
Joint Statistics Seminar

The Hong Kong University of Science and Technology

Hoeffding Decompositions and Urn Sequences

by

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Time: 11:00 a.m. - 12:00 noon

Venue: Room 3501 (Lift 25/26)

Abstract

Let $(X_n)_{n \geq 1}$ be a sequence of random variables. We say that the sequence is Hoeffding-decomposable if, for all $n \geq 2$, any symmetric statistic $T(X_1, \dots, X_n)$ admits a unique representation as a sum of $(n + 1)$ uncorrelated U-Statistics with completely degenerated kernels of increasing orders. Introduced for the first time in a seminal paper by W. Hoeffding in 1948, the so called Hoeffding decompositions grew to become one of the fundamental techniques for proving asymptotic distributional results. This has definitely been the case for i.i.d. sequences; as for dependent sequences, only extractions without replacement from finite populations (see Bloznelis and Gotze [2001; 2002]) had been investigated before the theory was generalized to arbitrary exchangeable sequences in Peccati [2004] : in fact, in the latter reference, a characterization of Hoeffding-decomposable sequences in terms of a technical condition the author named weak independence is obtained. In El-Dakkak and Peccati [2008] ; we seek a more transparent characterization of Hoeffding decomposable exchangeable sequences and focus on sequences taking values in a finite set $*D*$: in the case in which $*D* = \{0,1\}$, we show that the only Hoeffding decomposable exchangeable sequences are i.i.d. sequences and two-colour Polya sequences. When $*D*$ is an arbitrary finite set, we obtain a partial generalization of the two-colour case. The full generalization is obtained in El-Dakkak, Peccati and Prunster [2010] : more precisely, we show that multicolour Polya sequences are the only $*D*$ -valued purely exchangeable sequences that are Hoeffding decomposable.

References:

- Bloznelis, M. et Gotze F. [2001] : Orthogonal decompositions of finite population statistics and its applications to distributional asymptotics. *The Annals of Statistics* 29 (3), 353-365.
- Bloznelis, M. et Gotze F. [2002] : An Edgeworth expansion for finite population statistics. *The Annals of Probability* 30, 1238-1265.
- El-Dakkak, O. et Peccati, G. [2008] : Hoeffding decompositions and urn sequences. *The Annals of Probability*, 36 (6), 2280-2310.
- El-Dakkak, O., Peccati, G. et Pruenster, I. [2010]: A Hoeffding-type characterization of multicolour Polya sequences. Manuscript.
- Hoeffding, W. [1948] : A class of statistics with asymptotically normal distributions. *The Annals of Mathematical Statistics* 19, 293-325.
- Peccati, G. [2004]: Hoeffding-ANOVA decompositions for symmetric statistics of exchangeable observations. *The Annals of Probability* 32 (3A), 1796-1829.

❖ *All interested are welcome!* ❖
For details, please contact ISOM Department.