x_2=0) = \Pr(u_1=u_h,u_2=u_h)\cdot\Pr_2(x_1=1,x_2=0|u_1=u_h,u_2=u_h) = p_h^2\cdot\Pr_2(x_1=1,x_2=0|u_1=u_h,u_2=u_h)$. As both products are of high value, the probability that consumer 1 purchases product 1 is the probability that she is willing to perform the first search and she searches product 1 first, which is $F(p_hu_h+p_lu_l)/2$. Similarly, we have $\Pr_2(u_1=u_h,u_2=u_l,x_1=1,x_2=0) = \Pr(u_1=u_h,u_2=u_l)\cdot\Pr_2(x_1=1,x_2=0|u_1=u_h,u_2=u_l) = p_hp_l\cdot\Pr_2(x_1=1,x_2=0|u_1=u_h,u_2=u_l)$. There are two possibilities for consumer 1 to purchase product 1 in this case. The first is that consumer 1 is only willing to perform the first search and searches product 1 first, which happens with probability $\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2}$. The second is that consumer 1 is willing to perform the second search. In this case it does not matter which product consumer 1 searches first and the probability is $F(p_h(u_h-u_l))$. So $\Pr_2(x_1=1,x_2=0|u_1=u_h,u_2=u_l) = F(p_h(u_h-u_l)) + \frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l)}{2}$.

For $\Pr_2(u_1=u_l,u_2=u_h,x_1=1,x_2=0) = \Pr(u_1=u_l,u_2=u_h)\cdot\Pr_2(x_1=1,x_2=0|u_1=u_l,u_2=u_h) = p_hp_l\cdot\Pr_2(x_1=1,x_2=0|u_1=u_l,u_2=u_h)$, the probability that consumer 1 purchases product 1 is $\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2}$ as it must be the case that consumer 1 is unwilling to perform the second search and searches product 1 first. Lastly, for $\Pr_2(u_1=u_l,u_2=u_l,x_1=1,x_2=0) = \Pr(u_1=u_l,u_2=u_l)\cdot\Pr_2(x_1=1,x_2=0|u_1=u_l,u_2=u_l) = p_l^2\cdot\Pr_2(x_1=1,x_2=0|u_1=u_l,u_2=u_l)$, we have $\Pr_2(x_1=1,x_2=0|u_1=u_l,u_2=u_l) = F(p_hu_h+p_lu_l)/2$ as consumer 1 must be willing to perform the first search and choose to purchase product 1 instead of the other product, which happens with probability 1/2.

Similarly,

$$
\begin{aligned}
\pi_{-1}^v(2,1,0) &= \Pr_2(u_2=u_h|x_1=1,x_2=0) \\
&= \frac{\Pr_2(u_2=u_h,x_1=1,x_2=0)}{\Pr(x_1=1,x_2=0)} \\
&= \frac{\Pr_2(u_1=u_h,u_2=u_h,x_1=1,x_2=0)+\Pr_2(u_1=u_l,u_2=u_h,x_1=1,x_2=0)}{\begin{pmatrix}\Pr_2(u_1=u_h,u_2=u_h,x_1=1,x_2=0)+\Pr_2(u_1=u_h,u_2=u_l,x_1=1,x_2=0)\\ +\Pr_2(u_1=u_l,u_2=u_h,x_1=1,x_2=0)+\Pr_2(u_1=u_l,u_2=u_l,x_1=1,x_2=0)\end{pmatrix}} \\
&= \frac{p_h^2F(p_hu_h+p_lu_l)/2 + p_hp_l\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2})}{\begin{pmatrix}p_h^2F(p_hu_h+p_lu_l)/2 + p_hp_l(F(p_h(u_h-u_l)) + \frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2})\\ +p_l^2F(p_hu_h+p_lu_l)/2 + p_hp_l\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2}\end{pmatrix}} \\
&= \frac{p_h^2F(p_hu_h+p_lu_l) + 2p_hp_l\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2}}{F(p_hu_h+p_lu_l)} \\
&= p_h(p_h + p_l\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{F(p_hu_h+p_lu_l)}) \\
&\leq p_h(p_h+p_l) = p_h
\end{aligned}
$$

For $\pi_2^v(2,1,0)$, we have

$$
\begin{aligned}
\pi_2^v(2,1,0) &= \Pr_2(u_2=u_h|u_1=u_l,x_1=1,x_2=0) \\
&= \frac{\Pr_2(u_2=u_h,u_1=u_l,x_1=1,x_2=0)}{\Pr_2(u_1=u_l,x_1=1,x_2=0)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\Pr_2(u_2=u_h,u_1=u_l,x_1=1,x_2=0)}{\Pr_2(u_2=u_h,u_1=u_l,x_1=1,x_2=0)+\Pr_2(u_2=u_l,u_1=u_l,x_1=1,x_2=0)}\\
&= \frac{p_h p_l \frac{F(p_h u_h+p_l u_l)-F(p_h(u_h-u_l))}{2}}{p_h p_l \frac{F(p_h u_h+p_l u_l)-F(p_h(u_h-u_l))}{2}+p_l^2 F(p_h u_h+p_l u_l)/2}\\
&= \frac{p_h}{p_h+p_l\frac{F(p_h u_h+p_l u_l)}{F(p_h u_h+p_l u_l)-F(p_h(u_h-u_l))}}\\
&\le \frac{p_h}{p_h+p_l}=p_h
\end{aligned}
$$

□

Proof of Lemma S.35 Let $\tau(i|j)$ be the probability that among the first $j-1$ consumers, i consumers make purchase, so $\sum_{i=0}^{j-1}\tau(i|j)=1$ for all $j\ge 1$. Then, conditioning on how many consumers have made purchases, we have

$$
\begin{aligned}
\pi_1^r(2)&=\tau(0|2)\pi_1^v(2,0,0)+\tau(1|2)\pi_1^v(2,1,0)\\
\pi_{-1}^r(2)&=\tau(0|2)\pi_{-1}^v(2,0,0)+\tau(1|2)\pi_{-1}^v(2,1,0)\\
\pi_2^r(2)&=\tau(0|2)\pi_2^v(2,0,0)+\tau(1|2)\pi_2^v(2,1,0)
\end{aligned}
$$

As $\pi_1^v(2,0,0)=\pi_{-1}^v(2,0,0)=\pi_2^v(2,0,0)=p_h$ (as consumer 1 does not purchase and consumer 2's beliefs stay the same as prior beliefs) and by Lemma S.34 $\pi_1^v(2,1,0)\ge p_h\ge \pi_{-1}^v(2,1,0)$ and $\pi_2^v(2,1,0)\le p_h$, it follows that $\pi_1^r(2)\ge p_h, \pi_{-1}^r(2)\le p_h, \pi_2^r(2)\le p_h$, which concludes the proof. □

Proof of Lemma S.37 We have

$$
\begin{aligned}
\pi_1^v(3,1,1) =& \Pr_3(u_1=u_h|x_1=1,x_2=1)\\
=& \frac{\Pr_3(u_1=u_h,x_1=1,x_2=1)}{\Pr_3(x_1=1,x_2=1)}\\
=& \frac{\Pr_3(u_1=u_h,u_2=u_l,x_1=1,x_2=1)}{\begin{pmatrix}\Pr_3(u_1=u_h,u_2=u_l,x_1=1,x_2=1)+\Pr_3(u_1=u_l,u_2=u_h,x_1=1,x_2=1)\\ +\Pr_3(u_1=u_l,u_2=u_l,x_1=1,x_2=1)\end{pmatrix}}\\
=& \frac{p_h p_l\cdot \frac{F(p_h u_h+p_l u_l)-F(p_h(u_h-u_l))}{2}\cdot F(\pi_2^v(2,1,0)(u_h-u_l))}{2p_h p_l\cdot \frac{F(p_h u_h+p_l u_l)-F(p_h(u_h-u_l))}{2}\cdot F(\pi_2^v(2,1,0)(u_h-u_l))+p_l^2 F(p_h u_h+p_l u_l)F(\pi_2^v(2,1,0)(u_h-u_l))/2}\\
=& \frac{p_h}{2p_h+p_l\frac{F(p_h u_h+p_l u_l)}{F(p_h u_h+p_l u_l)-F(p_h(u_h-u_l))}}\\
<& \frac{p_h}{p_h+p_l}=p_h
\end{aligned}
$$

The third equality follows as we know one product must be of low value when $x_1=1$ and $x_2=1$, as explained in the text. The fourth equality follows from $\Pr_3(u_1=u_h,u_2=u_l,x_1=1,x_2=1)=\Pr(u_1=u_h,u_2=u_l)\cdot\Pr_3(x_1=1,x_2=1|u_1=u_h,u_2=u_l)=p_h p_l\cdot\Pr_3(x_1=1,x_2=1|u_1=u_h,u_2=u_l)$. For $\Pr_3(x_1=1,x_2=1|u_1=u_h,u_2=u_l)$, it must be the case that the first consumer purchases the low-value product and the second consumer purchases the high-value product. It follows that first consumer searches once and is unwilling to perform the second search while the second consumer performs the second search. Hence, the first consumer must have search cost lower than $p_h u_h+p_l u_l$ and higher than $p_h(u_h-u_l)$ and the second consumer has search cost lower than $\pi_2^v(2,1,0)(u_h-u_l)$. $\Pr_3(x_1=1,x_2=1|u_1=u_h,u_2=u_l)=\frac{F(p_h u_h+p_l u_l)-F(p_h(u_h-u_l))}{2}\cdot F(\pi_2^v(2,1,0)(u_h-u_l))$ as the first consumer searches the low-value product first with probability 1/2. Same analysis applies for $\Pr_3(u_1=u_l,u_2=u_h,x_1=1,x_2=1)$.

Similarly, $\Pr_3(u_1=u_l,u_2=u_l,x_1=1,x_2=1)=\Pr(u_1=u_l,u_2=u_l)\cdot\Pr_3(x_1=1,x_2=1|u_1=u_l,u_2=u_l)=p_l^2\cdot\Pr_3(x_1=1,x_2=1|u_1=u_l,u_2=u_l)$. For $\Pr_3(x_1=1,x_2=1|u_1=u_l,u_2=u_l)$, consumer 1 must be willing to perform the first search and consumer 2 must be willing to perform the second search. The probability is $F(p_hu_h+p_lu_l)F(\pi_2^v(2,1,0)(u_h-u_l))/2$ as consumer 2 purchases different product than consumer 1 with probability 1/2.

For $\pi_2^v(3,1,1)$, we have

$$\begin{aligned}
&\pi_2^v(3,1,1)\\
=&\Pr_3(u_2=u_h|u_1=u_l,x_1=1,x_2=1)\\
=&\frac{\Pr_3(u_2=u_h,u_1=u_l,x_1=1,x_2=1)}{\Pr_3(u_1=u_l,x_1=1,x_2=1)}\\
=&\frac{\Pr_3(u_2=u_h,u_1=u_l,x_1=1,x_2=1)}{\Pr_3(u_2=u_h,u_1=u_l,x_1=1,x_2=1)+\Pr_3(u_2=u_l,u_1=u_l,x_1=1,x_2=1)}\\
=&\frac{p_hp_l\cdot\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2}\cdot F(\pi_2^v(2,1,0)(u_h-u_l))}{p_hp_l\cdot\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2}\cdot F(\pi_2^v(2,1,0)(u_h-u_l))+p_l^2F(p_hu_h+p_lu_l)F(\pi_2^v(2,1,0)(u_h-u_l))/2}\\
=&\frac{p_h}{p_h+p_l\frac{F(p_hu_h+p_lu_l)}{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}}\\
\leq&\frac{p_h}{p_h+p_l}=p_h
\end{aligned}$$

□

Proof of Lemma S.38 We have

$$\begin{aligned}
\pi_1^v(3,2,0)=&\Pr_3(u_1=u_h|x_1=2,x_2=0)\\
=&\frac{\Pr_3(u_1=u_h,x_1=2,x_2=0)}{\Pr_3(x_1=2,x_2=0)}\\
=&\frac{\Pr_3(u_1=u_h,u_2=u_h,x_1=2,x_2=0)+\Pr_3(u_1=u_h,u_2=u_l,x_1=2,x_2=0)}{\begin{pmatrix}\Pr_3(u_1=u_h,u_2=u_h,x_1=2,x_2=0)+\Pr_3(u_1=u_h,u_2=u_l,x_1=2,x_2=0)\\+\Pr_3(u_1=u_l,u_2=u_h,x_1=2,x_2=0)+\Pr_3(u_1=u_l,u_2=u_l,x_1=2,x_2=0)\end{pmatrix}}\\
=&\frac{\begin{pmatrix}p_h^2F(p_hu_h+p_lu_l)/2\cdot F(\pi_1^v(2,1,0)u_h+(1-\pi_1^v(2,1,0))u_l)\\+p_hp_l(\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2}+F(p_h(u_h-u_l)))F(\pi_1^v(2,1,0)u_h+(1-\pi_1^v(2,1,0))u_l)\end{pmatrix}}{\begin{pmatrix}p_h^2F(p_hu_h+p_lu_l)/2\cdot F(\pi_1^v(2,1,0)u_h+(1-\pi_1^v(2,1,0))u_l)\\+p_hp_l(\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2}+F(p_h(u_h-u_l)))F(\pi_1^v(2,1,0)u_h+(1-\pi_1^v(2,1,0))u_l)+T\end{pmatrix}}
\end{aligned}$$

where

$$\begin{aligned}
T=&p_hp_l\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2}(F(\pi_1^v(2,1,0)u_h+(1-\pi_1^v(2,1,0))u_l)-F(\pi_2^v(2,1,0)(u_h-u_l)))\\
&+p_l^2F(p_hu_h+p_lu_l)/2\cdot(F(\pi_1^v(2,1,0)u_h+(1-\pi_1^v(2,1,0))u_l)-F(\pi_2^v(2,1,0)(u_h-u_l))+F(\pi_2^v(2,1,0)(u_h-u_l))/2)
\end{aligned}$$

For the fourth equality, notice that $\Pr_3(u_1=u_h,u_2=u_h,x_1=2,x_2=0)=\Pr(u_1=u_h,u_2=u_h)\cdot\Pr_3(x_1=2,x_2=0|u_1=u_h,u_2=u_h)=p_h^2\cdot\Pr_3(x_1=2,x_2=0|u_1=u_h,u_2=u_h)$. For $\Pr_3(x_1=2,x_2=0|u_1=u_h,u_2=u_h)$, as consumer 2 always searches the product purchased by consumer 1 first, this is the probability that both consumer 1 and consumer 2 are willing to perform the first search and consumer 1 purchase product 1 (this occurs with probability 1/2 as the two products are symmetric), which is $F(p_hu_h+p_lu_l)/2\cdot F(\pi_1^v(2,1,0)u_h+(1-\pi_1^v(2,1,0))u_l)$. Similarly, $\Pr_3(u_1=u_h,u_2=u_l,x_1=2,x_2=0)=\Pr(u_1=u_h,u_2=u_l)\cdot\Pr_3(x_1=2,x_2=0|u_1=u_h,u_2=u_l)=p_hp_l\cdot\Pr_3(x_1=2,x_2=0|u_1=u_h,u_2=u_l)$. $\Pr_3(x_1=2,x_2=0|u_1=u_h,u_2=u_l)$ is the probability that the first consumer purchases the high-value product and the second-consumer makes purchase, which is $(\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2}+F(p_h(u_h-u_l)))F(\pi_1^v(2,1,0)u_h+(1-\pi_1^v(2,1,0))u_l)$.

For $\Pr_3(u_1=u_l,u_2=u_h,x_1=2,x_2=0)=\Pr(u_1=u_l,u_2=u_h)\cdot\Pr_3(x_1=2,x_2=0|u_1=u_l,u_2=u_h)=p_hp_l\cdot\Pr_3(x_1=2,x_2=0|u_1=u_l,u_2=u_h)$, where $\Pr_3(x_1=2,x_2=0|u_1=u_l,u_2=u_h)$ is the probability that the first two consumers purchase the low-value product when there is a high-value product. So both consumers must have refused to perform the second search and consumer 1 searches the low-value product first. The probability is $\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2}(F(\pi_1^v(2,1,0)u_h+(1-\pi_1^v(2,1,0))u_l)-F(\pi_2^v(2,1,0)(u_h-u_l)))$.

For $\Pr_3(u_1=u_l,u_2=u_l,x_1=2,x_2=0)=\Pr(u_1=u_l,u_2=u_l)\cdot\Pr_3(x_1=2,x_2=0|u_1=u_l,u_2=u_l)=p_hp_l\cdot\Pr_3(x_1=2,x_2=0|u_1=u_l,u_2=u_l)$, where $\Pr_3(x_1=2,x_2=0|u_1=u_l,u_2=u_l)$ is the probability that the first two consumers purchase the same low-value product. So either consumer 2 refuses to perform the second search, or consumer 2 performs the second search and purchases the same product as consumer 1. Moreover, consumer 1 purchase product 1 with probability 1/2. Hence, the probability is $F(p_hu_h+p_lu_l)/2\cdot(F(\pi_1^v(2,1,0)u_h+(1-\pi_1^v(2,1,0))u_l)-F(\pi_2^v(2,1,0)(u_h-u_l))+F(\pi_2^v(2,1,0)(u_h-u_l))/2)$.

Note that we have

$$\pi_1^v(3,2,0)=\frac{p_h}{p_h+p_l\bar{T}}$$

with

$$\bar{T}=\frac{\begin{pmatrix} p_h\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2}(F(\pi_1^v(2,1,0)u_h+(1-\pi_1^v(2,1,0))u_l)-F(\pi_2^v(2,1,0)(u_h-u_l))) \\ +p_lF(p_hu_h+p_lu_l)/2\cdot(F(\pi_1^v(2,1,0)u_h+(1-\pi_1^v(2,1,0))u_l)-F(\pi_2^v(2,1,0)(u_h-u_l))+F(\pi_2^v(2,1,0)(u_h-u_l))/2)\end{pmatrix}}{\begin{pmatrix} p_hF(p_hu_h+p_lu_l)/2\cdot F(\pi_1^v(2,1,0)u_h+(1-\pi_1^v(2,1,0))u_l) \\ +p_l(\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2}+F(p_h(u_h-u_l)))F(\pi_1^v(2,1,0)u_h+(1-\pi_1^v(2,1,0))u_l)\end{pmatrix}}$$
$$\leq 1$$

The inequality follows from

$$\begin{aligned}&\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2}(F(\pi_1^v(2,1,0)u_h+(1-\pi_1^v(2,1,0))u_l)-F(\pi_2^v(2,1,0)(u_h-u_l)))\\ \leq& F(p_hu_h+p_lu_l)/2\cdot F(\pi_1^v(2,1,0)u_h+(1-\pi_1^v(2,1,0))u_l)\end{aligned}$$

and

$$\begin{aligned}&F(p_hu_h+p_lu_l)/2\cdot(F(\pi_1^v(2,1,0)u_h+(1-\pi_1^v(2,1,0))u_l)-F(\pi_2^v(2,1,0)(u_h-u_l))+F(\pi_2^v(2,1,0)(u_h-u_l))/2)\\ \leq&(\frac{F(p_hu_h+p_lu_l)-F(p_h(u_h-u_l))}{2}+F(p_h(u_h-u_l)))F(\pi_1^v(2,1,0)u_h+(1-\pi_1^v(2,1,0))u_l)\end{aligned}$$

Hence $\pi_1^v(3,2,0)\geq p_h$. □

Proof of Lemma S.39 To prove the lemma, we first show that sales ranking information does not change consumers' belief about the number of high-value products. Let $1>2$ and $2>1$ represent the events that product 1 and product 2 has higher sales ranking, respectively. Denote HH,HL,LL as the events that there are two high-value products, one high-value product, and no high-value product, respectively. For instance, $\Pr_3(1>2)$ is the probability that among the first two consumer, product 1 has higher sales ranking.

We first introduce the following lemma, for which the proof is provided after the current proof.

Lemma S.40. *When sales ranking information is released in real time, there is*

$$Pr_t(HH|1>2) = Pr_t(HH|2>1) = Pr(HH) = p_h^2$$
$$Pr_t(HL|1>2) = Pr_t(HL|2>1) = Pr(HL) = 2p_hp_l$$
$$Pr_t(LL|1>2) = Pr_t(LL|2>1) = Pr(LL) = p_l^2$$

for all $t \geq 1$.

Notice that $\pi_1^r(3) + \pi_{-1}^r(3)$ is the expected number of high-value product after sales ranking information of the first two consumers is released. It follows that $\pi_1^r(3) + \pi_{-1}^r(3) = 2 \cdot \mathrm{Pr}_3(HH|1>2) + 1 \cdot \mathrm{Pr}_3(HL|1>2) = 2p_h$. Hence, to show $\pi_1^r(3) \geq p_h \geq \pi_{-1}^r(3)$, it suffices to show $\pi_1^r(3) \geq p_h$. Conditioning on the number of high-value products, we have

$$\begin{aligned}
\pi_1^r(3) =& \mathrm{Pr}_3(u_1 = u_h|1>2) \\
=& \mathrm{Pr}_3(HH|1>2) \cdot \mathrm{Pr}_3(u_1 = u_h|HH, 1>2) + \mathrm{Pr}_3(HL|1>2) \cdot \mathrm{Pr}_3(u_1 = u_h|HL, 1>2) \\
&+ \mathrm{Pr}_3(LL|1>2) \cdot \mathrm{Pr}_3(u_1 = u_h|LL, 1>2) \\
=& p_h^2 \cdot 1 + 2p_hp_l \cdot \mathrm{Pr}_3(u_1 = u_h|HL, 1>2) + p_l^2 \cdot 0 \\
=& p_h^2 + 2p_hp_l \cdot \mathrm{Pr}_3(u_1 = u_h|HL, 1>2) \\
=& p_h(p_h + p_l \cdot 2\mathrm{Pr}_3(u_1 = u_h|HL, 1>2))
\end{aligned}$$

To show $\pi_1^r(3) \geq p_h$, it is equivalent to show $p_h + p_l \cdot 2\mathrm{Pr}_3(u_1 = u_h|HL, 1>2) \geq 1$, which is $\mathrm{Pr}_3(u_1 = u_h|HL, 1>2) \geq 1/2$ (as $p_h + p_l = 1$). Notice that

$$\begin{aligned}
\mathrm{Pr}_3(u_1 = u_h|HL, 1>2) =& \frac{\mathrm{Pr}_3(u_1 = u_h, HL, 1>2)}{\mathrm{Pr}_3(HL, 1>2)} \\
=& \frac{\mathrm{Pr}_3(u_1 = u_h, u_2 = u_l, 1>2)}{\mathrm{Pr}_3(HL, 1>2)} \\
=& \frac{\mathrm{Pr}_3(u_1 = u_h, u_2 = u_l) \cdot \mathrm{Pr}_3(1>2|u_1 = u_h, u_2 = u_l)}{\mathrm{Pr}_3(HL|1>2) \cdot \mathrm{Pr}(1>2)} \\
=& \frac{p_hp_l \cdot \mathrm{Pr}_3(1>2|u_1 = u_h, u_2 = u_l)}{2p_hp_l \cdot 1/2} \\
=& \mathrm{Pr}_3(1>2|u_1 = u_h, u_2 = u_l)
\end{aligned}$$

and it is equivalent to show $\mathrm{Pr}_3(1>2|u_1 = u_h, u_2 = u_l) \geq 1/2$. To prove this we consider two different cases depending on whether consumer 1 makes purchase.

The first case is that consumer 1 purchases a product. If consumer 1 is only willing to search once, then she purchases each product with equal probabilities as the two products are ex ante homogeneous. If consumer 1 is willing to perform the second search when the first search reveals a low-value product, then she purchases product 1 with probability one as product 1 is of high value while product 2 is of low value. Therefore, when consumer 1 makes purchases, she purchases product 1 with probability at least 1/2. Next we show that if consumer 1 purchases product 1, then product 1 must have higher sales ranking after consumer 2. As there is only one consumer before consumer 2, consumer 1 can be considered as the early consumer and consumer 2 can be considered as the late consumer, which is a special case of the base model. By the result in the base model, when product 1 has higher sales ranking, it is optimal for consumer 2 to search product 1 first. As product 1 is of high value, consumer 2 would purchase product 1 (if search) without performing the second search. Thus, if consumer 2 decides to search, product 1 must have higher sales ranking after consumer 2 purchases. If consumer 2 decides not to search, then product 1 still has higher sales ranking as it has one unit of sales while product 2 has no sales. It follows that when consumer 1 makes purchase, the probability that product 1 has higher sales ranking after consumer 2 is at least 1/2.

The second case is that consumer 1 does not purchase. In this case the two products are symmetric for consumer 2. If consumer 2 is only willing to search once, then she purchases each product with equal probabilities. If consumer 2 is willing to perform the second search when the first search reveals a low-value product, then she purchases product 1 with probability one as product 1 is of high value while product 2 is of low value. It follows that the probability that consumer 2 purchase product 1 is at least 1/2 and product 1 has higher sales ranking with probability at least 1/2. Combining the two cases we have established that $\Pr_3(1>2|u_1=u_h,u_2=u_l)\geq 1/2$, which finishes the proof. □

Proof of Lemma S.40 It suffices to prove for the event HL, events HH and LL can be proved similarly and is omitted here. We have

$$\Pr_t(HL|1>2)=\frac{\Pr_t(HL,1>2)}{\Pr_t(1>2)}=\frac{\Pr(HL)\cdot\Pr_t(1>2|HL)}{\Pr_t(1>2)}$$

where $\Pr_t(HL,1>2)$ is the probability that there is one high-value product and consumer t observes sales ranking $1>2$, $\Pr_t(1>2)$ is the probability that consumer t observes sales ranking $1>2$, and $\Pr_t(1>2|HL)$ is the probability that consumer t observes sales ranking $1>2$ conditioning on there is one high-value product. Notice that the two products are symmetric under all events HH,HL, and LL. In particular, the two products are symmetric when there is one high-value product as the two products are equally likely to be the high-value product (specifically, since the products are ex ante homogeneous, we can swap product labels). Therefore $\Pr_t(1>2|HL)=\Pr_t(1>2)=1/2$ and

$$\Pr_t(HL|1>2)=\frac{\Pr(HL)\cdot\Pr_t(1>2|HL)}{\Pr_t(1>2)}=\Pr(HL)=2p_hp_l$$

□

Proof of Proposition S.29 Let $q(x_1,x_2|j)$ be the probability that the sales of the two products are x_1 and x_2, respectively, with $x_1\geq x_2$, among the first $j-1$ consumers. Then

$$\sum_{x_1\geq x_2,x_1+x_2<j} q(x_1,x_2|j)=1$$

and

$$\pi_1^r(j)=\sum_{x_1\geq x_2} q(x_1,x_2|j)\pi_1^v(j,x_1,x_2)=\mathbb{E}_{x_1,x_2}[\pi_1^v(j,x_1,x_2)]$$

□

Proof of Proposition S.31 We prove the proposition by constructing the following example. Let $p_h=0.4$, $u_h=5$, $u_l=1$. Consider a F with $F(x)=1,x\geq 2$, $F(x)=0.5,1\leq x<2$ and $F(x)=0,x<1$. Consider a case with three consumers. When there is no sales information, as $p_hu_h+p_lu_l=2.6>2$, all the three consumers make purchase. When real time sales volume information is released, there is a positive probability that the first two consumer purchase different products. As

$$\begin{aligned}\pi_1^v(3,1,1) &= \frac{p_h}{2p_h + p_l \frac{F(p_h u_h + p_l u_l)}{F(p_h u_h + p_l u_l) - F(p_h(u_h - u_l))}} \\ &= \frac{0.4}{0.8 + 0.6 * \frac{1}{1-0.5}} \\ &= 0.2\end{aligned}$$

The third consumer makes purchase with probability $F(\pi_1^v(3,1,1)u_h + (1-\pi_1^v(3,1,1))u_l) = 0.5$ when the first two consumers purchase different products. It follows that the expected sales under real time volume information is less than 3 and the proof is completed. □

Proof of Proposition S.32 We prove the proposition by constructing the following example. Let $p_h = 0.4$, $u_h = 5$, $u_l = 1$. Consider a distribution function $F(\cdot)$ with $F(x) = 1, x \geq 2$, $F(x) = 0.5, 1 \leq x < 2$ and $F(x) = 0, x < 1$. Consider a case with three consumers. First consider the case where sales volume information is only released once and after the first consumer makes the purchasing decision. As $p_h u_h + p_l u_l = 2.6 > 2$, the first consumer makes purchase. As the two products are ex ante homogeneous, assume without loss of generality that the first consumer purchases product 1. The second and third consumers' belief that the product with one unit of sales is of high value is

$$\begin{aligned}&\frac{\Pr[u_1 = u_2 = u_h, X_1 = 1] + \Pr[u_1 = u_h, u_2 = u_l, X_1 = 1]}{\begin{pmatrix}\Pr[u_1 = u_2 = u_h, X_1 = 1] + \Pr[u_1 = u_h, u_2 = u_l, X_1 = 1] \\ + \Pr[u_1 = u_l, u_2 = u_h, X_1 = 1] + \Pr[u_1 = u_2 = u_l, X_1 = 1]\end{pmatrix}} \\ &= \frac{p_h^2/2 + p_h p_l((F(p_h u_h + p_l u_l) - F(p_h(u_h - u_l)))/2 + F(p_h(u_h - u_l)))}{p_h^2/2 + p_h p_l((F(p_h u_h + p_l u_l) - F(p_h(u_h - u_l)))/2 + F(p_h(u_h - u_l))) + p_h p_l(F(p_h u_h + p_l u_l) - F(p_h(u_h - u_l)))/2 + p_l^2/2} \\ &= \frac{0.16/2 + 0.24 * 0.75}{0.16/2 + 0.24 * 0.75 + 0.24 * 0.25 + 0.36/2} \\ &= 0.52 > p_h\end{aligned}$$

It follows that both the second consumer and the third consumer make purchase and the expected sales is 3 when sales volume information is only released once and after the first consumer makes the purchasing decision.

When real time sales volume information is released, there is a positive probability that the first two consumers purchase different products. As

$$\begin{aligned}\pi_1^v(3,1,1) &= \frac{p_h}{2p_h + p_l \frac{F(p_h u_h + p_l u_l)}{F(p_h u_h + p_l u_l) - F(p_h(u_h - u_l))}} \\ &= \frac{0.4}{0.8 + 0.6 * \frac{1}{1-0.5}} \\ &= 0.2\end{aligned}$$

The third consumer makes purchase with probability $F(\pi_1^v(3,1,1)u_h + (1-\pi_1^v(3,1,1))u_l) = 0.5$ when the first two consumers purchase different products. It follows that the expected sales under real time volume information is less than 3 and the proof is completed. □

SQ. Learning-Induced Pricing Competition

So far we have abstracted away from the possibility that the selling prices of the products may be adjusted after bestseller information is released. This is because product pricing is typically a decision of product providers and beyond the control of an e-commerce platform. When product pricing can be adapted to bestseller information, how does it impact the platform's incentive to provide the information? To address this question, we extend the model to consider the situation that the two products are offered by two different product providers (referred to as "sellers"), one product by one seller. The sellers simultaneously and independently choose the second-period price of their own product upon observing the bestseller information (if any) released by the platform. That is, the two sellers are engaged in a pricing competition induced by social learning. The platform anticipates the pricing competition and aims to maximize the expected total commission collected from the two sellers, which is equivalent to maximizing the expected total revenue from the product sales, as the per-sale commission is typically proportional to the selling price.

Specifically, let r_1 and r_2 denote the second-period selling price of product 1 and product 2, respectively, after the bestseller information (if any) is released. Same as in the base model, assume without loss of generality that product 1 is the bestseller. We focus on the case that $r_1, r_2 \leq u_l$ so that the prices can not be higher than the utility of a low-type product.

SQ.1. Model Analysis

The first-period analysis is similar to that in the base model. Thus, we focus on the second period and start from second-period consumers' search and purchasing decisions. To this end, define the following beliefs in the second period given information type $t \in \{\phi, r, v\}$:

- π_1^t: Probability that product 1 is of high value before any search in the second period
- ν_1^t: Probability that product 2 is of high value before any search in the second period
- π_2^{tH}: Probability that product 2 is of high value after a first search reveals high value in product 1
- π_2^{tL}: Probability that product 2 is of high value after a first search reveals low value in product 1
- ν_2^{tH}: Probability that product 1 is of high value after a first search reveals high value in product 2
- ν_2^{tL}: Probability that product 1 is of high value after a first search reveals low value in product 2

SQ.1.1. Consumer Beliefs

We first analyze the beliefs under sales ranking information. We have

$$\begin{aligned}
\pi_1^r =& \Pr[u_1 = u_h | X_1 \geq \frac{n_1}{2}] \\
=& \frac{\Pr[u_1 = u_2 = u_h, X_1 \geq \frac{n_1}{2}] + \Pr[u_1 = u_h, u_2 = u_l, X_1 \geq \frac{n_1}{2}]}{\begin{pmatrix} \Pr[u_1 = u_2 = u_h, X_1 \geq \frac{n_1}{2}] + \Pr[u_1 = u_h, u_2 = u_l, X_1 \geq \frac{n_1}{2}] \\ +\Pr[u_1 = u_l, u_2 = u_h, X_1 \geq \frac{n_1}{2}] + \Pr[u_1 = u_2 = u_l, X_1 \geq \frac{n_1}{2}] \end{pmatrix}} \\
=& \frac{p_h^2 \bar{G}_s(\frac{n_1}{2}) + p_h p_l \bar{G}_a(\frac{n_1}{2})}{p_h^2 \bar{G}_s(\frac{n_1}{2}) + p_h p_l \bar{G}_a(\frac{n_1}{2}) + p_h p_l G_a(\frac{n_1}{2}) + p_l^2 \bar{G}_s(\frac{n_1}{2})} \\
=& p_h^2 + 2 p_h p_l (1 - G_a(\frac{n_1}{2}))
\end{aligned}$$

$$\begin{aligned}
\nu_1^r =& \Pr[u_2 = u_h | X_1 \geq \frac{n_1}{2}] \\
=& \frac{\Pr[u_1 = u_2 = u_h, X_1 \geq \frac{n_1}{2}] + \Pr[u_1 = u_l, u_2 = u_h, X_1 \geq \frac{n_1}{2}]}{\begin{pmatrix} \Pr[u_1 = u_2 = u_h, X_1 \geq \frac{n_1}{2}] + \Pr[u_1 = u_h, u_2 = u_l, X_1 \geq \frac{n_1}{2}] \\ +\Pr[u_1 = u_l, u_2 = u_h, X_1 \geq \frac{n_1}{2}] + \Pr[u_1 = u_2 = u_l, X_1 \geq \frac{n_1}{2}] \end{pmatrix}}
\end{aligned}$$

$$
\begin{aligned}
=&\frac{p_h^2\bar{G}_s(\frac{n_1}{2})+p_hp_lG_a(\frac{n_1}{2})}{p_h^2\bar{G}_s(\frac{n_1}{2})+p_hp_l\bar{G}_a(\frac{n_1}{2})+p_hp_lG_a(\frac{n_1}{2})+p_l^2\bar{G}_s(\frac{n_1}{2})}\\
=&p_h^2+2p_hp_lG_a(\tfrac{n_1}{2})\\
\pi_2^{rH}=&\Pr[u_2=u_h|X_1\geq\frac{n_1}{2},u_1=u_h]\\
=&\frac{\Pr[u_1=u_h,u_2=u_h,X_1\geq\frac{n_1}{2}]}{\Pr[u_1=u_h,u_2=u_h,X_1\geq\frac{n_1}{2}]+\Pr[u_1=u_h,u_2=u_l,X_1\geq\frac{n_1}{2}]}\\
=&\frac{p_hG_s(\frac{n_1}{2})}{p_hG_s(\frac{n_1}{2})+p_l(1-G_a(\frac{n_1}{2}))}\\
\pi_2^{rL}=&\Pr[u_2=u_h|X_1\geq\frac{n_1}{2},u_1=u_l]\\
=&\frac{\Pr[u_1=u_l,u_2=u_h,X_1\geq\frac{n_1}{2}]}{\Pr[u_1=u_l,u_2=u_h,X_1\geq\frac{n_1}{2}]+\Pr[u_1=u_2=u_l,X_1\geq\frac{n_1}{2}]}\\
=&\frac{p_hG_a(\frac{n_1}{2})}{p_hG_a(\frac{n_1}{2})+p_l\bar{G}_s(\frac{n_1}{2})}\\
\nu_2^{rH}=&\Pr[u_1=u_h|X_1\geq\frac{n_1}{2},u_2=u_h]\\
=&\frac{\Pr[u_1=u_h,u_2=u_h,X_1\geq\frac{n_1}{2}]}{\Pr[u_1=u_h,u_2=u_h,X_1\geq\frac{n_1}{2}]+\Pr[u_1=u_l,u_2=u_h,X_1\geq\frac{n_1}{2}]}\\
=&\frac{p_hG_s(\frac{n_1}{2})}{p_hG_s(\frac{n_1}{2})+p_lG_a(\frac{n_1}{2})}\\
\nu_2^{rL}=&\Pr[u_1=u_h|X_1\geq\frac{n_1}{2},u_2=u_l]\\
=&\frac{\Pr[u_1=u_h,u_2=u_l,X_1\geq\frac{n_1}{2}]}{\Pr[u_1=u_h,u_2=u_l,X_1\geq\frac{n_1}{2}]+\Pr[u_1=u_2=u_l,X_1\geq\frac{n_1}{2}]}\\
=&\frac{p_h(1-G_a(\frac{n_1}{2}))}{p_h(1-G_a(\frac{n_1}{2}))+p_l\bar{G}_s(\frac{n_1}{2})}
\end{aligned}
$$

Lemma S.41 shows a nice ordering between the six beliefs under sales ranking information.

LEMMA S.41. $\nu_2^{rH}\geq\pi_1^r\geq\nu_2^{rL}\geq\pi_2^{rH}\geq\nu_1^r\geq\pi_2^{rL}$

The intuition of Lemma S.41 is that whenever a product is revealed to be high type in the first search, the probability of the other product being high type increases. Similarly, whenever a product is revealed to be low type in the first search, the probability of the other product being high type decreases. Essentially, the fact that a product outsells a high type increases the probability that the product itself is of high value; and the fact that a product is outsold by a high type (instead of an unknown type or a low type) also increases the probability that the product itself is of high value. Conversely, the fact that a product outsells a low type decreases the probability that the product itself is of high value; and the fact that a product is outsold by a low type (instead of an unknown type or a high type) also decreases the probability that the product itself is of high value.

Next, we present the belies under sales volume information. When volume information is released, we have

$$
\begin{aligned}
\pi_1^v(x)=&\Pr[u_1=u_h|X_1=x]\\
=&\frac{\Pr[u_1=u_2=u_h,X_1=x]+\Pr[u_1=u_h,u_2=u_l,X_1=x]}{\begin{pmatrix}\Pr[u_1=u_2=u_h,X_1=x]+\Pr[u_1=u_h,u_2=u_l,X_1=x]\\+\Pr[u_1=u_l,u_2=u_h,X_1=x]+\Pr[u_1=u_2=u_l,X_1=x]\end{pmatrix}}\\
=&\frac{p_h^2g_s(x)+p_hp_lg_a(x)}{p_h^2g_s(x)+p_hp_lg_a(x)+p_hp_lg_a(n_1-x)+p_l^2g_s(x)}\\
\nu_1^v(x)=&\Pr[u_2=u_h|X_1=x]
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\Pr[u_1 = u_2 = u_h, X_1 = x] + \Pr[u_1 = u_l, u_2 = u_h, X_1 = x]}{\begin{pmatrix} \Pr[u_1 = u_2 = u_h, X_1 = x] + \Pr[u_1 = u_h, u_2 = u_l, X_1 = x] \\ +\Pr[u_1 = u_l, u_2 = u_h, X_1 = x] + \Pr[u_1 = u_2 = u_l, X_1 = x] \end{pmatrix}} \\
&= \frac{p_h^2 g_s(x) + p_h p_l g_a(n_1 - x)}{p_h^2 g_s(x) + p_h p_l g_a(x) + p_h p_l g_a(n_1 - x) + p_l^2 g_s(x)} \\
\pi_2^{vH}(x) &= \Pr[u_2 = u_h | X_1 = x, u_1 = u_h] \\
&= \frac{\Pr[u_1 = u_h, u_2 = u_h, X_1 = x]}{\Pr[u_1 = u_h, u_2 = u_h, X_1 = x] + \Pr[u_1 = u_h, u_2 = u_l, X_1 = x]} \\
&= \frac{p_h g_s(x)}{p_h g_s(x) + p_l g_a(x)} \\
\pi_2^{vL}(x) &= \Pr[u_2 = u_h | X_1 = x, u_1 = u_l] \\
&= \frac{\Pr[u_1 = u_l, u_2 = u_h, X_1 = x]}{\Pr[u_1 = u_l, u_2 = u_h, X_1 = x] + \Pr[u_1 = u_2 = u_l, X_1 = x]} \\
&= \frac{p_h g_a(n_1 - x)}{p_h g_a(n_1 - x) + p_l g_s(x)} \\
\nu_2^{vH}(x) &= \Pr[u_1 = u_h | X_1 = x, u_2 = u_h] \\
&= \frac{\Pr[u_1 = u_h, u_2 = u_h, X_1 = x]}{\Pr[u_1 = u_h, u_2 = u_h, X_1 = x] + \Pr[u_1 = u_l, u_2 = u_h, X_1 = x]} \\
&= \frac{p_h g_s(x)}{p_h g_s(x) + p_l g_a(n_1 - x)} \\
\nu_2^{vL}(x) &= \Pr[u_1 = u_h | X_1 = x, u_2 = u_l] \\
&= \frac{\Pr[u_1 = u_h, u_2 = u_l, X_1 = x]}{\Pr[u_1 = u_h, u_2 = u_l, X_1 = x] + \Pr[u_1 = u_2 = u_l, X_1 = x]} \\
&= \frac{p_h g_a(x)}{p_h g_a(x) + p_l g_s(x)}
\end{aligned}
$$

Lemma S.42 confirms an ordering of the posterior beliefs for volume information, similar to that in Lemma S.41 for ranking information. The underlying logic is aligned with that for Lemma S.41.

LEMMA S.42. $\nu_2^{vH}(x) \geq \pi_1^v(x) \geq \nu_2^{vL}(x)$ *and* $\pi_2^{vH}(x) \geq \nu_1^v(x) \geq \pi_2^{vL}(x), \forall x \geq n_1/2$.

SQ.1.2. Consumers' Search and Purchase Behavior We first derive the optimal search sequence of consumers. Given search cost s, a consumer's expected utility of first searching product 1 is:

$$
\begin{aligned}
U_1^t(s) = &- s + \pi_1^t \max(0, u_h - r_1, -s + \pi_2^{tH} \max(u_h - r_2, u_h - r_1, 0) + (1 - \pi_2^{tH}) \max(u_l - r_2, u_h - r_1, 0)) \\
&+ (1 - \pi_1^t) \max(0, u_l - r_1, -s + \pi_2^{tL} \max(u_h - r_2, u_l - r_1, 0) + (1 - \pi_2^{tL}) \max(u_l - r_2, u_l - r_1, 0)) \\
= &- s + \pi_1^t \max(u_h - r_1, -s + \pi_2^{tH} \max(u_h - r_2, u_h - r_1) + (1 - \pi_2^{tH}) \max(u_l - r_2, u_h - r_1)) \\
&+ (1 - \pi_1^t) \max(u_l - r_1, -s + \pi_2^{tL} \max(u_h - r_2, u_l - r_1) + (1 - \pi_2^{tL}) \max(u_l - r_2, u_l - r_1))
\end{aligned}
$$

that of first searching product 2 is:

$$
\begin{aligned}
U_2^t(s) = &- s + \nu_1^t \max(0, u_h - r_2, -s + \nu_2^{tH} \max(u_h - r_1, u_h - r_2, 0) + (1 - \nu_2^{tH}) \max(u_l - r_1, u_h - r_2, 0)) \\
&+ (1 - \nu_1^t) \max(0, u_l - r_2, -s + \nu_2^{tL} \max(u_h - r_1, u_l - r_2, 0) + (1 - \nu_2^{tL}) \max(u_l - r_1, u_l - r_2, 0)) \\
= &- s + \nu_1^t \max(u_h - r_2, -s + \nu_2^{tH} \max(u_h - r_1, u_h - r_2) + (1 - \nu_2^{tH}) \max(u_l - r_1, u_h - r_2)) \\
&+ (1 - \nu_1^t) \max(u_l - r_2, -s + \nu_2^{tL} \max(u_h - r_1, u_l - r_2) + (1 - \nu_2^{tL}) \max(u_l - r_1, u_l - r_2))
\end{aligned}
$$

A consumer searches product 1 first if and only if $U_1^t(s) \geq U_2^t(s)$ and searches product 2 first if and only if $U_1^t(s) < U_2^t(s)$.

To further characterize consumers' first-search strategy, we consider below two cases: $r_1 \geq r_2$ and $r_1 < r_2$.

- $\underline{r_1 \geq r_2}$: we have

$$\begin{aligned}
U_1^t(s) =& -s + \pi_1^t \max(u_h - r_1, -s + \pi_2^{tH}(u_h - r_2) + (1 - \pi_2^{tH})(u_h - r_1)) \\
&+ (1 - \pi_1^t) \max(0, u_l - r_1, -s + \pi_2^{tL}(u_h - r_2) + (1 - \pi_2^{tL}) \max(u_l - r_2, 0)) \\
=& -s + \pi_1^t \max(u_h - r_1, -s + \pi_2^{tH}(u_h - r_2) + (1 - \pi_2^{tH})(u_h - r_1)) \\
&+ (1 - \pi_1^t) \max(u_l - r_1, -s + \pi_2^{tL}(u_h - r_2) + (1 - \pi_2^{tL})(u_l - r_2)) \\
U_2^t(s) =& -s + \nu_1^t(u_h - r_2) + (1 - \nu_1^t) \max(0, u_l - r_2, -s + \nu_2^{tL}(u_h - r_1) + (1 - \nu_2^{tL}) \max(u_l - r_2, 0)) \\
=& -s + \nu_1^t(u_h - r_2) + (1 - \nu_1^t) \max(u_l - r_2, -s + \nu_2^{tL}(u_h - r_1) + (1 - \nu_2^{tL})(u_l - r_2))
\end{aligned}$$

To derive the condition for $U_1^t(s) \geq U_2^t(s)$, we start by considering $U_1^t(s)$. When $-s + \pi_2^{tL}(u_h - r_2) + (1 - \pi_2^{tL})(u_l - r_2) < u_l - r_1$, which is $s > \pi_2^{tL}(u_h - u_l) + r_1 - r_2$, consumers do not perform the second search when product 1 is revealed to be of low value. Similarly, when $u_h - r_1 > -s + \pi_2^{tH}(u_h - r_2) + (1 - \pi_2^{tH})(u_h - r_1)$, which is $s > \pi_2^{tH}(r_1 - r_2)$, consumers do not perform the second search when product 1 is revealed to be of high value. Hence,

$$U_1^t(s) = \begin{cases} -s + \pi_1^t u_h + (1 - \pi_1^t)u_l - r_1, s > \pi_2^{tL}(u_h - u_l) + r_1 - r_2 \\ -s + \pi_1^t(u_h - r_1) + (1 - \pi_1^t)(-s + \pi_2^{tL}(u_h - r_2) + (1 - \pi_2^{tL})(u_l - r_2)), \pi_2^{tH}(r_1 - r_2) < s \leq \pi_2^{tL}(u_h - u_l) + r_1 - r_2 \\ -s + \pi_1^t(-s + \pi_2^{tH}(u_h - r_2) + (1 - \pi_2^{tH})(u_h - r_1)) + (1 - \pi_1^t)(-s + \pi_2^{tL}(u_h - r_2) + (1 - \pi_2^{tL})(u_l - r_2)), s \leq \pi_2^{tH}(r_1 - r_2) \end{cases}$$

Then consider $U_2^t(s)$. When $u_l - r_2 > -s + \nu_2^{tL}(u_h - r_1) + (1 - \nu_2^{tL})(u_l - r_2)$, which is $s > \nu_2^{tL}(u_h - u_l - r_1 + r_2)$, consumers do not perform the second search when product 2 is revealed to be of low value. Hence,

$$U_2^t(s) = \begin{cases} -s + \nu_1^t(u_h - r_2) + (1 - \nu_1^t)(u_l - r_2), & s > \nu_2^{tL}(u_h - u_l - r_1 + r_2) \\ -s + \nu_1^t(u_h - r_2) + (1 - \nu_1^t)(-s + \nu_2^{tL}(u_h - r_1) + (1 - \nu_2^{tL})(u_l - r_2)), & s \leq \nu_2^{tL}(u_h - u_l - r_1 + r_2) \end{cases}$$

To compare $U_1^t(s)$ and $U_2^t(s)$, we consider the following six cases.

Case 1 When $s > \nu_2^{tL}(u_h - u_l - r_1 + r_2)$ and $s > \pi_2^{tL}(u_h - u_l) + r_1 - r_2$,

$$\begin{aligned}
U_1^t(s) &= -s + \pi_1^t u_h + (1 - \pi_1^t)u_l - r_1 \\
U_2^t(s) &= -s + \nu_1^t(u_h - r_2) + (1 - \nu_1^t)(u_l - r_2)
\end{aligned}$$

and $U_1^t(s) \geq U_2^t(s)$ when $r_1 - r_2 \leq (\pi_1^t - \nu_1^t)(u_h - u_l)$.

Case 2 When $s \leq \nu_2^{tL}(u_h - u_l - r_1 + r_2)$ and $s > \pi_2^{tL}(u_h - u_l) + r_1 - r_2$

$$\begin{aligned}
U_1^t(s) &= -s + \pi_1^t u_h + (1 - \pi_1^t)u_l - r_1 \\
U_2^t(s) &= -s + \nu_1^t(u_h - r_2) + (1 - \nu_1^t)(-s + \nu_2^{tL}(u_h - r_1) + (1 - \nu_2^{tL})(u_l - r_2))
\end{aligned}$$

and $U_1^t(s) \geq U_2^t(s)$ when

$$(1 - (1 - \nu_1^t)\nu_2^{tL})r_1 \leq \pi_1^t u_h + (1 - \pi_1^t)u_l - \nu_1^t(u_h - r_2) - (1 - \nu_1^t)(-s + \nu_2^{tL}u_h + (1 - \nu_2^{tL})(u_l - r_2)).$$

which is

$$(1 - \nu_1^t)s \geq (1 - (1 - \nu_1^t)\nu_2^{tL})r_1 - \pi_1^t u_h - (1 - \pi_1^t)u_l + \nu_1^t(u_h - r_2) + (1 - \nu_1^t)(\nu_2^{tL}u_h + (1 - \nu_2^{tL})(u_l - r_2))$$

Case 3 When $s > \nu_2^{tL}(u_h - u_l - r_1 + r_2)$ and $\pi_2^{tH}(r_1 - r_2) < s \leq \pi_2^{tL}(u_h - u_l) + r_1 - r_2$,

$$U_1^t(s) = -s + \pi_1^t(u_h - r_1) + (1 - \pi_1^t)(-s + \pi_2^{tL}(u_h - r_2) + (1 - \pi_2^{tL})(u_l - r_2)))$$
$$U_2^t(s) = -s + \nu_1^t(u_h - r_2) + (1 - \nu_1^t)(u_l - r_2)$$

and $U_1^t(s) \geq U_2^t(s)$ when

$$\pi_1^t r_1 \leq \pi_1^t u_h + (1 - \pi_1^t)(-s + \pi_2^{tL}(u_h - r_2) + (1 - \pi_2^{tL})(u_l - r_2))) - \nu_1^t(u_h - r_2) - (1 - \nu_1^t)(u_l - r_2),$$

which is

$$(1 - \pi_1^t)s \leq \pi_1^t(u_h - r_1) + (1 - \pi_1^t)(\pi_2^{tL}(u_h - r_2) + (1 - \pi_2^{tL})(u_l - r_2))) - \nu_1^t(u_h - r_2) - (1 - \nu_1^t)(u_l - r_2)$$

Case 4 When $s \leq \nu_2^{tL}(u_h - u_l - r_1 + r_2)$ and $\pi_2^{tH}(r_1 - r_2) < s \leq \pi_2^{tL}(u_h - u_l) + r_1 - r_2$,

$$U_1^t(s) = -s + \pi_1^t(u_h - r_1) + (1 - \pi_1^t)(-s + \pi_2^{tL}(u_h - r_2) + (1 - \pi_2^{tL})(u_l - r_2))$$
$$U_2^t(s) = -s + \nu_1^t(u_h - r_2) + (1 - \nu_1^t)(-s + \nu_2^{tL}(u_h - r_1) + (1 - \nu_2^{tL})(u_l - r_2))$$

and $U_1^t(s) \geq U_2^t(s)$ when

$$\begin{aligned}(\pi_1^t - (1 - \nu_1^t)\nu_2^{tL})r_1 \leq &\pi_1^t u_h + (1 - \pi_1^t)(-s + \pi_2^{tL}(u_h - r_2) + (1 - \pi_2^{tL})(u_l - r_2)) \\ &- \nu_1^t(u_h - r_2) - (1 - \nu_1^t)(-s + \nu_2^{tL}u_h + (1 - \nu_2^{tL})(u_l - r_2))\end{aligned}$$

which is

$$\begin{aligned}(\pi_1^t - \nu_1^t)s \geq &- \pi_1^t(u_h - r_1) - (1 - \pi_1^t)(\pi_2^{tL}(u_h - r_2) + (1 - \pi_2^{tL})(u_l - r_2)) \\ &+ \nu_1^t(u_h - r_2) + (1 - \nu_1^t)(\nu_2^{tL}(u_h - r_1) + (1 - \nu_2^{tL})(u_l - r_2))\end{aligned}$$

Case 5 When $s > \nu_2^{tL}(u_h - u_l - r_1 + r_2)$ and $s \leq \pi_2^{tH}(r_1 - r_2)$,

$$U_1^t(s) = -s + \pi_1^t(-s + \pi_2^{tH}(u_h - r_2) + (1 - \pi_2^{tH})(u_h - r_1)) + (1 - \pi_1^t)(-s + \pi_2^{tL}(u_h - r_2) + (1 - \pi_2^{tL})(u_l - r_2))$$
$$U_2^t(s) = -s + \nu_1^t(u_h - r_2) + (1 - \nu_1^t)(u_l - r_2)$$

and $U_1^t(s) \geq U_2^t(s)$ when

$$\begin{aligned}\pi_1^t(1 - \pi_2^{tH})r_1 \leq &\pi_1^t(-s + \pi_2^{tH}(u_h - r_2) + (1 - \pi_2^{tH})u_h) + (1 - \pi_1^t)(-s + \pi_2^{tL}(u_h - r_2) + (1 - \pi_2^{tL})(u_l - r_2)) \\ &- \nu_1^t(u_h - r_2) - (1 - \nu_1^t)(u_l - r_2)\end{aligned}$$

which is

$$\begin{aligned}s \leq &\pi_1^t(\pi_2^{tH}(u_h - r_2) + (1 - \pi_2^{tH})(u_h - r_1)) + (1 - \pi_1^t)(\pi_2^{tL}(u_h - r_2) + (1 - \pi_2^{tL})(u_l - r_2)) \\ &- \nu_1^t(u_h - r_2) - (1 - \nu_1^t)(u_l - r_2)\end{aligned}$$

Case 6 When $s \leq \nu_2^{tL}(u_h - u_l - r_1 + r_2)$ and $s \leq \pi_2^{tH}(r_1 - r_2)$,

$$U_1^t(s) = -s + \pi_1^t(-s + \pi_2^{tH}(u_h - r_2) + (1 - \pi_2^{tH})(u_h - r_1)) + (1 - \pi_1^t)(-s + \pi_2^{tL}(u_h - r_2) + (1 - \pi_2^{tL})(u_l - r_2))$$
$$U_2^t(s) = -s + \nu_1^t(u_h - r_2) + (1 - \nu_1^t)(-s + \nu_2^{tL}(u_h - r_1) + (1 - \nu_2^{tL})(u_l - r_2))$$

and $U_1^t(s) \geq U_2^t(s)$ when

$$\begin{aligned}(\pi_1^t(1-\pi_2^{tH})-(1-\nu_1^t)\nu_2^{tL})r_1 \leq &\pi_1^t(-s+\pi_2^{tH}(u_h-r_2)+(1-\pi_2^{tH})u_h)+(1-\pi_1^t)(-s+\pi_2^{tL}(u_h-r_2)+(1-\pi_2^{tL})(u_l-r_2)) \\ &-\nu_1^t(u_h-r_2)-(1-\nu_1^t)(-s+\nu_2^{tL}u_h+(1-\nu_2^{tL})(u_l-r_2))\end{aligned}$$

which is

$$\begin{aligned}&\pi_1^t(-s+\pi_2^{tH}(u_h-r_2)+(1-\pi_2^{tH})(u_h-r_1))+(1-\pi_1^t)(-s+\pi_2^{tL}(u_h-r_2)+(1-\pi_2^{tL})(u_l-r_2)) \\ \geq &\nu_1^t(u_h-r_2)+(1-\nu_1^t)(-s+\nu_2^{tL}(u_h-r_1)+(1-\nu_2^{tL})(u_l-r_2))\end{aligned}$$

or

$$\begin{aligned}\nu_1^t s \leq &\pi_1^t(\pi_2^{tH}(u_h-r_2)+(1-\pi_2^{tH})(u_h-r_1))+(1-\pi_1^t)(\pi_2^{tL}(u_h-r_2)+(1-\pi_2^{tL})(u_l-r_2)) \\ &-\nu_1^t(u_h-r_2)-(1-\nu_1^t)(\nu_2^{tL}(u_h-r_1)+(1-\nu_2^{tL})(u_l-r_2))\end{aligned}$$

- $\underline{r_1 < r_2}$: we have

$$\begin{aligned}U_2^t(s) = &-s+\nu_1^t\max(u_h-r_2, -s+\nu_2^{tH}(u_h-r_1)+(1-\nu_2^{tH})(u_h-r_2)) \\ &+(1-\nu_1^t)\max(0, u_l-r_2, -s+\nu_2^{tL}(u_h-r_1)+(1-\nu_2^{tL})\max(u_l-r_1, 0)) \\ = &-s+\nu_1^t\max(u_h-r_2, -s+\nu_2^{tH}(u_h-r_1)+(1-\nu_2^{tH})(u_h-r_2)) \\ &+(1-\nu_1^t)\max(u_l-r_2, -s+\nu_2^{tL}(u_h-r_1)+(1-\nu_2^{tL})(u_l-r_1))\end{aligned}$$

$$\begin{aligned}U_1^t(s) = &-s+\pi_1^t(u_h-r_1)+(1-\pi_1^t)\max(0, u_l-r_1, -s+\pi_2^{tL}(u_h-r_2)+(1-\pi_2^{tL})\max(u_l-r_1, 0)) \\ = &-s+\pi_1^t(u_h-r_1)+(1-\pi_1^t)\max(u_l-r_1, -s+\pi_2^{tL}(u_h-r_2)+(1-\pi_2^{tL})(u_l-r_1))\end{aligned}$$

To derive the condition for $U_2^t(s) \geq U_1^t(s)$, we start by considering $U_2^t(s)$. When $-s+\nu_2^{tL}(u_h-r_1)+(1-\nu_2^{tL})(u_l-r_1) < u_l-r_2$, which is $s > \nu_2^{tL}(u_h-u_l)+r_2-r_1$, consumers do not perform the second search when product 1 is revealed to be of low value. Similarly, when $u_h-r_2 > -s+\nu_2^{tH}(u_h-r_1)+(1-\nu_2^{tH})(u_h-r_2)$, which is $s > \nu_2^{tH}(r_2-r_1)$, consumers do not perform the second search when product 1 is revealed to be of high value. Hence,

$$U_2^t(s) = \begin{cases} -s+\nu_1^t u_h+(1-\nu_1^t)u_l-r_2, s > \nu_2^{tL}(u_h-u_l)+r_2-r_1 \\ -s+\nu_1^t(u_h-r_2)+(1-\nu_1^t)(-s+\nu_2^{tL}(u_h-r_1)+(1-\nu_2^{tL})(u_l-r_1)), \nu_2^{tH}(r_2-r_1) < s \leq \nu_2^{tL}(u_h-u_l)+r_2-r_1 \\ -s+\nu_1^t(-s+\nu_2^{tH}(u_h-r_1)+(1-\nu_2^{tH})(u_h-r_2))+(1-\nu_1^t)(-s+\nu_2^{tL}(u_h-r_1)+(1-\nu_2^{tL})(u_l-r_1)), s \leq \nu_2^{tH}(r_2-r_1) \end{cases}$$

Then consider $U_1^t(s)$. When $u_l-r_1 > -s+\pi_2^{tL}(u_h-r_2)+(1-\pi_2^{tL})(u_l-r_1)$, which is $s > \pi_2^{tL}(u_h-u_l-r_2+r_1)$, consumers do not perform the second search when product 2 is revealed to be of low value. Hence,

$$U_1^t(s) = \begin{cases} -s+\pi_1^t(u_h-r_1)+(1-\pi_1^t)(u_l-r_1), & s > \pi_2^{tL}(u_h-u_l-r_2+r_1) \\ -s+\pi_1^t(u_h-r_1)+(1-\pi_1^t)(-s+\pi_2^{tL}(u_h-r_2)+(1-\pi_2^{tL})(u_l-r_1)), & s \leq \pi_2^{tL}(u_h-u_l-r_2+r_1) \end{cases}$$

Case 1 When $s > \pi_2^{tL}(u_h-u_l-r_2+r_1)$ and $s > \nu_2^{tL}(u_h-u_l)+r_2-r_1$,

$$\begin{aligned}U_2^t(s) &= -s+\nu_1^t u_h+(1-\nu_1^t)u_l-r_2 \\ U_1^t(s) &= -s+\pi_1^t(u_h-r_1)+(1-\pi_1^t)(u_l-r_1)\end{aligned}$$

and $U_2^t(s) \geq U_1^t(s)$ when $r_2 - r_1 \leq (\nu_1^t - \pi_1^t)(u_h - u_l)$.

Case 2 When $s \leq \pi_2^{tL}(u_h - u_l - r_2 + r_1)$ and $s > \nu_2^{tL}(u_h - u_l) + r_2 - r_1$

$$U_2^t(s) = -s + \nu_1^t u_h + (1-\nu_1^t)u_l - r_2$$
$$U_1^t(s) = -s + \pi_1^t(u_h - r_1) + (1-\pi_1^t)(-s + \pi_2^{tL}(u_h - r_2) + (1-\pi_2^{tL})(u_l - r_1))$$

and $U_2^t(s) \geq U_1^t(s)$ when

$$(1-(1-\pi_1^t)\pi_2^{tL})r_2 \leq \nu_1^t u_h + (1-\nu_1^t)u_l - \pi_1^t(u_h - r_1) - (1-\pi_1^t)(-s + \pi_2^{tL}u_h + (1-\pi_2^{tL})(u_l - r_1)).$$

which is

$$(1-\pi_1^t)s \geq (1-(1-\pi_1^t)\pi_2^{tL})r_2 - \nu_1^t u_h - (1-\nu_1^t)u_l + \pi_1^t(u_h - r_1) + (1-\pi_1^t)(\pi_2^{tL}u_h + (1-\pi_2^{tL})(u_l - r_1))$$

Case 3 When $s > \pi_2^{tL}(u_h - u_l - r_2 + r_1)$ and $\nu_2^{tH}(r_2 - r_1) < s \leq \nu_2^{tL}(u_h - u_l) + r_2 - r_1$,

$$U_2^t(s) = -s + \nu_1^t(u_h - r_2) + (1-\nu_1^t)(-s + \nu_2^{tL}(u_h - r_1) + (1-\nu_2^{tL})(u_l - r_1)))$$
$$U_1^t(s) = -s + \pi_1^t(u_h - r_1) + (1-\pi_1^t)(u_l - r_1)$$

and $U_2^t(s) \geq U_1^t(s)$ when

$$\nu_1^t r_2 \leq \nu_1^t u_h + (1-\nu_1^t)(-s + \nu_2^{tL}(u_h - r_1) + (1-\nu_2^{tL})(u_l - r_1))) - \pi_1^t(u_h - r_1) - (1-\pi_1^t)(u_l - r_1),$$

which is

$$(1-\nu_1^t)s \leq \nu_1^t(u_h - r_2) + (1-\nu_1^t)(\nu_2^{tL}(u_h - r_1) + (1-\nu_2^{tL})(u_l - r_1))) - \pi_1^t(u_h - r_1) - (1-\pi_1^t)(u_l - r_1)$$

Case 4 When $s \leq \pi_2^{tL}(u_h - u_l - r_2 + r_1)$ and $\nu_2^{tH}(r_2 - r_1) < s \leq \nu_2^{tL}(u_h - u_l) + r_2 - r_1$,

$$U_2^t(s) = -s + \nu_1^t(u_h - r_2) + (1-\nu_1^t)(-s + \nu_2^{tL}(u_h - r_1) + (1-\nu_2^{tL})(u_l - r_1))$$
$$U_1^t(s) = -s + \pi_1^t(u_h - r_1) + (1-\pi_1^t)(-s + \pi_2^{tL}(u_h - r_2) + (1-\pi_2^{tL})(u_l - r_1))$$

and $U_2^t(s) \geq U_1^t(s)$ when

$$\begin{aligned}(\nu_1^t - (1-\pi_1^t)\pi_2^{tL})r_2 \leq &\nu_1^t u_h + (1-\nu_1^t)(-s + \nu_2^{tL}(u_h - r_1) + (1-\nu_2^{tL})(u_l - r_1)) \\ &- \pi_1^t(u_h - r_1) - (1-\pi_1^t)(-s + \pi_2^{tL}u_h + (1-\pi_2^{tL})(u_l - r_1))\end{aligned}$$

which is

$$\begin{aligned}(\nu_1^t - \pi_1^t)s \geq &- \nu_1^t(u_h - r_2) - (1-\nu_1^t)(\nu_2^{tL}(u_h - r_1) + (1-\nu_2^{tL})(u_l - r_1)) \\ &+ \pi_1^t(u_h - r_1) + (1-\pi_1^t)(\pi_2^{tL}(u_h - r_2) + (1-\pi_2^{tL})(u_l - r_1))\end{aligned}$$

Case 5 When $s > \pi_2^{tL}(u_h - u_l - r_2 + r_1)$ and $s \leq \nu_2^{tH}(r_2 - r_1)$,

$$U_2^t(s) = -s + \nu_1^t(-s + \nu_2^{tH}(u_h - r_1) + (1-\nu_2^{tH})(u_h - r_2)) + (1-\nu_1^t)(-s + \nu_2^{tL}(u_h - r_1) + (1-\nu_2^{tL})(u_l - r_1))$$
$$U_1^t(s) = -s + \pi_1^t(u_h - r_1) + (1-\pi_1^t)(u_l - r_1)$$

and $U_2^t(s) \geq U_1^t(s)$ when

$$\begin{aligned}\nu_1^t(1-\nu_2^{tH})r_2 \leq &\nu_1^t(-s + \nu_2^{tH}(u_h - r_1) + (1-\nu_2^{tH})u_h) + (1-\nu_1^t)(-s + \nu_2^{tL}(u_h - r_1) + (1-\nu_2^{tL})(u_l - r_1)) \\ &- \pi_1^t(u_h - r_1) - (1-\pi_1^t)(u_l - r_1)\end{aligned}$$

which is

$$\begin{aligned} s \leq & \nu_1^t(\nu_2^{tH}(u_h - r_1) + (1-\nu_2^{tH})(u_h - r_2)) + (1-\nu_1^t)(\nu_2^{tL}(u_h - r_1) + (1-\nu_2^{tL})(u_l - r_1)) \\ & - \pi_1^t(u_h - r_1) - (1-\pi_1^t)(u_l - r_1) \end{aligned}$$

Case 6 When $s \leq \pi_2^{tL}(u_h - u_l - r_2 + r_1)$ and $s \leq \nu_2^{tH}(r_2 - r_1)$,

$$U_2^t(s) = -s + \nu_1^t(-s + \nu_2^{tH}(u_h - r_1) + (1-\nu_2^{tH})(u_h - r_2)) + (1-\nu_1^t)(-s + \nu_2^{tL}(u_h - r_1) + (1-\nu_2^{tL})(u_l - r_1))$$
$$U_1^t(s) = -s + \pi_1^t(u_h - r_1) + (1-\pi_1^t)(-s + \pi_2^{tL}(u_h - r_2) + (1-\pi_2^{tL})(u_l - r_1))$$

and $U_2^t(s) \geq U_1^t(s)$ when

$$\begin{aligned} (\nu_1^t(1-\nu_2^{tH}) - (1-\pi_1^t)\pi_2^{tL})r_2 \leq & \nu_1^t(-s + \nu_2^{tH}(u_h - r_1) + (1-\nu_2^{tH})u_h) + (1-\nu_1^t)(-s + \nu_2^{tL}(u_h - r_1) + (1-\nu_2^{tL})(u_l - r_1)) \\ & - \pi_1^t(u_h - r_1) - (1-\pi_1^t)(-s + \pi_2^{tL}u_h + (1-\pi_2^{tL})(u_l - r_1)) \end{aligned}$$

which is

$$\begin{aligned} \pi_1^t s \leq & \nu_1^t(\nu_2^{tH}(u_h - r_1) + (1-\nu_2^{tH})(u_h - r_2)) + (1-\nu_1^t)(\nu_2^{tL}(u_h - r_1) + (1-\nu_2^{tL})(u_l - r_1)) \\ & - \pi_1^t(u_h - r_1) - (1-\pi_1^t)(\pi_2^{tL}(u_h - r_2) + (1-\pi_2^{tL})(u_l - r_1)) \end{aligned}$$

SQ.1.3. Consumers' Search and Purchasing Behavior after First Search Now we analyze consumers' search and purchasing behavior after the first search. Consider the following two cases, each consisting of three sub-cases:

• **Search Product 1 First**

- **Sub-case** $r_1 > r_2$

If product 1 is of high value, then consumers with $s \leq \pi_2^{rH}u_h + (1-\pi_2^{rH})u_l - r_2 - (u_h - r_1)$ perform the second search, and the remaining consumers purchase product 1 without performing the second search. For those consumers who perform the second search, if product 2 is of high value, they purchase product 2; if product 2 is of low value, they purchase product 1 if $u_h - r_1 > u_l - r_2$, purchase product 2 if $u_h - r_1 < u_l - r_2$, and randomly choose a product to purchase if $u_h - r_1 = u_l - r_2$. If product 1 is of low value, then consumers with $s \leq \pi_2^{rL}u_h + (1-\pi_2^{rL})u_l - r_2 - (u_l - r_1)$ perform the second search and always purchase product 2. Consumers with higher search costs purchase product 1 directly.

- **Sub-case** $r_1 < r_2$

If product 1 is of high value, no consumer performs the second search and all the consumers who perform the first search purchase product 1. If product 1 is of low value and $u_h - r_2 > u_l - r_1$, then consumers with $s \leq \pi_2^{rL}u_h + (1-\pi_2^{rL})u_l - r_2 - (u_l - r_1)$ perform the second search. Consumers with higher search costs purchase product 1 directly. For the consumers who conduct the second search, if product 2 is of low value, they purchase product 1; otherwise (i.e., if product 2 is of high value), they purchase product 2.

- **Sub-case** $r_1 = r_2$

If product 1 is of high value, no consumer performs the second search and all the consumers who perform the first search purchase product 1. If product 1 is of low value, then consumers with $s \leq \pi_2^{rL}u_h + (1-\pi_2^{rL})u_l - r_2 - (u_l - r_1)$ perform the second search. Consumers with higher search costs purchase product 1 directly. If product 2 is found to be of low value through the second search, the consumers who conduct the second search randomly choose a product to purchase; otherwise (i.e., if product 2 is of high value), they purchase product 2.

• Search Product 2 First

- Sub-case $r_1 > r_2$

If product 2 is of high value, no consumer performs the second search and all the consumers who perform the first search purchase product 2. If product 2 is of low value and $u_h - r_1 > u_1 - r_2$, then consumers with $s \le \nu_2^{rL} u_h + (1-\nu_2^{rL})u_l - r_1 - (u_l - r_2)$ perform the second search. Consumers with higher search costs purchase product 2 directly. For the consumers who perform the second search, if product 1 is of low value, they purchase product 2; otherwise (i.e., if product 1 is of high value), they purchase product 1.

- Sub-case $r_1 < r_2$

If product 2 is of high value, then consumers with $s \le \nu_2^{rH} u_h + (1-\nu_2^{rH})u_l - r_1 - (u_h - r_2)$ perform the second search, and the remaining consumers purchase product 2 without performing the second search. For those consumers who perform the second search, if product 1 is of high value, they purchase product 1; if product 1 is of low value, they purchase product 2 if $u_h - r_2 > u_l - r_1$, purchase product 1 if $u_h - r_2 < u_l - r_1$, and randomly choose a product to purchase if $u_h - r_2 = u_l - r_1$. If product 2 is of low value, then consumers with $s \le \nu_2^{rL} u_h + (1-\nu_2^{rL})u_l - r_1 - (u_l - r_2)$ perform the second search and purchase product 1. Consumers with higher search costs purchase product 2 directly.

- Sub-case $r_1 = r_2$

If product 2 is of high value, no consumer performs the second search and all the consumers who perform the first search purchase product 1. If product 2 is of low value, then consumers with $s \le \nu_2^{rL} u_h + (1-\nu_2^{rL})u_l - r_1 - (u_l - r_2)$ perform the second search. Consumers with higher search costs purchase product 2 directly. If product 1 is of low value, the consumers who perform the second search randomly choose a product to purchase; otherwise (i.e., if product 1 is of high value), they purchase product 1.

The consumers' purchasing behavior as detailed above determines the sellers' second-period sales for given prices r_1, r_2. For $i \in \{1,2\}$ and $t \in \{\phi, r, v\}$, let $S_i^t(r_i, r_{3-i})$ denote product i's expected second-period sales under bestseller information t. For given $t \in \{\phi, r, v\}$ and r_{3-i}, seller i's second-period profit is then $r_i S_i^t(r_i, r_{3-i})$ and its best response is $r_i^{t,*}(r_{3-i}) = \arg\max_{r_i} r_i S_i^t(r_i, r_{3-i})$. For $t \in \{\phi, r, v\}$, a Nash equilibrium is attained if there exists a price pair, $(r_1^{t,**}, r_2^{t,**})$, such that $r_1^{t,**} = r_1^{t,*}(r_2^{t,**})$ and $r_2^{t,**} = r_2^{t,*}(r_1^{t,**})$. If multiple Nash equilibria exist for the pricing game between the two sellers, we select a Pareto-dominant one from the sellers' perspective. If multiple Pareto dominant equilibria exist, we apply a second selection criterion of choosing the one maximizing the total profit of the two sellers. In case a tie remains after applying the second criterion, we choose the pricing equilibrium that is lexicographically lowest.

The platform chooses an information type $t \in \{\phi, r, v\}$ to maximize the total expected second-period profit in equilibrium, i.e., $r_1^{t,**} S_1^t(r_1^{t,**}, r_2^{t,**}) + r_2^{t,**} S_2^t(r_2^{t,**}, r_1^{t,**})$.

SQ.2. Numerical Experiments and Results

As consumers' search and purchasing behavior becomes very complicated under sellers' pricing competition, it is difficult to obtain analytical results and thus we resort to numerical experiments. In the numerical experiments, the basic subroutine is to compute the sales for either product under given prices r_1 and r_2. Specifically, we compute the profit of both sellers for different combinations of r_1, r_2, obtain the best response of either seller, and, subsequently, search for the Nash equilibrium.

For numerical tractability, we consider discrete price points and assume that either seller selects its price from a finite price menu. Specifically, we focus on the case where the domain of r_1 and r_2 is $\{u_l/4, u_l/2, 3u_l/4, u_l\}$. Throughout the numerical study, we adopt the same search cost distribution as in the base model. We assume that the first-period price of either product is the same as the equilibrium second-period price in the no-information case, as no bestseller information is available in the first period.

Robustness of Key Insights in Base Model

Table S.15 reports the equilibrium prices and profits under different types of sales information. We first notice that when sales ranking information is provided, the total profit for the two sellers increases compared to the case without sales information. This is consistent with our result in the base model, where ranking information is always beneficial to the platform. Furthermore, we note that the total profit under sales volume information is sometimes lower than that under sales ranking information, as exemplified in the cases of $\delta_u = 2$. This is again in line with our finding in the base model whereby sales volume information may be outperformed for the platform by sales ranking information.

Table S.15 **Second-period profits and prices under pricing equilibrium for $t \in \{\phi, r, v\}$, where the first (second) number in the column "profits" represents the second-period profit for the product with higher (lower) sales ranking and the last number represents total profits of the two products:** $\mu = 4.5$, $\sigma = 1.5$, $n_0 = 40$, $n_2 = 60$, $p_h = 0.2$, $\alpha = 0.08$, $u_h = 10 + \delta_u$, $u_l = 6 + \delta_u$.

	$t=\phi$		$t=r$		$t=v$
δ_u	profits	prices	profits	prices	profits
0	32.71, 32.71, 65.42	1.5, 1.5	66.93, 3.61, 70.54	1.5, 1.5	67.19, 3.73, 70.92
1	45.22, 45.22, 90.44	1.75, 1.75	88.72, 4.25, 92.97	1.75, 1.75	84.19, 9.58, 93.78
1.5	51.09, 51.09, 102.18	1.875, 1.875	99.57, 4.56, 104.13	1.875, 1.875	94.23, 10.47, 104.70
2	56.54, 56.54, 113.09	2, 2	110.27, 4.82, 115.09	2, 2	110.09, 4.83, 114.92

Impact of Flexible Pricing on Profits

To examine the impact of learning-induced pricing competition, we compare the equilibrium outcome (profits, sales, prices) when prices are adjusted according to the bestseller information and its counterpart when prices are fixed at the no-information equilibrium levels. For expositional convenience, we refer to the first scenario as "flexible pricing" and the second as "fixed pricing". Tables S.16 and S.17 exemplify the comparative results under sales volume information for each first-period sales realization.

We first note in Table S.16, where prices are adjusted according to the first-period sales volume, that both products' prices (weakly) increase in the bestseller's sales volume. In particular, when the products' sales difference is great (as indicated by a high sales volume of the bestseller, x), consumers assign a high belief for the bestseller being of high value, allowing the higher-ranked seller to capitalize by setting a high price, compared to when the sales difference is low or when the price is fixed at the no-information level. Thus, the flexibility to adapt price to bestseller information enhances the profitability of the higher-ranked seller. Interestingly, so does it for the lower-ranked seller: as the higher-ranked seller raises price, some consumers shift to first search the lower-ranked product and may eventually purchase it, which benefits the lower-ranked seller. Essentially, the pricing flexibility enables the sellers to differentiate from each other along a second dimension, i.e., pricing, in addition to the dimension of the first-period sales difference. It leads to a finer market segmentation, rendering more consumer surplus extracted and thus a win-win for the two sellers.

Table S.16 Equilibrium outcome in the second period under volume information for different sales realization $n_0 = 40$, $n_2 = 60$, $p_h = 0.2$, $u_h = 10$, and $u_l = 6$, where x denotes the first-period sales of the bestseller.

x	profits	prices	sales
15	62.22, 3.67, 65.89	1.5, 1.5	41.48, 2.44
16	64.90, 3.62, 68.50	1.5, 1.5	43.26, 2.41
17	67.53, 3.59, 71.12	1.5, 1.5	45.02, 2.39
⋮			
27	123.00, 7.56, 130.56	3, 1.5	41.00, 5.04
28	126.49, 7.56, 134.05	3, 1.5	42.16, 5.04
29	129.60, 7.56, 137.16	3, 1.5	43.20, 5.04
Expected	67.19, 3.73, 70.92	1.5433, 1.5	43.88, 2.49

Table S.17 Equilibrium outcome in the second period under volume information for different sales realization with fixed prices $r_1 = r_2 = 1.5$ (i.e., the equilibrium prices under no information $t = \phi$) and $n_0 = 40, n_2 = 60, p_h = 0.2$, $u_h = 10$, and $u_l = 6$, where x denotes the first-period sales of the bestseller.

x	profits	sales
15	62.22, 3.67, 65.89	41.48, 2.44
16	64.90, 3.62, 68.50	43.26, 2.41
17	67.53, 3.59, 71.12	45.02, 2.39
⋮		
27	82.52, 3.51, 86.03	55.02, 2.34
28	83.05, 3.51, 86.56	55.37, 2.34
29	83.49, 3.51, 87.00	55.66, 2.34
Expected	66.67, 3.61, 70.28	44.45, 2.41

Similar comparative results under another problem instance, with $n_0 = 60$ and $n_2 = 40$, are presented in Tables S.18 and S.19. In particular, we observe the profit advantage of flexible pricing for both sellers.

Table S.18 Equilibrium outcome in the second period under volume information for different sales realization $n_0 = 60, n_2 = 40, p_h = 0.2$, $u_h = 10$, and $u_l = 6$, where x denotes the first-period sales of the bestseller.

x	profits	prices	sales
22	40.88, 2.42, 43.30	1.5, 1.5	27.25, 1.62
23	42.83, 2.40, 45.23	1.5, 1.5	28.56, 1.60
24	44.83, 2.38, 47.21	1.5, 1.5	29.88, 1.59
⋮			
41	99.25, 5.04, 104.29	3, 1.5	33.08, 3.36
42	99.95, 5.04, 104.99	3, 1.5	33.32, 3.36
43	100.54, 5.04, 105.58	3, 1.5	33.51, 3.36
Expected	46.66, 2.63, 49.29	1.6299, 1.5	29.11, 1.75

Impact of Flexible Pricing on Granularity of Bestseller Information Provision

Now we examine how sellers' flexible pricing impacts the granularity of the platform's bestseller information provision. Table S.20 illustrates the total-profit-maximizing type of sales information among the three information types (no information, ranking, and volume) under flexible pricing and fixed pricing, respectively.

Table S.19 Equilibrium outcome in the second period under volume information for different sales realization with fixed prices $r_1 = r_2 = 1.5$ (i.e., the equilibrium prices under no information $t = \phi$) and $n_0 = 60, n_2 = 40, p_h = 0.2$, $u_h = 10$, and $u_l = 6$, where x denotes the first-period sales of the bestseller.

x	profits	sales
22	$40.88, 2.42, 43.30$	$27.25, 1.62$
23	$42.83, 2.40, 45.23$	$28.56, 1.60$
24	$44.83, 2.38, 47.21$	$29.88, 1.59$
⋮		
41	$57.07, 2.34, 59.41$	$38.05, 1.56$
42	$57.13, 2.34, 59.47$	$38.08, 1.56$
43	$57.17, 2.34, 59.51$	$38.12, 1.56$
Expected	$45.24, 2.38, 47.62$	$30.16, 1.59$

Table S.20 Two products' total profits in the two periods as functions of n_0: $\mu = 4.5$, $\sigma = 1.5$, $n = 1000$, $p_h = 0.1$, $\alpha = 0.08$, $u_h = 10$, and $u_l = 6$. The underlined number in either panel corresponds to the optimal granularity and timing of the bestseller information provision.

	Flexible Pricing				Fixed Pricing			
n_0	*Ranking*	*Volume*	*No Info.*	*Opt. Info.*	*Ranking*	*Volume*	*No Info.*	*Opt. Info.*
60	1001.5	1012.4	955.1	Volume	1001.5	1012.2	955.1	Volume
110	1031.8	1048.9	955.1	Volume	1031.8	1028.8	955.1	Ranking
160	1041.6	1061.6	955.1	Volume	**<u>1041.6</u>**	1027.3	955.1	Ranking
210	1041.2	1070.1	955.1	Volume	1041.2	1021.9	955.1	Ranking
260	1038.7	**<u>1073.7</u>**	955.1	Volume	1038.7	1015.1	955.1	Ranking
310	1035.0	1073.7	955.1	Volume	1035.0	1008.0	955.1	Ranking
360	1030.4	1070.9	955.1	Volume	1030.4	1001.1	955.1	Ranking
410	1025.3	1066.1	955.1	Volume	1025.3	994.5	955.1	Ranking
460	1019.9	1062.5	955.1	Volume	1019.9	987.0	955.1	Ranking
510	1014.2	1054.4	955.1	Volume	1014.2	981.8	955.1	Ranking
560	1008.4	1045.6	955.1	Volume	1008.4	977.1	955.1	Ranking
610	1002.5	1036.3	955.1	Volume	1002.5	973.0	955.1	Ranking
660	996.5	1026.5	955.1	Volume	996.5	969.3	955.1	Ranking
710	990.4	1015.6	955.1	Volume	990.4	966.2	955.1	Ranking
760	984.4	1005.9	955.1	Volume	984.4	963.4	955.1	Ranking
810	978.3	995.7	955.1	Volume	978.3	960.8	955.1	Ranking
860	972.2	985.1	955.1	Volume	972.2	958.9	955.1	Ranking
910	966.1	974.4	955.1	Volume	966.1	957.3	955.1	Ranking
960	960.0	963.7	955.1	Volume	960.0	956.0	955.1	Ranking
1000	955.1	955.1	955.1	Tie	955.1	955.1	955.1	Tie

A comparison of the two panels of Table S.20 reveals that, compared to fixed pricing, flexible pricing renders volume information increasingly likely to be the most preferred one by the platform among the three information types: specifically, when n_0 is greater than or equal to 110, the optimal information type switches from ranking to volume as the pricing scenario shifts from "fixed pricing" to "flexible pricing". In particular, it appears from the table that the profit enhancement due to pricing flexibility is more pronounced under volume information than ranking information. This could be attributed to the fact that, under volume information, prices can be adjusted to each possible sales realization of the bestseller product, while pricing according to ranking information is less flexible in the sense that it is based on coarser information.

Impact of Flexible Pricing on Timing of Bestseller Information Provision

Similar to that in the base model, the optimal timing of the platform's bestseller information provision is represented by the value of n_0 (i.e., the number of first-period consumers) which maximizes the total expected two-period profit in equilibrium. We further observe from Table S.20 that, due to sellers' pricing flexibility, the platform optimally postpones its provision of the bestseller information. Specifically, the optimal value for n_0 increases from 160 to 260, as the scenario shifts from fixed pricing to flexible pricing. That is, we observe that flexible pricing tilts the platform's preference over more exploration in the exploration-exploitation tradeoff.

To see why pricing flexibility may prolong the optimal duration of exploration, we perform a detailed analysis of the products' total profit in the two-period horizon by breaking it down into the total profit in the first period and that in the second period, and examine how the profits vary as functions of the value of n_0 and the pricing flexibility (or lack thereof). Table S.21 presents an example, which illustrates the impact of pricing flexibility on the platform's choice of n_0.

Table S.21 Two products' total profits as functions of n_0**:** $\mu = 4.5, \sigma = 1.5, n = 1000, p_h = 0.1, \alpha = 0.08,$ $u_h = 10, u_l = 6$

n_0	First Period	Second Period					
		Flexible Pricing			Fixed Pricing		
		$t=v$	$t=r$	$t=\phi$	$t=v$	$t=r$	$t=\phi$
60	57.3	955.1	944.2	897.8	954.9	944.2	897.8
110	105.1	943.9	926.8	850.0	923.7	926.8	850.0
160	152.8	908.8	888.8	802.3	874.5	888.8	802.3
210	200.6	869.5	840.6	754.5	821.3	840.6	754.5
260	248.3	825.4	790.4	706.8	766.8	790.4	706.8
310	296.1	777.6	738.9	659.0	712.0	738.9	659.0
360	343.8	727.1	686.6	611.3	657.2	686.6	611.3
410	391.6	674.4	633.7	563.5	603.0	633.7	563.5
460	439.3	623.2	580.6	515.7	547.7	580.6	515.7
510	487.1	567.3	527.1	468.0	494.7	527.1	468.0
560	534.8	510.8	473.5	420.0	442.2	473.5	420.0
610	582.6	453.7	419.9	372.5	390.4	419.9	372.5
660	630.4	396.1	366.1	324.7	339.0	366.1	324.7
710	678.1	337.5	312.3	277.0	288.1	312.3	277.0
760	725.9	280.0	258.5	229.2	237.6	258.5	229.2
810	773.6	222.0	204.7	181.5	187.2	204.7	181.5
860	821.3	163.7	150.8	133.7	137.5	150.8	133.7
910	869.1	105.3	97.0	86.0	88.2	97.0	86.0
960	916.9	46.8	43.1	38.2	39.1	43.1	38.2
1000	955.1	0	0	0	0	0	0

Specifically, as n_0 increases, the first-period profit increases as either product's first-period sales is proportional to n_0. In particular, it increases at a constant rate as the per-consumer first-period profit is independent of n_0. On the other hand, for given pricing scenario and information type $t \in \{r, v\}$, how the second-period profit varies in n_0 is determined by the joint effect of two opposing forces, similar to those discussed in the base model. The first force is that, as n_0 increases, sales information becomes more informative, which enables the platform to extract more surplus from each second-period buyer, whose search and purchasing decisions are now better informed. The second force is that an increase in n_0 reduces the number of consumers in the second period, naturally limiting the platform's profit potential from exploiting the information.

As the optimal n_0 maximizes the sum of profits in the two periods, it is such that the increasing rate of the first-period profit equals the decreasing rate of the second-period profit. Due to the sellers' pricing flexibility, the platform is able to better capitalize on the sales information and extract more value from it. As a result, the first force in the second period tends to be strengthened, which may slow down the decrease in the second-period profit (compared to the case with fixed pricing). Thus, the optimal n_0 may increase because of pricing flexibility.

SQ.3. Appendix

Proof of Lemma S.41: Clearly we have $\pi_1^r \geq \nu_1^r$. Then,

$$\begin{aligned}
&\pi_2^{rH} - \pi_2^{rL} \\
=&\frac{p_h G_s(\frac{n_1}{2})}{p_h G_s(\frac{n_1}{2}) + p_l(1-G_a(\frac{n_1}{2}))} - \frac{p_h G_a(\frac{n_1}{2})}{p_h G_a(\frac{n_1}{2}) + p_l \bar{G}_s(\frac{n_1}{2})} \\
=&\frac{p_h^2 \cdot G_a(\frac{n_1}{2})/2 + p_h p_l/4 - p_h^2 \cdot G_a(\frac{n_1}{2})/2 - p_h p_l G_a(\frac{n_1}{2})(1-G_a(\frac{n_1}{2}))}{(p_h G_s(\frac{n_1}{2}) + p_l(1-G_a(\frac{n_1}{2})))(p_h G_a(\frac{n_1}{2}) + p_l \bar{G}_s(\frac{n_1}{2}))} \\
\geq&0
\end{aligned}$$

as $G_a(\frac{n_1}{2}) \leq 1/2$. Also,

$$\begin{aligned}
&\nu_2^{rH} - \nu_2^{rL} \\
=&\frac{p_h G_s(\frac{n_1}{2})}{p_h G_s(\frac{n_1}{2}) + p_l G_a(\frac{n_1}{2})} - \frac{p_h(1-G_a(\frac{n_1}{2}))}{p_h(1-G_a(\frac{n_1}{2})) + p_l \bar{G}_s(\frac{n_1}{2})} \\
=&\frac{p_h^2 \cdot (1-G_a(\frac{n_1}{2}))/2 + p_h p_l/4 - p_h^2 \cdot (1-G_a(\frac{n_1}{2}))/2 - p_h p_l G_a(\frac{n_1}{2})(1-G_a(\frac{n_1}{2}))}{(p_h G_s(\frac{n_1}{2}) + p_l G_a(\frac{n_1}{2}))(p_h(1-G_a(\frac{n_1}{2})) + p_l \bar{G}_s(\frac{n_1}{2}))} \\
\geq&0
\end{aligned}$$

We can rewrite

$$\begin{aligned}
\nu_1^r =&\Pr[u_2 = u_h | X_1 \geq \frac{n_1}{2}] \\
=&\Pr(u_1 = u_h | X_1 \geq \frac{n_1}{2}) \cdot \Pr[u_2 = u_h | u_1 = u_h, X_1 \geq \frac{n_1}{2}] + \Pr(u_1 = u_l | X_1 \geq \frac{n_1}{2}) \cdot \Pr[u_2 = u_h | u_1 = u_l, X_1 \geq \frac{n_1}{2}] \\
=&\Pr(u_1 = u_h | X_1 \geq \frac{n_1}{2}) \cdot \pi_2^{rH} + \Pr(u_1 = u_l | X_1 \geq \frac{n_1}{2}) \cdot \pi_2^{rL}
\end{aligned}$$

As $\pi_2^{rH} \geq \pi_2^{rL}$, it follows that $\pi_2^{rH} \geq \nu_1^r \geq \pi_2^{rL}$.

Similarly, we have

$$\pi_1^r = \Pr[u_1 = u_h | X_1 \geq \frac{n_1}{2}] \tag{S.38}$$

$$= \Pr(u_2 = u_h | X_1 \geq \frac{n_1}{2}) \cdot \Pr[u_1 = u_h | u_2 = u_h, X_1 \geq \frac{n_1}{2}] + \Pr(u_2 = u_l | X_1 \geq \frac{n_1}{2}) \cdot \Pr[u_1 = u_h | u_2 = u_l, X_1 \geq \frac{n_1}{2}] \tag{S.39}$$

$$= \Pr(u_2 = u_h | X_1 \geq \frac{n_1}{2}) \cdot \nu_2^{rH} + \Pr(u_2 = u_l | X_1 \geq \frac{n_1}{2}) \cdot \nu_2^{rL} \tag{S.40}$$

As $\nu_2^{rH} \geq \nu_2^{rL}$, $\nu_2^{rH} \geq \pi_1^r \geq \nu_2^{rL}$.

Finally, we have

$$\begin{aligned}
&\nu_2^{rL} - \pi_2^{rH} \\
=&\frac{p_h(1-G_a(\frac{n_1}{2}))}{p_h(1-G_a(\frac{n_1}{2})) + p_l \bar{G}_s(\frac{n_1}{2})} - \frac{p_h G_s(\frac{n_1}{2})}{p_h G_s(\frac{n_1}{2}) + p_l(1-G_a(\frac{n_1}{2}))}
\end{aligned}$$

$$
\begin{aligned}
&=\frac{p_h^2\cdot(1-G_a(\frac{n_1}{2}))/2+p_hp_l(1-G_a(\frac{n_1}{2}))^2-p_h^2\cdot(1-G_a(\frac{n_1}{2}))/2-p_hp_l/4}{(p_h(1-G_a(\frac{n_1}{2}))+p_l\bar{G}_s(\frac{n_1}{2}))(p_hG_s(\frac{n_1}{2})+p_l(1-G_a(\frac{n_1}{2})))}\\
&=\frac{p_hp_l(1-G_a(\frac{n_1}{2}))^2-p_hp_l/4}{(p_h(1-G_a(\frac{n_1}{2}))+p_l\bar{G}_s(\frac{n_1}{2}))(p_hG_s(\frac{n_1}{2})+p_l(1-G_a(\frac{n_1}{2})))}\\
&\geq 0
\end{aligned}
$$

Hence the ordering of the beliefs is $\nu_2^{rH}\geq\pi_1^r\geq\nu_2^{rL}\geq\pi_2^{rH}\geq\nu_1^r\geq\pi_2^{rL}$. □

Proof of Lemma S.42: From our result (Lemma 3 2) in the base model we know that $\nu_1^v(x)\geq\pi_2^{vL}(x),\forall x\geq n_1/2$. We first show that $\nu_2^{vH}(x)\geq\pi_1^v(x),\forall x\geq n_1/2$, which is

$$\frac{p_hg_s(x)}{p_hg_s(x)+p_lg_a(n_1-x)}\geq\frac{p_h^2g_s(x)+p_hp_lg_a(x)}{p_h^2g_s(x)+p_hp_lg_a(x)+p_hp_lg_a(n_1-x)+p_l^2g_s(x)},$$

which is equivalent to

$$
\begin{aligned}
\frac{p_hg_s(x)}{p_hg_s(x)+p_lg_a(n_1-x)}&=\frac{p_h^2g_s(x)}{p_h^2g_s(x)+p_hp_lg_a(n_1-x)}\\
\geq\frac{p_hp_lg_a(x)}{p_hp_lg_a(x)+p_l^2g_s(x)}&=\frac{p_hg_a(x)}{p_hg_a(x)+p_lg_s(x)}
\end{aligned}
$$

or

$$\frac{p_h}{p_h+p_lg_a(n_1-x)/g_s(x)}\geq\frac{p_h}{p_h+p_lg_s(x)/g_a(x)}.$$

So it suffices to show $g_a(n_1-x)/g_s(x)=g_a(n_1-x)/g_s(n_1-x)\leq g_s(x)/g_a(x)$, which is $g_a(n_1-x)g_a(x)\leq g_s(n_1-x)g_s(x)$, which has been proved in the proof of Lemma 3 (2). So $\nu_2^{vH}(x)\geq\pi_1^v(x),\forall x\geq n_1/2$.

Next, we show that $\pi_1^v(x)\geq\nu_2^{vL}(x),\forall x\geq n_1/2$, which is

$$\frac{p_h^2g_s(x)+p_hp_lg_a(x)}{p_h^2g_s(x)+p_hp_lg_a(x)+p_hp_lg_a(n_1-x)+p_l^2g_s(x)}\geq\frac{p_hg_a(x)}{p_hg_a(x)+p_lg_s(x)}$$

This is equivalent to

$$
\begin{aligned}
\frac{p_h^2g_s(x)}{p_h^2g_s(x)+p_hp_lg_a(n_1-x)}&=\frac{p_hg_s(x)}{p_hg_s(x)+p_lg_a(n_1-x)}\\
\geq\frac{p_hp_lg_a(x)}{p_hp_lg_a(x)+p_l^2g_s(x)}&=\frac{p_hg_a(x)}{p_hg_a(x)+p_lg_s(x)}
\end{aligned}
$$

which is what we have proved above. So $\pi_1^v(x)\geq\nu_2^{vL}(x)$.

Finally, we show that $\pi_2^{vH}(x)\geq\nu_1^v(x),\forall x\geq n_1/2$, which is

$$\frac{p_hg_s(x)}{p_hg_s(x)+p_lg_a(x)}\geq\frac{p_h^2g_s(x)+p_hp_lg_a(n_1-x)}{p_h^2g_s(x)+p_hp_lg_a(x)+p_hp_lg_a(n_1-x)+p_l^2g_s(x)}.$$

This is equivalent to

$$
\begin{aligned}
\frac{p_hg_s(x)}{p_hg_s(x)+p_lg_a(x)}&=\frac{p_h^2g_s(x)}{p_h^2g_s(x)+p_hp_lg_a(x)}\\
\geq\frac{p_hp_lg_a(n_1-x)}{p_hp_lg_a(n_1-x)+p_l^2g_s(x)}&=\frac{p_hg_a(n_1-x)}{p_hg_a(n_1-x)+p_lg_s(x)},
\end{aligned}
$$

which is equivalent to $\frac{p_h}{p_h+p_lg_a(x)/g_s(x)}\geq\frac{p_h}{p_h+p_lg_s(x)/g_a(n_1-x)}$, or $g_a(x)/g_s(x)\leq g_s(x)/g_a(n_1-x)=g_s(n_1-x)/g_a(n_1-x)$. This is equivalent to $g_a(x)g_a(n_1-x)\leq g_s(x)g_s(n_1-x)$, which we know is true from above. Therefore, $\pi_2^{vH}(x)\geq\nu_1^v(x)$ and the lemma has been proved. □

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