

Joint Statistics Seminar

The Hong Kong University of Science and Technology

Sudden Death Versus Slow Extinction of Branching Processes in Random Environment

by

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Time: 4:00 p.m. – 5:00 p.m.

Venue: Room 3416 (near Lift 17/18)

Abstract

A branching process $Z(n), n = 0, 1, \dots$, is considered which evolves in a random environment generated by a sequence of iid generating functions $f_0(s), f_1(s), \dots$. Let $T = \min\{k : Z(k-1) > 0, Z(k) = 0\}$ be the extinction moment of the process, $S_0 = 0, S_k = \log f'_0(1) + \dots + \log f'_{k-1}(1), k \geq 1$, be the associated random walk and $\tau(m, n)$ be the left-most point of minimum of $\{S_k, k \geq 0\}$ on the interval $[m, n]$.

Assuming that the distribution of $\log f'_0(1)$ belongs to the domain of attraction of a stable law with parameter $\alpha \in (0, 2]$ and zero mean if $\alpha > 1$, we show that (under the quenched approach and given $T = n \rightarrow \infty$) the process approaches the extinction moment by passing through a number of bottlenecks, when the size of the population becomes very small, and periods of recovering, when the size of the population is of order $e^{S_m - S_{\tau(0, m)}}$ if $m < \tau(0, n)$ and of order $e^{S_m - S_{\tau(m, n)}}$ if $m \geq \tau(0, n)$. In particular, the size Z_{n-1} of the population is "not too big" and, therefore, the process dies out in a "natural" way. Similar phenomenon takes place under the annealed approach if $\text{Var} \log f'_0(1) < \infty$. If, however, $\text{Var} \log f'_0(1) = \infty$ then (under the annealed approach) $\log Z_{n-1}$ is very big, namely, of order $n^{1/\alpha}$. In this case the extinction of the process at moment n may be treated as it's sudden death under very unfavorable environment.

Jointly with Elena Dyakonova (Russia), Andreas Kyprianou (UK), and Vitaly Wachtel (Germany)

❖ *All interested are welcome!* ❖

For details, please contact ISOM Department.