

Product Launches and Buying Frenzies: A Dynamic Perspective

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Buying frenzies caused by a firm's intentional undersupplying of a new product are frequently evident in several industries including electronics (cell phones, video games), luxury automobiles, and fashion goods. We develop a dynamic model of buying frenzies that incorporates the firm's manufacturing and sale of a product over time and characterizes the conditions under which inducing such frenzies is an optimal strategy. We find that buying frenzies occur when customers are sufficiently uncertain about their valuations of the product and when they discount the future to a certain extent. We propose measures of "customer desperation" and of the extent of scarcity to measure the depth and breadth of buying frenzies, respectively. We also demonstrate that such frenzies can have a significantly positive effect on firm profits and partially recover the loss due to non-commitment to future prices. This paper provides managerial insights on how firms can influence market response to a new product through production, pricing, and inventory decisions to induce profitable frenzies.

Key words: advance selling; buying frenzy; customer desperation; strategic customer behavior

1. Introduction

Long queues of enthusiastic customers were common when Apple's iPad 2 hit the Hong Kong markets on April 29, 2010. Given the inclement weather, Apple stores were glad to provide umbrellas and raincoats (bearing Apple's logo) to waiting customers. Each store received a limited number of iPads and distributed them on a first-come, first-served basis. Some customers were willing to pay considerably higher prices to obtain the product in the "gray" market. Bitter complaints about long queues and active scalping led Apple to require that potential customers participate in a daily lottery—and present e-mail confirmation of winning that lottery (along with an ID) to the store—to purchase no more than two iPads (*AppleInsider* 2012). Apple used this procedure also when launching the iPhone 5. Such shortages are not confined to Apple products. The shortage of Nintendo's game console Wii, for example, lasted from its initial introduction in 2006 until 2009

(Liu and Schiraldi 2014). Sony’s PlayStation 2 faced the same problems when entering the US market eight months after entering Japan’s markets (Stock and Balachander 2005).

A *buying frenzy* happens when a firm intentionally undersupplies a market and leaves the rationed customers strictly worse-off. This practice is common in markets for such diverse products as luxury cars, fashion, and especially electronics (cell phones, video games, game consoles). Shortages could be attributed to demand forecasting errors, issues in component supplies, or production problems; however, their repeated occurrence—particularly during the launch phase of innovative products—suggests that the true explanation involves a deliberate strategy. The few studies to investigate this issue have considered mainly static models. These models capture situations where firms sell the good only once; examples include tickets for sporting or music events, limited edition products, and one-off auctions. Yet the predictions of static models collapse when the firm produces repeatedly over time, as is the case for most manufactured products. The reason is the firm’s desire to serve also the customers who were excluded from the early sales. In that case, why should customers be desperate to buy early when the product will still be available later? We propose a dynamic model that explains an initial buying frenzy followed by a period of sales without frenzies.

In this paper, we develop a model in which production and sales occur in two periods and characterize the dynamics of sales, prices, and scarcity. The two-period model is a stylized representation that features the dichotomy between the launch phase of a new product and its subsequent mature phase. We show that the firm’s gains from inducing a buying frenzy (relative to matching supply and demand) can be economically substantial, and we investigate the conditions under which buying frenzies are optimal. Finally, we compute customers’ loss from being excluded during the initial launch phase, which is a proxy for “customer desperation”, and show that this loss can be significant. This explains why customers may invest resources to obtain the good early (e.g., wait in queues) and why prices can be significantly higher in resale markets.

There are several ingredients to our analysis. We assume that customers are initially uncertain about their preferences for the product (see e.g. Xie and Shugan 2001, Gallego and Şahin 2010, Yu et al. 2011). This assumption applies to the innovative products that have been the object of buying frenzies. Following DeGraba (1995) and Denicolò and Garella (1999), we assume that the firm cannot commit to future prices and quantities. This assumption implies that the firm discounts prices when inventories build up and is consistent with the response of car and electronics manufacturers when sales of their products are slower than expected. For example, Hewlett-Packard had to slash prices to clear unsold inventory of the TouchPad only two months after its launch (*Wall Street Journal* 2011). Finally, we use Pareto dominance as the selection criterion among the game’s equilibria (Cachon and Netessine 2004). In other words, given the firm policy, the equilibrium selected is the one that gives customers the highest payoff.

In a dynamic model, how does one account for uninformed customers rushing to buy early? Customers can always obtain the good if they wait. They benefit from waiting as well because they can then incorporate the information learned (about their preferences) before making a purchase decision. If the firm produces a large quantity in the first period, then customers anticipate that they can obtain the product at a lower price in the second period if they all wait. Thus, for a product to sell in the first period, its price must be sufficiently low to prevent such strategic waiting. This determines the maximum price the firm can charge in the first period. At that price, the equilibrium where all customers intend to buy early yields the same expected utility as the equilibrium where all customers wait. If customers do not value the waiting option, that is, they discount future utility significantly, the firm does not have to discount much the first period price and sells to all customers in the first period. If, on other hand, customers are willing to wait to make informed decisions, the firm has to discount the first period price so much that an individual customer is indifferent between buying and waiting. In this case, the firm rations the early demand but those rationed are not worse off. Finally, if customers are sufficiently, but not excessively, patient, the firm's optimal policy is to induce a frenzy. In a frenzy, an individual customer who is rationed and has to wait is strictly worse-off because the price the firm charges in the second period is higher than the price the customer could have obtained had all customers waited. This is why customers strictly prefer to obtain the good early.

If customers are relatively informed about the product and do not value the waiting option, the firm does not have any incentives to ration demand. Buying frenzies are more likely to occur when customers are initially uncertain about their preferences for the product and benefit from waiting to learn their preferences. Such uncertainty is more likely with respect to an innovative product that will match the needs of some customers but not others which may explain why frenzies are common for new electronic products (e.g., the iPhone) than for similar, "me too" products (e.g., Android phones) that are released later. It is therefore reasonable to assume that customers have more uncertainty about new products than about knockoffs produced later.

There are two main theories of buying frenzies. The first is based, as in our model, on intertemporal price discrimination. Denicolò and Garella (1999) show that a monopolist may ration customers, with known heterogenous preferences, to prevent strategic waiting; DeGraba (1995) considers a static model with individual preference uncertainty.¹ In both of these models, unlike ours, buying frenzies occur only with specific rationing rules. Moreover, our analysis delivers a tractable model that describes how preference uncertainty affects the existence of buying frenzies and formally measures customer desperation.

¹ What we mean by static is that, on the equilibrium path, there is no production and sales in the second period.

Other research on intertemporal price discrimination and scarcity policies that is closely related to ours includes Liu and van Ryzin (2008), Cachon and Swinney (2009), and Liu and Schiraldi (2014). However, those papers address different issues than we do. In particular, Liu and van Ryzin show that the possibility of a stock-out in the second period can prevent customers with known preferences from waiting. Cachon and Swinney suggest that, by limiting the initial stocking level and offering optimal markdowns, the firm may be able to constrain the strategic purchase behavior of its customers. Liu and Schiraldi show that the existence of resale markets can induce the firm to understock a product and thereby increase its equilibrium price.

The second theory of buying frenzies is based on asymmetric information. Stock and Balachander (2005) argue that a high-quality firm employs scarcity to signal quality to uninformed customers. Rationing as a signal of quality has also been studied in Debo and van Ryzin (2009) and in Allen and Faulhaber (1991), among others. Papanastasiou et al. (2014) develop a model of “boundedly rational social learning” and show that a firm may restrict a product’s availability to elicit favorable reviews from early adopters, which have a positive effect on the preferences of other customers. Our model is not based on information asymmetry and does not rely on irrationalities to explain frenzies. Neither do we assume that customers are myopic or that the firm must charge a fixed price over time.

Other models of buying frenzies are based on demand externality (Becker 1991) and psychological drives (Verhallen and Robben 1994). Becker’s model accounts for social interactions and shows that an upward-sloping demand curve might result in an unstable equilibrium with arbitrarily small frenzies. Verhallen and Robben argue that scarcity itself can increase customers’ willingness to pay—provided they attribute that scarcity to demand-side variables (such as popularity) and not to supply-side variables (such as the firm intentionally limiting supply). Finally, buying frenzies are distinct from herding (Debo and Veeraraghavan 2009), in which customers ignore their private noisy information about a product and merely follow what previous customers did. In contrast, imperfect information is absent from our model of frenzies.

There is little empirical work that addresses buying frenzies. An important exception is Balachander et al. (2009), who present a thorough analysis of buying frenzies in automobile markets and show that their findings are not consistent with DeGraba’s (1995) static model of buying frenzies. We however argue that their results are consistent with a dynamic model. This difference demonstrates the importance of distinguishing between static and dynamic models of buying frenzies.

The rest of the paper is organized as follows. Section 2 outlines the general model. Section 3 analyzes the dynamic model when the firm can produce over time and derives the conditions under which frenzies are optimal; it also defines measures of customer desperation and the extent of a

buying frenzy. Section 4 illustrates the conceptual framework and the most important results by a simple numerical example. Section 5 extends the model to include second-period arrivals. Section 6 concludes the paper.

2. A Model of Buying Frenzies

A monopolist sells to a population of N_1 customers who arrive in the first period of a two-period horizon. In period 1, customers are uncertain about their valuations and decide whether to buy or wait until period 2 where they learn their valuations. The uncertain customer valuation in period 1 has density $f_1(v)$ and survival function $\bar{F}_1(v)$. We assume that f_1 and F_1 are continuous with support $[\underline{v}, \bar{v}] \subset [0, \infty)$ and that $E[\mathbf{v}] = \mu$. We also assume that $f_1(x) > 0$ is log-concave.² This demand specification approximates a large market in which customers are infinitesimal³ and have idiosyncratic preferences that are discovered over time (cf. Lewis and Sappington 1994, Yu et al. 2014). The two-period horizon is a stylized representation of a product launch phase followed by a mature phase of sales. It is therefore unnecessary for the two periods to have equal length; the second period can be arbitrarily longer than the first. What matters is that individual customer uncertainty is resolved by the end of the first period. Customers discount future utility by δ^c , and the monopolist discounts future profits by δ^m . Although we do not restrict the parameter space for (δ^c, δ^m) , it is reasonable to assume that customers have a lower discount factor than the firm (Liu and van Ryzin 2008).

The monopolist can produce in both periods, so we use q_1 and q_2 to denote the production quantities in (respectively) the first and second periods. To simplify the exposition, we assume that the marginal cost of production is zero. The firm announces (q_1, p_1) at the start of the first period. Customers are strategic and form expectations about what will happen in period 2; they buy or wait depending on which option maximizes their discounted utility given those expectations. We assume that all inventory available in the second period (period-2 production plus leftover inventory from period 1) is sold at the market-clearing price.

The market-clearing assumption may seem restrictive because one could argue that the firm should set the static profit-maximizing price in period 2. However, that alternative assumption is not time consistent because customers will not buy at the static profit-maximizing price in period 2 if they expect prices to drop later (Coase 1972). In the absence of commitment, the firm will eventually sell all its inventory and the price of the last unit sold will be the market clearing price. As discussed in the Introduction, the sellout assumption is consistent with the literature

² This technical assumption is typically made to ensure the equilibrium is unique. However, our results do not depend on it. Widely applied parametric families—including the uniform, exponential, normal, and logistic—have log-concave density (Bagnoli and Bergstrom 2005).

³ This means that whether a customer decides to purchase or to wait does not affect the payoff of other customers.

(DeGraba 1995, Denicolò and Garella 1999) and with practices observed for manufactured products. This simplifying assumption captures the firm’s distress when inventories pile up and avoids the complications of modeling the Coase price dynamics after period 2. Our key assumption is the lack of commitment, which is realistic given that product launches are rare and isolated in time, and prevents the firm from developing a reputation for withholding or destroying excess inventory.⁴ The market-clearing assumption implies that the firm cannot commit to a pure advance selling strategy.

We assume throughout that the game structure is common knowledge, and solve for the symmetric rational expectations equilibria (REE).⁵ In our model this means that, given (q_1, p_1) and their expectations of period-2 price and availability, customers decide whether to buy early or to wait. The firm then sets the price and production volumes given its expectations of customers’ willingness to pay. Moreover, the expectations of customers and the firm are consistent with the actual outcomes.

Denote a customer’s decision to buy or wait in an REE by the probability $x \in [0, 1]$. A customer waits if $x = 0$ or buys if $x = 1$; her strategy is mixed if $x \in (0, 1)$. We focus on symmetric equilibria—that is, we assume x is the same for all infinitesimal customers. The price that a customer expects to face in period 2 depends on the fraction of customers who have attempted to buy, x , and on the firm’s initial production quantity, q_1 . We use $p_2^e(x|q_1)$ to denote the expected period-2 price. The expected period-2 surplus is then

$$\delta^c E[\mathbf{v} - p_2^e(x|q_1)]^+ \triangleq \delta^c T_1(p_2^e(x|q_1)).$$

Definition 1 *Assume the firm announces (q_1, p_1) in period 1. (a) $x = 0$ is a pure strategy REE if and only if $\mu - p_1 \leq \delta^c T_1(p_2^e(0|q_1))$. (b) $x = 1$ is a pure strategy REE if and only if $\mu - p_1 \geq \delta^c T_1(p_2^e(1|q_1))$. (c) $x \in (0, 1)$ is a mixed strategy REE if and only if $\mu - p_1 = \delta^c T_1(p_2^e(x|q_1))$.*

We select the Pareto-dominant equilibrium if there are multiple equilibria. If there are multiple Pareto-dominant equilibria—that is, when customers’ expected surplus is the same—then we select the equilibrium that maximizes the firm’s profits (customers would strictly prefer the buying equilibrium if the firm lowers the period-1 price by an arbitrarily small amount). This procedure is consistent with other works in the literature; see Cachon and Netessine (2004) and the references therein.

⁴ In contrast, firms selling perishable products (e.g., in the airline or hotel industry) sometimes commit to policies that do not necessarily discount excess inventories when sales are low; see Liu and van Ryzin (2008) for an extended discussion of this assumption.

⁵ The concept of a rational expectations equilibrium is commonly applied in operations management; see, for example, Liu and van Ryzin (2008), Su and Zhang (2008; this work includes a detailed description), and Cachon and Swinney (2009).

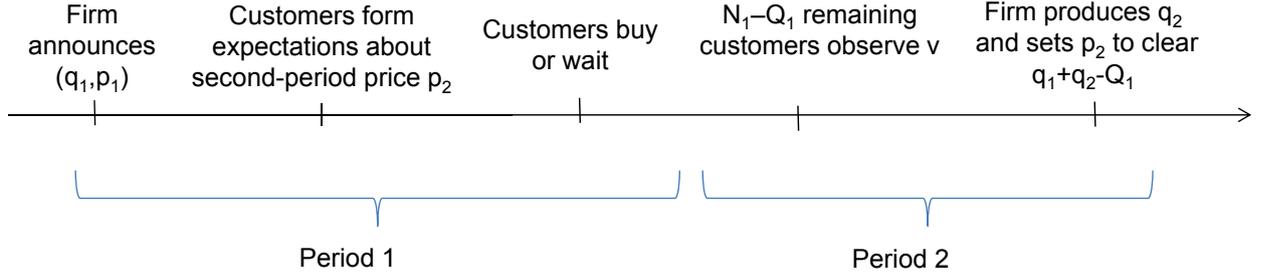


Figure 1 Sequence of events.

At the beginning of period 2, the firm observes sales in period 1, $Q_1 = \min(xN_1, q_1)$, and sets q_2 . The second-period price p_2 is such that the second-period supply $q_2 + q_1 - \min(xN_1, q_1)$ equals second-period demand. The rational expectations assumption implies that $p_2 = p_2^e(x|q_1)$. In period 2, customers buy if their valuation is greater than the market-clearing price. Figure 1 shows the sequence of events.

We let $p_2^m = \operatorname{argmax}_p p \bar{F}_1(p)$ be the unconstrained period-2 profit-maximizing price. Further, define $\bar{\pi} \triangleq p_2^m \bar{F}_1(p_2^m)$ as the maximum profit per customer obtained by selling only in period 2. We assume that $p_2^m > \underline{v}$. Without this assumption, the firm would serve the entire market in period 2 and frenzies would never happen.

3. Model Analysis

Profit maximization in period 2 implies that

$$p_2^e(x|q_1) = \operatorname{argmax}_p p \bar{F}_1(p) \quad (1)$$

$$\text{s.t.} \quad (N_1 - \min(xN_1, q_1)) \bar{F}_1(p) \geq q_1 - \min(xN_1, q_1),$$

where the constraint is the market-clearing assumption. This price is unique for a given announcement (q_1, p_1) because f_1 is log-concave. Our first result establishes the properties of $p_2^e(x|q_1)$.

Lemma 1 *Assume that $x \in [0, 1]$ is part of an REE. (a) If $q_1 \leq N_1 \bar{F}_1(p_2^m)$ then $p_2^e(x|q_1) = p_2^m$ for all $x \in [0, 1]$. (b) If $q_1 > N_1 \bar{F}_1(p_2^m)$ then $p_2^e(x|q_1) = \bar{F}_1^{-1}\left(\frac{q_1 - xN_1}{(1-x)N_1}\right)$ for $x \in \left[0, \frac{q_1 - N_1 \bar{F}_1(p_2^m)}{N_1 \bar{F}_1(p_2^m)}\right]$ and is strictly increasing in x , and $p_2^e(x|q_1) = p_2^m$ for $x \in \left[\frac{q_1 - N_1 \bar{F}_1(p_2^m)}{N_1 \bar{F}_1(p_2^m)}, 1\right]$ and is constant in x . (c) $p_2^e(x|q_1)$ is decreasing in q_1 .*

Lemma 1 shows that the expected period-2 price is decreasing in q_1 and increasing in x . In other words, the more the firm produces in period 1, the lower the price it can charge in period 2. This result follows from the market-clearing assumption and is essential for our analysis. We emphasize that what motivates the market-clearing assumption and drives Lemma 1 is the fact that the firm cannot commit to future prices. In a multi-period model, non-commitment would imply that the

optimal price decreases over time and the last unit is sold at the market clearing price (Coase 1972). Indeed, in such a model the expected period-2 price will not be the market-clearing price. However, as long as $q_1 > N_1 \bar{F}_1(p_2^m)$, that price would still be decreasing in q_1 and increasing in x which is what is essential for the analysis. For simplicity, we consider a two-period model and impose market clearing in period 2.

Define $p_1^b(q_1) \triangleq \mu - \delta^c T_1(p_2^e(1|q_1))$ and $p_1^w(q_1) \triangleq \mu - \delta^c T_1(p_2^e(0|q_1))$.⁶ Buying, $x = 1$, is an REE for any price $p_1 \leq p_1^b(q_1)$; waiting, $x = 0$, is an REE for any price $p_1 \geq p_1^w(q_1)$. Because $T_1(p)$ is decreasing in p , Lemma 1 implies that $p_1^w(q_1) \leq p_1^b(q_1)$. This reasoning establishes that an REE exists for any announcement (q_1, p_1) . Our next result shows that, whenever $p_2^e(x|q_1)$ is strictly increasing at $x = 0$, mixed strategy equilibria are not Pareto dominant.

Lemma 2 *Any $x \in (0, 1)$ such that $p_2^e(x|q_1) > p_2^e(0|q_1)$ cannot be part of a Pareto-dominant REE.*

To see why, assume that $x \in (0, 1)$ is a mixed strategy, Pareto-dominant REE for a firm announcement (q_1, p_1) . Then customers are indifferent between buying and waiting; that is, $\mu - p_1 = \delta^c T_1(p_2^e(x|q_1))$. Since $\mu - p_1 = \delta^c T_1(p_2^e(x|q_1)) < \delta^c T_1(p_2^e(0|q_1))$, it follows that the pure strategy waiting REE ($x = 0$) exists and yields greater expected surplus than x —a contradiction.

For any announcement (q_1, p_1) , we show in Appendix that any mixed strategy REE is either strictly Pareto dominated by the waiting pure strategy equilibrium $x = 0$ (by Lemma 2) or Pareto equivalent to the buying pure strategy REE $x = 1$, which is weakly preferred by the firm. We prove the latter by comparing firm profits in the mixed strategy REE x and the REE $x = 1$. Subsequently, we focus only on the set of pure strategy REE that could be $x = 0$, $x = 1$, or both. We then fix q_1 and solve for the price p_1 that maximizes firm profits. That is, we consider all possible prices p_1 , compute the profits in all Pareto-dominant REE associated with (q_1, p_1) , and select the price associated with the highest profits. Denote that price $p_1(q_1)$.

Proposition 1 *Assume that the period-1 production is q_1 . Then the maximum price the firm can charge in period-1 is $p_1(q_1) = p_1^w(q_1)$ and the associated Pareto-dominant REE is $x = 1$.*

Proposition 1 implies that the firm cannot produce $q_1 > N_1 \bar{F}_1(p_2^m)$ and charge $p_1^b(q_1)$ because then waiting would be the Pareto-dominant equilibrium. To sell q_1 , the firm can charge at most $p_1^w(q_1) < p_1^b(q_1)$. At this price, the two pure REEs (buying and waiting) are Pareto equivalent and one that maximizes the firm's profits is selected. This establishes a key ingredient to our model: a customer's expected utility from obtaining the good in period 1 ($\mu - p_1(q_1) = \delta^c T_1(p_2^e(0|q_1))$) and from having to wait until period 2 ($\delta^c T_1(p_2^e(1|q_1))$) may differ. When $q_1 > N_1 \bar{F}_1(p_2^m)$, buying early

⁶ 'b' and 'w' stand for buying and waiting, respectively.

strictly dominates waiting because Lemma 1 implies that in this case $p_1^w(q_1) < p_1^b(q_1)$ which in turn implies $\mu - p_1(q_1) > \delta^c T_1(p_2^e(1|q_1))$. However, there will not be enough supply for all customers if $q_1 < 1$. The rationed customers will be strictly worse off. We say that a buying frenzy occurs when this happens.

The customers' expected loss from being "rationed out" is

$$L(q_1) \triangleq \mu - p_1(q_1) - \delta^c T_1(p_2^e(1|q_1)) = p_1^b(q_1) - p_1^w(q_1). \quad (2)$$

It is instructive to distinguish two scarcity strategies. A buying frenzy takes place when there is excess demand (i.e., $N_1 > q_1$) and customers are strictly worse off being rationed out (i.e., $L(q_1) > 0$). If, on the other hand, $N_1 > q_1$ and $L(q_1) = 0$, we say the firm employs a 'rationing policy'. Here, $L(q_1)$ is a measure of the *intensity* of the frenzy or its *depth*—in other words of customer desperation. In our model, $L(q_1)$ quantifies the amount a customer is willing to invest to secure the good early. Conditional on this measure being positive, a measure of the *extent* or *breadth* of the buying frenzy is $N_1 - q_1$.

By Proposition 1, the period-1 price must be less than customers' expected valuation, μ , which is the profit per customer when the firm can credibly commit *not* to produce and sell in period 2. Therefore, $\delta^c T_1(p_2^e(0|q_1))$ is a measure of the firm's cost associated with not being able to commit. Proposition 1 also shows that the firm's maximization problem is well-defined. In particular, the firm chooses q_1 to maximize

$$\pi(q_1) = q_1 p_1(q_1) + \delta^m (N_1 - q_1) p_2^m \bar{F}_1(p_2^m). \quad (3)$$

Equation (3) covers the case where the firm sells only in period 2, which occurs when $q_1 = 0$ and customers have to wait. Further, note that on the equilibrium path the firm charges the static profit-maximizing price in period-2, p_2^m . The market clearing assumption determines what happens off the equilibrium path and hence the maximum price the firm can charge in period-1.

We next characterize the conditions under which a frenzy occurs.

Proposition 2 *The firm's optimal policy induces a unique buying frenzy if $\delta_2^c < \delta^c < \delta_1^c$, where $\delta_1^c = \frac{\mu - \delta^m \bar{\pi}}{T_1(p_2^m) + \frac{[\bar{F}_1(p_2^m)]^2}{f_1(p_2^m)}}$ and $\delta_2^c = \frac{\mu - \delta^m \bar{\pi}}{\mu - \underline{v} + \frac{1}{f_1(\underline{v})}}$.*

Proposition 2 distinguishes three cases. (1) When customers are impatient and excessively discount future surplus ($\delta^c \leq \delta_2^c$), the firm is better off serving the entire market early and rationing is not profitable. In other words, rationing myopic customers who do not value learning their preferences over time is not an optimal policy. (2) When customers are patient and willing to wait to make informed decisions ($\delta^c \geq \delta_1^c$), the best the firm could do is to make them indifferent between buying

and waiting. The firm does not have an incentive to serve the entire market early, i.e., set $q_1 = N_1$, because strategic customers would choose to wait and buy at a low market-clearing price. The firm then rations the supply in period 1 ($q_1^* < N_1$) and sets the early price to make customers indifferent to buy or wait. In the second period, the firm charges the maximum price p_2^m which leads to production and sale of $q_2^* = (N_1 - q_1^*)\bar{F}_1(p_2^m)$. In other words, unlike case (1), the firm does not serve all the customers over two periods. (3) When customers are sufficiently but not excessively patient ($\delta_2^c < \delta^c < \delta_1^c$), the firm's optimal policy is to induce a buying frenzy in the period-1 market ($N_1\bar{F}_1(p_2^m) < q_1 < N_1$). In the frenzy equilibrium, customers are indifferent between buying early and waiting *as a group*. That is: an individual customer's expected surplus from buying early when everyone else intends to buy early, $\mu - p_1(q_1)$, is equal to that from waiting when everyone else waits $\delta^c T_1(p_2^e(0|q_1))$. However, an individual customer who is rationed out and must therefore wait for period 2 is strictly worse-off ($L(q_1) > 0$). This is because her expected surplus upon waiting is $\delta^c T_1(p_2^e(1|q_1)) = \delta^c T_1(p_2^m) < \mu - p_1(q_1) = \delta^c T_1(p_2^e(0|q_1))$. This explains why customers are willing to invest resources (e.g., wait in queues) to obtain the good early.

To demonstrate the relevance of our dynamic framework, we revisit the finding by Balachander et al. (2009) that the introductory price of a new car is positively correlated with its scarcity.⁷ The concept of scarcity corresponds to $N_1 - q_1$ in our framework. From Proposition 1 it follows that the introductory price is decreasing in q_1 ; yet the extent of a frenzy, $N_1 - q_1$, is also decreasing in q_1 . Therefore, a decrease in q_1 (as might be caused by a shift in one of the model's primitives) will increase both the extent of a buying frenzy and the product's introductory price.

Our next result shows how the optimal production and profit change as a function of the customers' and firm's discount factors. The corollary will be useful in the rest of this section.

Corollary 1 (a) *The firm's optimal profit is increasing in δ^m and decreasing in δ^c . (b) The optimal period-1 production q_1^* is decreasing in both δ^c and δ^m .*

Figure 2 shows the optimal policy for uniform valuation distributions with mean μ and standard deviation σ indexed by their coefficient of variation (C.V.), defined as σ/μ . The optimal policy does not induce a buying frenzy if $\sigma/\mu \leq \sqrt{3}/9 \simeq 0.20$. A low C.V. implies that the market consists of customers with relatively known and homogenous preferences. The best the firm could do in period 2 then is to charge $p_2^m = \underline{v}$ at which any waiting customer would buy; see Proposition 2. As a result, the maximum price the firm can charge in period 1 is $p_1(q_1) = \mu - \delta^c T_1(p_2^e(0|q_1)) = \mu - \delta^c(\mu - \underline{v}) \geq \underline{v}$ which implies the optimal policy is to set $q_1^* = N_1$ (i.e., to serve the entire market early). So a buying frenzy can occur only when customers are sufficiently uncertain about their valuations, which is typically the case for new or innovative products.

⁷ Hypothesis 3B, p. 1627

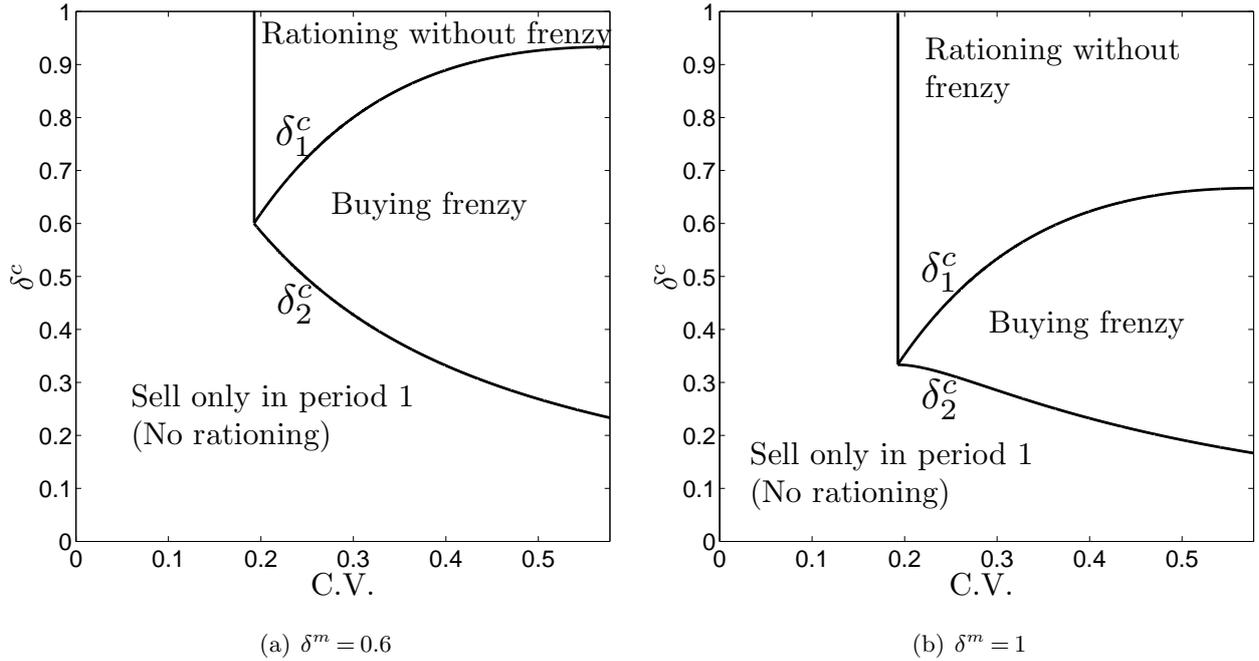


Figure 2 The firm’s optimal policy when the firm can produce in both periods and when customers have valuations that are uniform and with coefficient of variation not exceeding $\sqrt{3}/3$ (to ensure that the support of v is nonnegative).

As δ^m increases, the range of customer discount factor that supports a buying frenzy shrinks.⁸ In particular, it is never optimal to induce a buying frenzy if $\delta^c = \delta^m = 1$. Our model distinguishes between rationing and frenzy policies; although for $\delta^c > \delta_1^c$ it is not optimal to induce a frenzy, the firm does ration customers (i.e., $q_1^* < N_1$). Customers are then indifferent between buying early and waiting, and customer desperation as defined in (2) is zero.

Figure 3(a) plots customer desperation, normalized as $L_1(q_1^*)/(\delta_c T_1(p_2^m))$, and Figure 3(b) plots the extent of a buying frenzy under the optimal policy. Customer desperation (i.e., the frenzy’s intensity) can be economically significant. The loss from not obtaining the good early is 20% of the expected value of consumption when, for example, $\delta^m = 1$ and $\delta^c = 0.5$. For a given value of δ^m , customers’ relative desperation decreases as they become more patient (Figure 3(a)). In other words, patient customers are less likely to invest in acquiring the product early. The firm then increases the breadth of the frenzy by producing less (Figure 3(b)) because q_1^* is decreasing in δ^c by Corollary 1. Moreover, for a given δ^c , customer desperation is also decreasing in δ^m whereas the extent of the frenzy increases because q_1^* is decreasing in δ^m (again by Corollary 1). The intuition is that a patient firm produces less but charges a higher price in period 1 and so customers’ expected surplus from an early purchase decreases making them less desperate to buy early. The firm however serves a larger market in period 2 as more customers have to wait.

⁸ This is because $\delta_1^c - \delta_2^c$ is decreasing in δ^m (note that $T_1(p) + [\bar{F}_1(p)]^2/f_1(p)$ is decreasing in p and that $p_2^m > \underline{v}$).

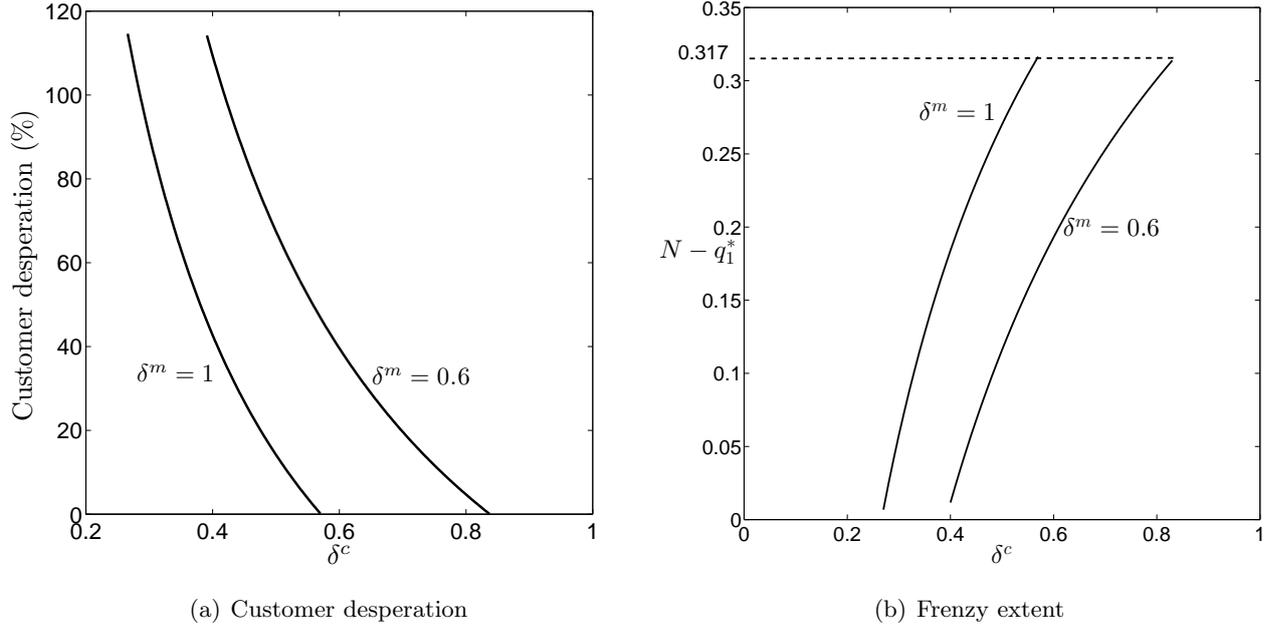


Figure 3 Customer desperation $L(q_1^*)$, normalized as $\frac{L(q_1^*)}{\delta^c T_1(p_2^m)} \times 100$, and the extent of a buying frenzy $N_1 - q_1^*$, when customers' valuation is uniform with mean $\mu = 12$ and standard deviation $\sigma = 4$. In this figure, $N_1 = 1$. For $\delta^c \geq \delta_1^c$ and $\delta^c \leq \delta_2^c$ we have $L(q_1^*) = 0$.

Figure 4 compares the firm's optimal profits with two benchmark cases. One case corresponds to the firm's profit under period-2 sales, $N_1 \bar{F}_1(p_2^m) p_2^m$; the other case corresponds to the commitment profit—that is, charging μ in period 1 to N_1 customers (and committing, credibly, not to sell in period 2). Figure 4(b) shows that, when $\sigma/\mu = 0.5$, the loss due to noncommitment is almost 100% whereas inducing a frenzy recovers from 36.2% to 66.7% of the profits (the more patient customers are the lower the percentage recovered). The equivalent profits recovered when $\sigma/\mu = 0.25$ range from 55.2% to 66.9%; thus the lower the coefficient of variation, the higher the percentage recovered. Furthermore, the percentage of profits recovered by inducing a frenzy decreases with increasing firm impatience (lower δ^m).

4. Illustration

In this section, we illustrate the conceptual framework in Section 2 and the key results in Section 3 by a simple numerical example.

A firm sells to a market that consists of 100 infinitesimal customers who arrive in the first period of a two-period horizon. Customers are uncertain about their valuations in period 1, \mathbf{v} . We assume \mathbf{v} is distributed uniformly over $[10, 90]$. The assumption of infinitesimal customers implies that in period 2 the firm faces inverse demand $\bar{F}_1(p) = \frac{90-p}{80}$. The period-2 profit-maximizing price is $p_2^m = \arg\max p \bar{F}_1(p) = 45$ at which the demand is $100 \bar{F}_1(45) = \frac{225}{4}$. The firm then is guaranteed

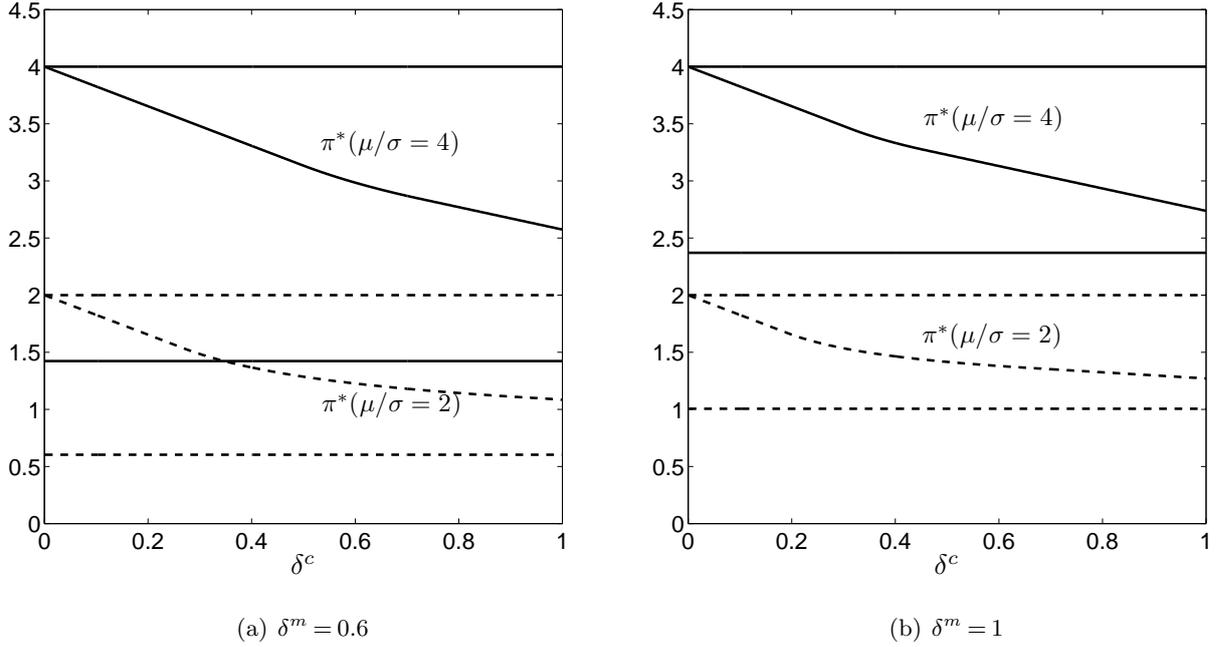


Figure 4 The optimal profit is bounded from above by commitment profits and from below by profits when selling only in period 2. Dashed and solid lines show the corresponding profits for $C.V. = 0.5$ and $C.V. = 0.25$, respectively. In panel (a), frenzy profits when $C.V. = 0.5$ correspond to the interval $\delta^c \in (0.27, 0.93)$; when $C.V. = 0.25$ they correspond to the interval $\delta^c \in (0.50, 0.72)$. In panel (b), frenzy profits when $C.V. = 0.5$ correspond to the interval $\delta^c \in (0.19, 0.66)$; when $C.V. = 0.25$ they correspond to the interval $\delta^c \in (0.31, 0.46)$. In this figure, $N_1 = 1$.

$\frac{10125}{4}$ in profits by producing and selling only in period 2, i.e., a pure spot selling strategy. Can the firm perform better by producing and selling in period 1 too?

For example, the firm could offer 90 units in period 1 and charge each customer her expected utility 50. The firm would earn 4500 *if* customers buy. However, is it reasonable to expect that customers will buy? Customers get a surplus of 0 if they buy. If instead all customers wait, the firm is left with an inventory of 90 units. Our first key assumption is that the firm cannot commit not to sell this inventory and sets the period-2 price to clear the market. In our example, The market-clearing price is 18 because $90 = 100\bar{F}_1(18)$. Customers' expected utility is $\delta^c E[\mathbf{v} - 18]^+ = \frac{162}{5}\delta^c$ if they wait. Waiting is then the equilibrium in which the firm earns $90 \times 18 = 1620$.

What is the highest period-1 price that induces customers to buy early? A customer will deviate and buy in period 1 only if her expected surplus $50 - p_1$ is at least as much as the expected surplus in the equilibrium where all others wait. Therefore, the maximum price the firm can charge in period 1 is such that $50 - p_1 = \frac{162}{5}\delta^c$ or $p_1 = 50 - \frac{162}{5}\delta^c$. The firm profit is then $90(50 - \frac{162}{5}\delta^c) + 10p_2^m \bar{F}_1(p_2^m) = \frac{76050}{16} - 2916\delta^c$. The second term in the profit calculation is the profit the firm obtains by producing and selling in period 2 to those customers who were rationed in period 1. Therefore, selling 90 units

| | | Period-1 production | |
|----------------------------------|--|--|--|
| | | $q_1 \leq \frac{225}{4}$ | $q_1 > \frac{225}{4}$ |
| Period-2 price if wait | p_2 | 45 | $90 - \frac{8}{10}q_1$ |
| Surplus if wait | $\delta^c E[\mathbf{v} - p_2]^+$ | $\frac{405\delta^c}{32}$ | $\frac{\delta^c}{250}q_1^2$ |
| Period-1 price | $p_1 = \mu - \delta^c E[\mathbf{v} - p_2]^+$ | $50 - \frac{405}{32}\delta^c$ | $50 - \frac{\delta^c}{250}q_1^2$ |
| Profit if sell q_1 in period 1 | $\pi(q_1)$ | $(\frac{395}{16} - \frac{405\delta^c}{32})q_1 + \frac{10125}{4}$ | $-\frac{\delta^c}{250}q_1^3 + \frac{395}{16}q_1 + \frac{10125}{4}$ |

Table 1 The ingredients of the model.

in period 1 at the price of $p_1 = 50 - \frac{162}{5}\delta^c$ is more profitable for the firm than charging a higher price which would induce customers to wait (because $\frac{76050}{16} - 2916\delta^c > 1620$ for any value of δ^c).

One may argue that the firm could slightly increase the price above p_1 and customers would still buy. It is true that an individual customer has no incentive to deviate and wait. This is because the price for a single deviating customer in period-2 is p_2^m and her expected surplus is $\delta^c E[\mathbf{v} - p_2^m]^+ < 50 - p_1 - \epsilon$ for sufficiently small ϵ . However this *buying* equilibrium is Pareto-dominated by the *waiting* equilibrium in which all customers wait and yields the surplus $E[\mathbf{v} - p_2^w]^+ = 50 - p_1 > 50 - p_1 - \epsilon$. The choice of the Pareto-dominant equilibrium is our second key assumption.

In summary, the firm prefers to produce 90 units and sell them at the price $p_1 = 50 - \frac{162}{5}\delta^c$ in period 1 to produce and sell only in period 2 (pure spot selling) if $\frac{76050}{16} - 2916\delta^c \geq \frac{10125}{4}$ or $\delta^c \lesssim 0.76$. In other words, producing and selling 90 units in period 1 is not in the firm's interest if customers are sufficiently patient. But the firm does not have to produce as much as 90 units in period 1.

Maintaining our two key assumptions, we can solve the firm's revenue maximization problem for any period-1 production quantity. Assume the firm produces quantity q_1 . What is the maximum price p_1 the firm can charge such that buying is a Pareto-dominant equilibrium? A customer who buys in period 1 obtains a surplus of $50 - p_1$. If customers wait, then two scenarios are possible (Table 1). The profit-maximizing period-2 price is $p_2^m = 45$ at which the demand is $100\bar{F}_1(p_2^m) = \frac{225}{4}$. Therefore, if $q_1 \leq \frac{225}{4}$, the firm will set the period-2 price $p_2^m = 45$ (row 1 in Table 1), sell q_1 and produce $\frac{225}{4} - q_1$ more units in period 2. If, on the other hand, $q_1 > \frac{225}{4}$, the firm has to set the price such that the demand in period 2, i.e., $100\bar{F}_1(p_2) = 100\frac{90-p_2}{80}$, is equal to the supply of q_1 . In this case, producing more will not benefit the firm. It follows that $p_2 = 90 - \frac{8}{10}q_1$. The waiting customers' surplus is $\delta^c E[\mathbf{v} - p_2^m]^+ = \frac{405\delta^c}{32}$ in the first scenario and $\delta^c E[\mathbf{v} - p_2]^+ = \frac{\delta^c}{250}q_1^2$ in the second scenario (row 2 in Table 1). By our second assumption, customers buy in period 1 if and only if $50 - p_1 > \frac{405\delta^c}{32}$ when the first scenario applies and $50 - p_1 > \frac{\delta^c}{250}q_1^2$ when the second scenario applies. As a tie-breaking rule, we assume that customers still buy when these inequalities bind. This is without loss of generality because customers would buy for any price arbitrarily lower. This establishes the period-1 price (row 3 in Table 1).

To sum up, the firm's profit is $\pi(q_1) = q_1 p_1 + (100 - q_1)p_2^m \bar{F}_1(p_2^m)$; row 4 in Table 1. Figure 5 shows the firm profits as a function of q_1 for three values of δ^c . As expected, there is a kink at

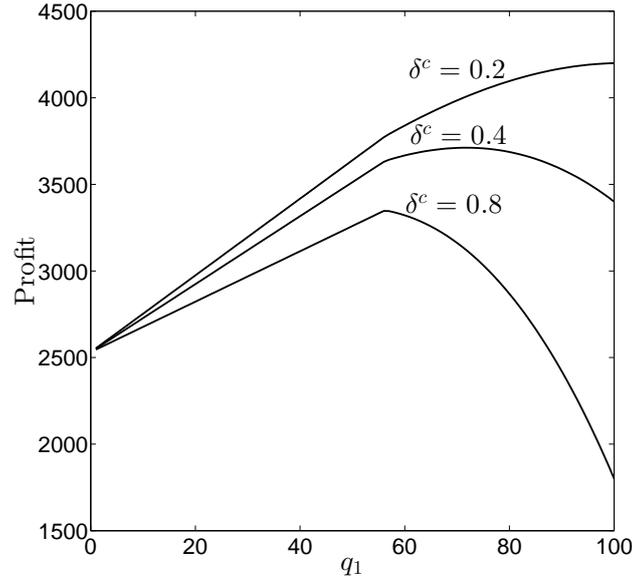


Figure 5 Firm profit as a function of q_1 for different values of customer discount rate δ^c .

$q_1 = \frac{225}{4}$. The linear part in Figure 5 captures the fact that the period-1 price does not depend on q_1 for $q_1 \leq \frac{225}{4}$. The reason is, in this case, the period-2 price is p_2^m whether customers wait or not. The concave part captures the fact that the period-1 price decreases with q_1 when $q_1 > \frac{225}{4}$. This follows because, when customers wait, the period-2 price is a decreasing function of q_1 and therefore customers are willing to pay less in period-1 as q_1 increases.

When customers discount the future significantly ($\delta^c = 0.2$), the firm does not have to discount much the period-1 price. It is optimal to sell to all customers in period 1, that is, to produce and sell 100 units in the first period. In this case, there is no buying frenzy. When customers are patient and value the waiting option to learn their preferences ($\delta^c = 0.8$), the firm has to discount the period-1 price so much that it is optimal to sell at the period-1 price such that $50 - p_1 = \delta^c E[\mathbf{v} - p_2^m]^+$. In our example, $p_1 \simeq 44.94$ and the firm produces and sells $q_1 = \frac{225}{4} = 56.25$ in period 1 and $q_2 = (100 - 56.25)\bar{F}_1(45) \simeq 24.61$ in period 2. Note that there is rationing in period 1 but no frenzy.⁹ This is because an individual customer is indifferent between buying and waiting. Buying yields $\mu - p_1 \simeq 5.06$. If the customer is rationed and has to wait then he obtains an expected surplus of $\delta^c E[\mathbf{v} - p_2^m] \simeq 5.06$. The firm does not serve the entire market over two periods ($56.25 + 24.61 = 80.86 < 100$).

When the customer discount rate takes an intermediate value ($\delta^c = 0.4$), the firm produces a quantity such that an individual customer is strictly better off buying in period 1 than to wait. For $\delta^c = 0.4$, the firm sells $q_1 \simeq 71.7$ at price $p_1 \simeq 41.8$ in period-1. To repeat, the reason the firm

⁹ We assume proportional or random rationing, i.e., when demand is in excess of supply all customers are equally likely to obtain the good.

sets the price $p_1 \simeq 41.8$ is that if customers decide to wait, the firm has to set a market-clearing price in period 2 which is $90 - \frac{8}{10}q_1 = 32.64$ (Table 1). The customers who succeed to purchase a unit obtain a surplus of $50 - p_1 = 8.2$. The remaining customers have to wait for the second period. They buy the product then if their valuation is greater than $p_2^m = 45$. Their expected surplus is $\delta^c E[\mathbf{v} - p_2^m]^+ \simeq 5.1$. Therefore, customers strictly prefer to buy in period 1 and there is a buying frenzy! In this case the firm produces and sells $(100 - 71.7)\bar{F}_1(45) \simeq 15.9$ units in period 2 which implies the firm does not serve the entire market across the two periods ($71.7 + 15.9 = 87.6 < 100$).

5. Second-Period Arrivals

A limitation of the analysis in Section 3 is that customers arrive only in period 1. A natural way to generalize the model is to distinguish between *early adopters*, who arrive in period 1 and face valuation uncertainty, and *followers*, who arrive in period 2 and know their valuations. In this section, we demonstrate that our results are robust to the arrival of followers.

Assume that N_2 new customers arrive in the second period. These customers have independently and identically distributed valuations with density f_2 and survival function \bar{F}_2 . The period-2 demand is a regular downward-sloping demand $N_2\bar{F}_2(p)$. This structure is common knowledge among market participants and is consistent with other models in the literature (see e.g., Xie and Shugan 2001, Gallego and Şahin 2010).

We need more notation to facilitate the exposition in this section. In particular, subscripts are used to denote period and superscripts are used to denote the arrival cohort. Thus $p_t^{i,m}$ denotes the monopoly price in period t for customer cohort i ; that is, $p_2^{1,m} \triangleq \operatorname{argmax}_p N_1 p \bar{F}_1(p)$ and $p_2^{2,m} \triangleq \operatorname{argmax}_p N_2 p \bar{F}_2(p)$. We assume that these two optimization problems are strictly concave. When Q_1 customers buy in period 1, we let $Q_2^b(p, Q_1) \triangleq (N_1 - Q_1)\bar{F}_1(p) + N_2\bar{F}_2(p)$ denote the demand in period 2. In line with Section 3, we let $p_2^m \triangleq \operatorname{argmax}_p p Q_2^b(p, 0)$ be the monopoly price when the firm sells only in period 2 and let $Q_2^m = Q_2^b(p_2^m, 0)$ be the corresponding quantity.

Suppose the firm offers q_1 units for sale in period 1 at a price p_1 . The period-2 price $p_2^e(x|q_1)$ maximizes the firms' period-2 profits subject to the market-clearing constraint. In other words,

$$\begin{aligned} p_2^e(x|q_1) &= \operatorname{argmax}_p p Q_2^b(p, \min(xN_1, q_1)) \\ \text{s.t. } Q_2^b(p, \min(xN_1, q_1)) &\geq q_1 - \min(xN_1, q_1). \end{aligned} \tag{4}$$

Define the unconstrained maximizer of (4) as $p_2^b(y) \triangleq \operatorname{argmax}_p p Q_2^b(p, y)$.

Lemma 3 $p_2^b(y)$ is increasing in y if $p_2^{1,m} < p_2^{2,m}$, is constant if $p_2^{1,m} = p_2^{2,m}$, and is decreasing otherwise.

Observe that the constraint in (4) is not binding for $x \in [\frac{q_1}{N_1}, 1]$. For $x < \frac{q_1}{N_1}$, define $\hat{p}_2(x, q_1)$ as the solution to

$$N_1(1-x)\bar{F}_1(p) + N_2\bar{F}_2(p) = q_1 - xN_1. \quad (5)$$

There is a unique solution to (5) because its left-hand side is decreasing in p , takes a maximum of $N_1(1-x) + N_2 \geq q_1 - xN_1$ at $p = 0$, and takes a minimum of $0 \leq q_1 - xN_1$ at $p = v_{\max}$. The following result characterizes $p_2^e(x|q_1)$.

Lemma 4 *Assume that $x \in [0, 1]$ is part of an REE. If $x \in [0, \frac{q_1}{N_1}]$, then $p_2^e(x|q_1) = \min(p_2^b(xN_1), \hat{p}_2(x, q_1))$; otherwise, $p_2^e(x|q_1) = p_2^b(q_1)$.*

We can show that $p_2^e(x|q_1)$ is (weakly) increasing only when $p_2^{1,m} \leq p_2^{2,m}$. The analysis in Section 3 extends easily when this is the case. We present this extension next and defer the analysis of the case $p_2^{1,m} > p_2^{2,m}$ to Appendix B where we show that the main results remain robust.

In the rest of this section, we assume that $p_2^{1,m} \leq p_2^{2,m}$. We first show that, much as in Section 3, $p_2^e(x|q_1)$ is monotonic in x . To do so, we define $\hat{x}(q_1)$ as the solution to

$$Q_2^b(p_2^b(xN_1), xN_1) = q_1 - xN_1. \quad (6)$$

The right-hand side of (6) is the left-over inventory from period 1 while the left-hand side is the demand in period 2 when the firm charges the price $p_2^b(xN_1)$. We show in Appendix that (6) has a unique solution when $Q_2^m \leq q_1 \leq N_1 + N_2\bar{F}_2(p_2^{2,m})$.

Lemma 5 *(a) Assume $p_2^{1,m} < p_2^{2,m}$. It follows that $p_2^e(x|q_1)$ is continuous and weakly increasing for $x \in [0, 1]$ with $\frac{\partial}{\partial x} p_2^e(x|q_1) > 0$ at $x = 0$. (b) Assume $p_2^{1,m} = p_2^{2,m}$. (b1) If $q_1 \leq Q_2^m$ then $p_2^e(x|q_1) = p_2^m$. (b2) If $Q_2^m \leq q_1 \leq N_1 + N_2\bar{F}_2(p_2^{2,m})$ then $p_2^e(x|q_1) = p_2^m$ for $x \geq \hat{x}(q_1)$ and $p_2^e(x|q_1) = \hat{p}_2(x, q_1)$ otherwise. (b3) If $q_1 > N_1 + N_2\bar{F}_2(p_2^{2,m})$ then $p_2^e(x|q_1) = \hat{p}_2(x, q_1)$. (b4) $p_2^e(x|q_1)$ is weakly increasing in x .*

Lemma 5 shows that $p_2^e(x|q_1)$ is weakly increasing in x . We leverage that property and apply Lemma 2 to eliminate all mixed strategies because they cannot be part of a Pareto-dominant REE. We then show that a result similar to Proposition 1 holds—namely, the profit-maximizing price for quantity q_1 is $p_1(q_1) = p_1^w(q_1)$ (see Appendix for proofs). Moreover, an early production quantity that exceeds the period-1 market size is never strictly profitable for the firm. The reason is that excess capacity in period 1 lowers the period-2 expected price (since $p_2^e(x|q_1)$ is decreasing in q_1); it therefore lowers the price the firm can charge in period 1 yet does not increase sales.

In summary, the monopolist chooses the early production quantity $q_1 \leq N_1$ so as to maximize

$$\pi(q_1) = q_1 p_1(q_1) + \delta^m p_2^e(1|q_1) Q_2^b(p_2^e(1|q_1), q_1). \quad (7)$$

We next investigate the conditions under which a buying frenzy equilibrium exists and is an optimal policy for the firm.

Lemma 6 *There is no buying frenzy equilibrium if $\frac{N_2}{N_1} \geq \frac{F_1(p_2^m)}{F_2(p_2^m)}$.*

Lemma 6 implies that buying frenzies are less likely to be optimal when the customer cohort arriving in period 2 is large relative to the one arriving in period 1 (i.e., when $N_2 \gg N_1$). In this case, the waiting equilibrium involves a relatively small number of period-1 customers who have little impact on the second-period price. For the rest of the section we assume that $\frac{N_2}{N_1} < \frac{F_1(p_2^m)}{F_2(p_2^m)}$; this assumption is equivalent to $Q_2^m < N_1$, which is analogous to the assumption $p_2^m > \underline{v}$ in Section 3. Frenzies never happen when this is not the case. The following proposition characterizes when a buying frenzy occurs.

Proposition 3 *Assume that $\frac{N_2}{N_1} < \frac{F_1(p_2^m)}{F_2(p_2^m)}$. Sufficient conditions for the firm's optimal policy to induce a buying frenzy are: (i) $p_2^{1,m} < p_2^{2,m}$ and $\delta^c > \delta_2^c$ or (ii) $p_2^{1,m} = p_2^{2,m}$ and $\delta_2^c < \delta^c < \delta_1^c$. Here*

$$\delta_1^c = \frac{\mu - \delta^m p_2^m \bar{F}_1(p_2^m)}{T_1(p_2^m) + \frac{Q_2^m \bar{F}_1(p_2^m)}{N_1 f_1(p_2^m) + N_2 f_2(p_2^m)}}, \quad \delta_2^c = \frac{\mu - \delta^m p_2^{2,m} \bar{F}_1(p_2^{2,m})}{T_1(p_2^w(N_1)) + \frac{N_1 \bar{F}_1(p_2^w(N_1))}{N_1 f_1(p_2^w(N_1)) + N_2 f_2(p_2^w(N_1))}}.$$

Proposition 3 establishes sufficient conditions under which $q_1^* < N_1$ and $L(q_1^*) > 0$ whenever $p_2^{1,m} < p_2^{2,m}$ or $p_2^{1,m} = p_2^{2,m}$.

Figure 6 generalizes Figure 2(b) to the case where new customers arrive in period 2. Overall, the main insights from Section 3 carry over. The shape of the parameter space where frenzies occur does not change much relative to Figure 2(b) with one exception. When $p_2^{1,m} < p_2^{2,m}$, Figure 6(a) reveals a parameter space that is larger than its counterpart in Figures 2(b) and 5(b). The areas labeled “rationing without frenzy” become “buying frenzy” in Figure 6(a). The area “rationing without frenzy” corresponds to the corner solution at Q_2^m with $p_2^e(1|Q_2^m) \leq p_2^e(0|Q_2^m)$; in Figure 6(a), this area disappears because we have $p_2^e(1|Q_2^m) > p_2^e(0|Q_2^m)$. Figure 6(b) considers the special case where $f_1(\cdot) = f_2(\cdot) = f(\cdot)$. In that case, we can derive a closed-form solution for the parameter space that generates buying frenzies. Moreover, the frenzy equilibrium is unique (see the proof of Proposition 3 in the Appendix).

To demonstrate the relevance of our analysis in this section we revisit one of the two key results of Balachander et al. (2009). They find a positive association between buying frenzies and intrinsic preferences for a product that lasts beyond its introductory period¹⁰. One needs a dynamic model of sales—such as the one presented here—to provide a proper interpretation of this finding. Assume that customers do not discount excessively. An increase in the strength of the second-period

¹⁰ Hypothesis 2A, p. 1626

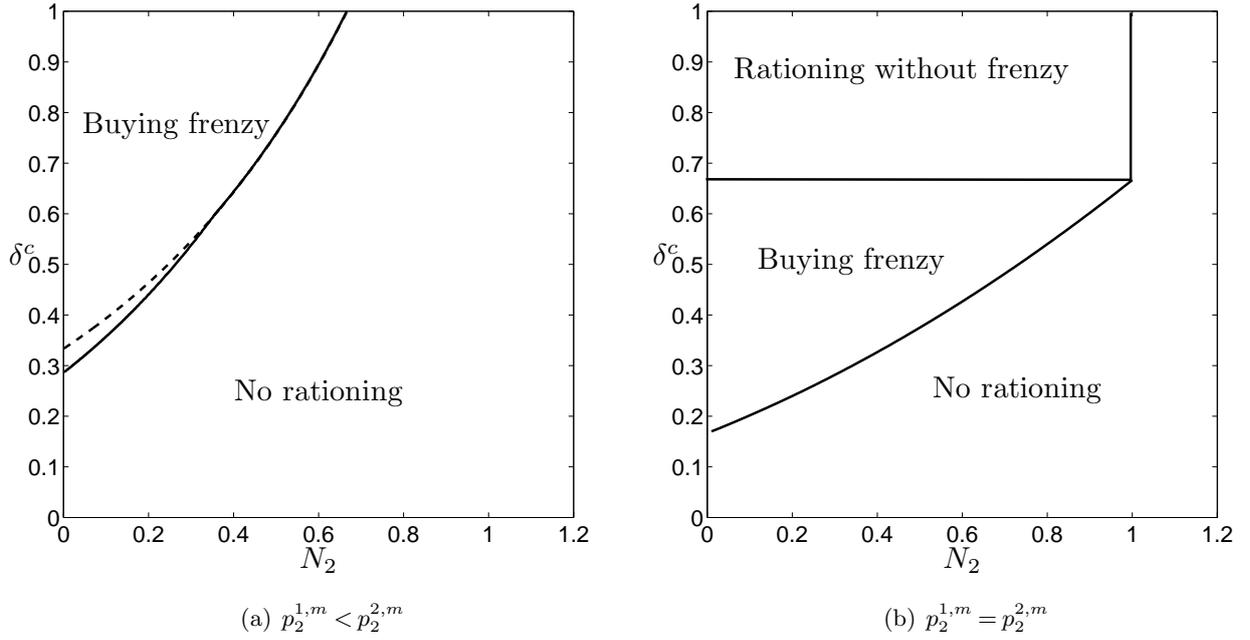


Figure 6 The firm’s optimal policy when the valuation distribution of early arrivals is $U[0, b_1]$ and that of late arrivals is $U[0, b_2]$. In this figure, $\delta^m = 1$ and $N_1 = 1$; also $(b_1, b_2) = (10, 20)$ in panel (a) and $(b_1, b_2) = (10, 10)$ in panel (b). The dashed line in panel (a) shows the sufficient conditions characterized in Proposition 3. The frenzy parameter space is slightly larger than the one implied by these conditions. Panel (b) shows the necessary and sufficient conditions for $f_1(\cdot) = f_2(\cdot)$.

demand can be interpreted as an increase in p_2^m . When p_2^m is low, the equilibrium is likely to be a “rationing without frenzy” (upper left area in Figure 6(b); see also Figure 8). For p_2^m sufficiently large, the equilibrium becomes a “buying frenzy” (upper left area in Figure 6(a)). Buying frenzies are therefore associated with a high level of p_2^m , which can be interpreted as strong and persistent intrinsic preferences.

6. Conclusion

This paper offers a tractable and convenient framework in which to analyze buying frenzies in a dynamic context. Unlike other models presented in the literature, in our model the existence of buying frenzies depends neither on the rule applied by the firm to allocate goods under rationing nor on information asymmetry among customers. Buying frenzies occur when customers are sufficiently uncertain about their product valuations and when customers discount the future but not excessively. Overly patient customers wait until establishing their preferences for the product, and overly impatient customers are served early. We demonstrated how the firm’s and customers’ impatience affects the depth and breadth of a frenzy and showed that the utility loss from not obtaining the good in a frenzy can be substantial. Similarly, the firm’s gain from a frenzy policy can be economically large and partially compensate for the firm’s lack of commitment to future prices.

The analysis presented here offers a rich framework for interpreting stylized facts and empirical findings about buying frenzies. We have illustrated this point by revisiting the two key results presented in Balachander et al. (2009): an introductory price that is positively correlated with product scarcity; and a positive association—that lasts beyond the introductory period—between buying frenzies and intrinsic preferences for a product. These findings are difficult to rationalize within static models, but both are consistent with our analysis.

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Appendix: Proofs

Proof of Lemma 1: The function $p\bar{F}_1(p)$ is unimodal, and $\operatorname{argmax} p\bar{F}_1(p) = p_2^m$. (a) In this case, the constraint in (1) is nonbinding. This is because $(N_1 - \min(xN_1, q_1))\bar{F}_1(p) \geq q_1 - \min(xN_1, q_1)$ implies that $N_1\bar{F}_1(p) + \min(xN_1, q_1)F_1(p) \geq q_1$. The inequality holds for $p = p_2^m$ since $q_1 \leq N_1\bar{F}_1(p_2^m)$ and so $p_2^e(x|q_1) = p_2^m$.

(b) Assume first that $x \in [0, \frac{q_1 - N_1\bar{F}_1(p_2^m)}{N_1\bar{F}_1(p_2^m)}]$. We have

$$\begin{aligned} x &\leq \frac{q_1 - N_1\bar{F}_1(p_2^m)}{N_1\bar{F}_1(p_2^m)}, \\ xN_1\bar{F}_1(p_2^m) &\leq q_1 - N_1\bar{F}_1(p_2^m), \\ xN_1 &\leq q_1 - N_1\bar{F}_1(p_2^m)(1-x), \\ xN_1 &\leq q_1. \end{aligned}$$

Therefore, $p_2^e(x|q_1) = \operatorname{argmax} p\bar{F}_1(p)$ subject to the constraint $(1-x)N_1\bar{F}_1(p) \geq q_1 - xN_1$. Since $(1-x)N_1\bar{F}_1(p_2^m) \leq q_1 - xN_1$, it follows that the constraint is binding. We conclude that $p_2^e(x|q_1)$ is the unique solution to $(1-x)N_1\bar{F}_1(p) = q_1 - xN_1$ (uniqueness follows because f_1 is log-concave);

that is, $p_2^e(x|q_1) = \bar{F}_1^{-1}\left(\frac{q_1 - xN_1}{(1-x)N_1}\right)$. Finally, because $\frac{q_1 - xN_1}{(1-x)N_1}$ is strictly decreasing in x , we have that $p_2^e(x|q_1)$ is strictly increasing in x_1 .

Now assume that $x \geq \frac{q_1 - N_1 \bar{F}_1(p_2^m)}{N_1 \bar{F}_1(p_2^m)}$ or, equivalently that, $xN_1 \geq q_1 - (1-x)N_1 \bar{F}_1(p_2^m)$. Two cases are possible: either $xN_1 \geq q_1$ or $q_1 \geq xN_1 \geq q_1 - (1-x)N_1 \bar{F}_1(p_2^m)$. In either case, the constraint in (1) is nonbinding and so $p_2^e(x|q_1) = p_2^m$, which is constant in x .

(c) The proof follows from parts (a) and (b). For a given x , if $x \in \left[0, \frac{q_1 - N_1 \bar{F}_1(p_2^m)}{N_1 \bar{F}_1(p_2^m)}\right]$, then for $q_1 \leq N_1 \bar{F}_1(p_2^m)$, we have $p_2^e(x|q_1) = p_2^m$ and constant in q_1 and for $q_1 > N_1 \bar{F}_1(p_2^m)$ we have $p_2^e(x|q_1) = \bar{F}_1^{-1}\left(\frac{q_1 - xN_1}{(1-x)N_1}\right)$ which is decreasing in q_1 . If, on the other hand, $x \in \left[\frac{q_1 - N_1 \bar{F}_1(p_2^m)}{N_1 \bar{F}_1(p_2^m)}, 1\right]$, then $p_2^e(x|q_1) = p_2^m$ and so is constant in q_1 . \square

Proof of Proposition 1: We first derive the Pareto-dominant REE preferred by the firm for a given (q_1, p_1) and then select the maximum price p_1 for a given q_1 . The analysis proceeds by way of two possible cases.

CASE 1: Assume that period-1 production is $q_1 \leq N_1 \bar{F}_1(p_2^m)$. We characterize the Pareto-dominant REE as follows.

Claim 1 (a) $p_1^w(q_1) = p_1^b(q_1)$. (b1) If $p_1 < p_1^w(q_1)$ then $x = 1$ is the unique Pareto-dominant REE. (b2) If $p_1 = p_1^w(q_1)$, then all $x \in [0, 1]$ are Pareto-equivalent REE and $x = 1$ is the Pareto-dominant REE preferred by the firm. (b3) If $p_1 > p_1^w(q_1)$ then $x = 0$ is the unique Pareto-dominant REE.

Proof: (a) This follows from Lemma 1 because $p_2^e(0|q_1) = p_2^e(1|q_1) = p_2^m$ for $q_1 \leq N_1 \bar{F}_1(p_2^m)$.

(b1) We have $\mu - p_1 > \mu - p_1^w(q_1) = \delta^c T_1(p_2^e(x|q_1)) = \delta^c T_1(p_2^m)$ for all $x \in [0, 1]$. Therefore, a customer is better-off buying the product and $x = 1$ is the unique Pareto-dominant REE.

(b2) Since $\mu - p_1 = \mu - p_1^w(q_1) = \delta^c T_1(p_2^e(x|q_1)) = \delta^c T_1(p_2^m)$ for all $x \in [0, 1]$, it follows that all $x \in [0, 1]$ are Pareto-dominant REE because the customer's expected surplus is the same. We show that $x = 1$ is the Pareto-dominant REE that maximizes the firm profits. The firm's profit for a given $x \in [0, 1]$ is $\pi(x) = Q_1 p_1^w(q_1) + (N_1 - Q_1) p_2^m \bar{F}_1(p_2^m) = Q_1 (p_1^w(q_1) - p_2^m \bar{F}_1(p_2^m)) + N_1 p_2^m \bar{F}_1(p_2^m)$, where $Q_1 = \min(xN_1, q_1)$ is the sales in period 1. The profit $\pi(x)$ is increasing in x because $p_1^w(q_1) = \mu - \delta^c T_1(p_2^m)$ and

$$p_1^w(q_1) - p_2^m \bar{F}_1(p_2^m) = \mu - \int_{p_2^m} (v - p_2^m) dF_1 - \int_{p_2^m} p_2^m dF_1 = \int_0^{p_2^m} v dF_1 > 0.$$

Hence $x = 1$ is the Pareto-dominant REE that maximizes the firm's profits.

(b3) Since $\mu - p_1 < \delta^c T_1(p_2^e(x|q_1))$, it follows that $x = 0$ is the unique Pareto-dominant REE. \square

CASE 2: Assume that period-1 production is $q_1 > N_1 \bar{F}_1(p_2^m)$. Then the next result characterizes the Pareto-dominant REE.

Claim 2 (a) $p_1^w(q_1) < p_1^b(q_1)$. (b) Any mixed strategy REE is Pareto dominated by the REE $x = 0$. (c1) If $p_1 < p_1^w(q_1)$, then $x = 1$ is the unique Pareto-dominant REE. (c2) If $p_1 = p_1^w(q_1)$, then there are two Pareto-dominant REE, $x = 0$ and $x = 1$, and the latter is preferred by the firm. (c3) If $p_1^w(q_1) < p_1$, then $x = 0$ is the unique Pareto-dominant REE.

Proof: Part (a) follows directly from Lemma 1. (b) The condition in Lemma 2 holds for any $x \in (0, 1)$. (c1) If $p_1 < p_1^w(q_1)$, then $\mu - p_1 > \mu - p_1^w(q_1) = \delta_c T_1(p_2^e(0|q_1)) \geq \delta_c T_1(p_2^e(x|q_1))$ for all x ; hence $x = 1$ is the unique Pareto-dominant REE. (c2) If $p_1 = p_1^w(q_1)$, then $x = 0$ and $x = 1$ are Pareto-dominant REE because $\mu - p_1 = \delta_c T_1(p_2^e(0|q_1))$. We show that the Pareto-dominant REE $x = 1$ is preferred by the firm. The firm's profit under the equilibrium $x = 0$ is $\pi(x = 0) = \delta^m N_1 p_2^e(0|q_1) \bar{F}_1(p_2^e(0|q_1))$ and under the equilibrium $x = 1$ is $\pi(x = 1) = p_1 q_1 + \delta^m (N_1 - q_1) p_2^e(1|q_1) \bar{F}_1(p_2^e(1|q_1)) = (p_1 - \delta^m p_2^m \bar{F}_1(p_2^m)) q_1 + \delta^m N_1 p_2^m \bar{F}_1(p_2^m)$. The result then follows because $p_1 - p_2^m \bar{F}_1(p_2^m) \geq 0$ and $p_2^m \bar{F}_1(p_2^m) > p_2^e(0|q_1) \bar{F}_1(p_2^e(0|q_1))$. The first inequality holds because $p_1 = p_1^w(q_1)$ and

$$\begin{aligned} p_1^w(q_1) - p_2^m \bar{F}_1(p_2^m) &= \mu - \int_{p_2^e(0|q_1)} (v - p_2^e(0|q_1)) dF_1 - \int_{p_2^m} p_2^m dF_1 \\ &> \mu - \int_{p_2^m} (v - p_2^e(0|q_1) + p_2^m) dF_1 \quad \text{because } p_2^e(0|q_1) > p_2^m \text{ (Lemma 1)} \\ &= \int_0^{p_2^m} v dF_1 + \int_{p_2^m} (p_2^e(0|q_1) - p_2^m) dF_1 \\ &> 0. \end{aligned}$$

(c3) We know that $x = 0$ is an REE because $\delta_c T_1(p_2^e(0|q_1)) > \mu - p_1$. Furthermore, $x = 1$ is not an REE because $\mu - p_1 < \delta^c T_1(p_2^e(1|q_1))$. Hence $x = 0$ is the only Pareto-dominant REE. \square

We can now conclude by collecting the Pareto-dominant REE preferred by the firm for an announcement (q_1, p_1) from the two cases just described. For a period-1 price such that $p_1 > p_1^w(q_1)$, customers wait in any Pareto-dominant REE and hence the firm's profit is $\pi(x = 0) = \delta^m N_1 p_2^e(0|q_1) \bar{F}_1(p_2^e(0|q_1))$. If the period-1 price is set to $p_1 = p_1^w(q_1)$, then the firm's profit is $\pi(x = 1) = p_1^w(q_1) q_1 + \delta^m (N_1 - q_1) p_2^m \bar{F}_1(p_2^m)$. Because $p_1^w(q_1) > p_2^m \bar{F}_1(p_2^m)$ (see (b2) in Case 1, (c2) in Case 2) and $p_2^m \bar{F}_1(p_2^m) \geq p_2^e(0|q_1) \bar{F}_1(p_2^e(0|q_1))$ (by definition of p_2^m), we have $\pi(x = 1) > \pi(x = 0)$. Therefore, any price $p_1 > p_1^w(q_1)$ is dominated by $p_1(q) = p_1^w(q_1)$. For a period-1 price such that $p_1 < p_1^w(q_1)$, customers buy in any Pareto-dominant REE and the firm's profit is $\pi(x = 1) = p_1 q_1 + \delta^m (N_1 - q_1) p_2^m \bar{F}_1(p_2^m)$. This profit is increasing in p_1 , so $p_1 = p_1^w(q_1)$ dominates $p_1 < p_1^w(q_1)$. Therefore, the firm's profits are maximized at $p_1(q_1) = p_1^w(q_1)$. \square

Proof of Proposition 2: If $\bar{F}_1(p_2^m) = 1$ then, by Lemma 1, $p_2^e(0|q_1) = p_2^m = \underline{v}$ and $p_1^w(q_1) = \mu - \delta^c T_1(p_2^m)$. It follows from (3) that $\pi(q_1) = q_1(\mu - \delta^c T_1(p_2^m)) + \delta^m (N_1 - q_1) p_2^m \bar{F}_1(p_2^m)$,

which is increasing in q_1 . The reason is that $\mu - \delta^c T_1(p_2^m) - \delta^m p_2^m \bar{F}_1(p_2^m) \geq \mu - T_1(p_2^m) - p_2^m \bar{F}_1(p_2^m) = 0$.¹¹ Hence $q_1^* = N_1$; in other words, the firm serves the entire market in period 1 and customers are indifferent between buying and waiting. A buying frenzy can therefore not occur in this case.

Assume for the rest of this proof that $\bar{F}_1(p_2^m) < 1$. Observe that if $q_1^* = 0$ then the firm's optimal policy is to sell only in period 2. We prove next that the optimal production is strictly positive, which implies that selling only in period 2 is never optimal when the firm has the option of producing over time.

If $q_1 \leq N_1 \bar{F}_1(p_2^m)$, then $p_2^e(x|q_1) = p_2^m$ and $p_1(q_1) = \mu - \delta^c T_1(p_2^m)$; that is, $p_1(q_1)$ becomes independent of q_1 by Lemma 1 and Proposition 1. The profit function in (3) is then linear and increasing in q_1 because

$$\begin{aligned} \frac{d\pi(q_1)}{dq_1} &= \mu - \delta^c T_1(p_2^m) - \delta^m p_2^m \bar{F}_1(p_2^m) \\ &\geq \mu - T_1(p_2^m) - p_2^m \bar{F}_1(p_2^m) \\ &= \int \min(v, p_2^m) dF_1 - p_2^m \bar{F}_1(p_2^m) \\ &= \int_v^{p_2^m} v dF_1 > 0. \end{aligned}$$

The last inequality follows because $\bar{F}_1(p_2^m) < 1$. Therefore, $q_1 = N_1 \bar{F}_1(p_2^m)$ yields greater profits than any lower q_1 . Yet the production quantity $q_1 = N_1 \bar{F}_1(p_2^m)$ cannot induce a buying frenzy because customers are indifferent between buying early and waiting. It follows that a necessary condition for a buying frenzy is that $q_1 > N_1 \bar{F}_1(p_2^m)$. We next characterize the sufficient conditions under which it is optimal for the firm to induce a buying frenzy.

If $N_1 \bar{F}_1(p_2^m) \leq q_1$, then $p_2^e(0|q_1) = \bar{F}_1^{-1}(\frac{q_1}{N_1})$ by Lemma 1. The firm's profit function is

$$\pi(q_1) = q_1(\mu - \delta^c T_1(p_2^e(0|q_1))) + \delta^m (N_1 - q_1) p_2^m \bar{F}_1(p_2^m),$$

which is strictly concave in q_1 because

$$\frac{d\pi(q_1)}{dq_1} = \mu - \delta^c T_1(p_2^e) - \delta^c \frac{q_1^2}{N_1^2 f_1(p_2^e)} - \delta^m q_1 p_2^m \bar{F}_1(p_2^m)$$

and

$$\frac{d^2\pi(q_1)}{dq_1^2} = -\delta^c \frac{q_1}{N_1^2 f_1(p_2^e)} \left(1 + \frac{2[f_1(p_2^e)]^2 + \bar{F}_1(p_2^e) f_1'(p_2^e)}{[f_1(p_2^e)]^2} \right) < 0$$

(we suppress the argument in $p_2^e(0|q_1)$ for brevity). The inequality holds because f_1 is log-concave and so $[f_1(p)]^2 + \bar{F}_1(p) f_1'(p) > 0$ (Bagnoli and Bergstrom 2005). The first-order conditions then characterize the optimal q_1 ; that is, q_1^* solves

$$\frac{d\pi}{dq_1} = \mu - \delta^c T_1(p_2^e(0|q_1)) - \delta^c \frac{q_1^2}{N_1^2 f_1(p_2^e(0|q_1))} - \delta^m p_2^m \bar{F}_1(p_2^m) = 0 \quad (8)$$

¹¹ For the particular case $\delta^c = \delta^m = 1$, the profit function is independent of q_1 and so *all* $q_1 \in [0, N_1]$ are optimal. In this case, customers are still indifferent between buying and waiting and a frenzy does not occur.

provided that $N_1 \bar{F}_1(p_2^m) \leq q_1 \leq N_1$. Since the left-hand side of (8) is strictly decreasing in q_1 (because π is strictly concave), it follows that if $\frac{d\pi}{dq_1}|_{q_1=N_1} < 0$ and $\frac{d\pi}{dq_1}|_{q_1=N_1 \bar{F}_1(p_2^m)} > 0$ then there is a unique interior solution in $(N_1 \bar{F}_1(p_2^m), N_1)$ and that otherwise we obtain a boundary solution. To summarize: (i) if $\delta^c \leq \frac{\mu - \delta^m \bar{\pi}}{\mu - \bar{v} + \frac{1}{f_1(\bar{v})}}$ then $q_1^* = N_1$; (ii) if $\frac{\mu - \delta^m \bar{\pi}}{\mu - \bar{v} + \frac{1}{f_1(\bar{v})}} < \delta^c < \frac{\mu - \delta^m \bar{\pi}}{T_1(p_2^m) + \frac{[\bar{F}_1(p_2^m)]^2}{f_1(p_2^m)}}$ then $q_1^* \in (N_1 \bar{F}_1(p_2^m), N_1)$ and is characterized by (8); (iii) otherwise, $q_1^* = N_1 \bar{F}_1(p_2^m)$.

A buying frenzy can occur only in the second case. It remains to show that, in this case, $L_1(q_1^*) > 0$. But this follows since $N_1 \bar{F}_1(p_2^m) < q_1^* < N_1$ and so $p_2^e(0|q_1^*) < p_2^m$ by Lemma 1. Therefore, $L_1(q_1^*) > 0$ because $T_1(p)$ is strictly decreasing in p . \square

Proof of Corollary 1: (a) This part follows from the envelope theorem when one considers that, by (3), we have $\frac{d\pi^*}{d\delta^c} = -q_1^* T_1(p_2^e(0|Q_1^*)) < 0$ and $\frac{d\pi^*}{d\delta^m} = (N_1 - q_1^*) p_2^m \bar{F}_1(p_2^m) \geq 0$.

(b) The quantity q_1^* solves (8). Differentiating with respect to δ^c and simplifying yields

$$\frac{dq_1^*}{d\delta^c} = \frac{T_1(p_2^e) + \frac{q_1^{*2}}{N_1^2} \frac{1}{f_1(p_2^e)}}{\delta^c \bar{F}_1(p_2^e) \frac{dp_2^e}{dq_1} - \frac{\delta^c q_1^*}{N_1^2} \frac{2f_1(p_2^e) - f_1'(p_2^e) \frac{dp_2^e}{dq_1} q_1^*}{[f_1(p_2^e)]^2}} < 0$$

(we suppress the argument in $p_2^e(0|q_1^*)$ for brevity). The inequality follows because $p_2^e(0|q_1) = \bar{F}_1^{-1}(\frac{q_1}{N_1})$ and so $\frac{dp_2^e(0|q_1)}{dq_1} = \frac{-1}{N_1 f_1(p_2^e)} < 0$. Moreover, since $\bar{F}_1(p_2^e(0|q_1)) = \frac{q_1}{N_1}$ we have $2f_1(p_2^e) - f_1'(p_2^e) \frac{dp_2^e}{dq_1} q_1^* = \frac{2[f_1(p_2^e)]^2 + f_1'(p_2^e) \bar{F}_1(p_2^e)}{f_1(p_2^e)} > 0$ because f_1 is log-concave (see the proof of Proposition 2). Similarly, it is straightforward to observe that

$$\frac{dq_1^*}{d\delta^m} = \frac{p_2^m \bar{F}_1(p_2^m)}{\delta^c \bar{F}_1(p_2^e) \frac{dp_2^e}{dq_1} - \frac{\delta^c q_1^*}{N_1^2} \frac{2f_1(p_2^e) - f_1'(p_2^e) \frac{dp_2^e}{dq_1} q_1^*}{[f_1(p_2^e)]^2}} < 0. \quad \square$$

Proof of Lemma 3: $p_2^b(y)$ solves the first-order condition

$$(N_1 - y) \bar{F}_1(p) - p(N_1 - y) f_1(p) + N_2 \bar{F}_2(p) - p N_2 f_2(p) = 0.$$

Differentiating both sides with respect to y , we obtain

$$-[\bar{F}_1(p_2^b) - p_2^b f_1(p_2^b)] + (N_1 - y) \left[-2f_1(p_2^b) \frac{dp_2^b}{dy} - p f_1'(p_2^b) \frac{dp_2^b}{dy} \right] + N_2 \left[-2f_2(p_2^b) \frac{dp_2^b}{dy} - p f_2'(p_2^b) \frac{dp_2^b}{dy} \right] = 0,$$

or

$$\frac{dp_2^b}{dy} = \frac{\bar{F}_1(p_2^b) - p_2^b f_1(p_2^b)}{(N_1 - y) [-2f_1(p_2^b) - p f_1'(p_2^b)] + N_2 [-2f_2(p_2^b) - p f_2'(p_2^b)]}.$$

The denominator is negative since $p \bar{F}_i(p)$, $i \in \{1, 2\}$ is strictly concave. Further, the numerator is negative if $p_2^{1,m} < p_2^{2,m}$, positive if $p_2^{1,m} > p_2^{2,m}$, and 0 otherwise. \square

Proof of Lemma 4: If $x \in [0, \frac{q_1}{N_1}]$, then the constraint is not binding if $Q_2^b(p_2^b(xN_1), xN_1) \geq q_1 - xN_1$ and $p_2^e(x|q_1) = p_2^b(xN_1)$. Otherwise, $p_2^e(x|q_1) = \hat{p}_2(x, q_1)$ because the objective function is strictly concave. In other words, the price that clears the inventory remaining from period 1 is

optimal. The second part of the lemma follows because if $x \in [\frac{q_1}{N_1}, 1]$ then the constraint in (4) is not binding. \square

Proof of Lemma 5: (a) On the one hand, if $x \in [\frac{q_1}{N_1}, 1]$ then by Lemma 4 we have $p_2^e(x|q_1) = p_2^b(q_1)$, which is strictly increasing in q_1 (by Lemma 3) and constant in x . On the other hand, if $x \in [0, \frac{q_1}{N_1}]$ then $p_2^e(x|q_1) = \min(p_2^b(xN_1), \hat{p}_2(x, q_1))$, which is continuous in x . Furthermore, $p_2^b(xN_1)$ and $\hat{p}_2(x, q_1)$ are strictly increasing in x —the former by Lemma 3. To prove the latter, recall that $\hat{p}_2(x, q_1)$ solves (5). Differentiating then yields

$$\frac{d\hat{p}_2}{dx} = \frac{N_1 F_1(\hat{p}_2)}{(N_1 - xN_1)f_1(\hat{p}_2) + N_2 f_2(\hat{p}_2)} > 0.$$

It follows that $p_2^e(x|q_1)$ is strictly increasing at $x=0$ and also weakly increasing in $(0, 1]$.

(b) We first establish the conditions under which (6) has a unique solution. We then characterize $p_2^e(x|q_1)$ and show that it is (weakly) increasing in x .

Claim 3 *There exists a unique solution $x \in [0, 1]$ to (6) if and only if $Q_2^m \leq q_1 \leq N_1 + N_2 \bar{F}_2(p_2^{2,m})$.*

Proof: Define

$$G(x) = Q_2^b(p_2^b(xN_1); xN_1) - (q_1 - xN_1). \quad (9)$$

Note that $G(x)$ is increasing in x because $\frac{dG(x)}{dx} = \frac{\partial Q_2^b}{\partial p} \frac{\partial p_2^b}{\partial x} + \frac{\partial Q_2^b}{\partial Q_1} \frac{\partial xN_1}{\partial x} + N_1 > 0$. The inequality follows because $\frac{\partial Q_2^b}{\partial p} \leq 0$ and $\frac{\partial p_2^b}{\partial x} \leq 0$ by Lemma 3. Moreover, $\frac{\partial Q_2^b}{\partial Q_1} \frac{\partial xN_1}{\partial x} + N_1 = N_1(-\bar{F}_1(p_2^b) + 1) > 0$. So if (6) has a solution, it must be unique.

The existence of a solution is guaranteed provided that $G(0) \leq 0 \leq G(1)$ or $Q_2^m \leq q_1 \leq N_1 + N_2 \bar{F}_2(p_2^{2,m})$. \square

We can now prove the results in part (b) of the lemma. Observe that, by the proof of Claim 3, the function $G(x)$ is increasing in x . We shall use this property to characterize $p_2^e(x|q_1)$.

(b1) If $q_1 \leq Q_2^m$ then $G(x) \geq 0$ for all x , which implies that $p_2^b(xN_1) \leq \hat{p}_2(x, q_1)$. Hence by Lemma 4 we have $p_2^e(x|q_1) = p_2^b(xN_1)$ if $x \leq \frac{q_1}{N_1}$ and $p_2^e(x|q_1) = p_2^b(q_1)$ otherwise. Since $p_2^{1,m} = p_2^{2,m}$, it follows that $p_2^{1,m} = p_2^{2,m} = p_2^b(xN_1) = p_2^b(q_1) = p_2^m$ (see (4)); that is, $p_2^e(x|q_1) = p_2^m$.

(b2) If $Q_2^m \leq q_1 \leq N_1 + N_2 \bar{F}_2(p_2^{2,m})$, then (6) has a unique solution $\hat{x}(q_1) \in [0, 1]$ by Claim 3. Hence if $x \geq \hat{x}(q_1)$ then $p_2^e(x|q_1) = p_2^m$; otherwise, $p_2^e(x|q_1) = \hat{p}_2(x, q_1)$. As a caveat, note that if $q_1 < N_1$ then for $x \geq \frac{q_1}{N_1}$ we have $p_2^e(x|q_1) = p_2^b(q_1)$ by Lemma 4. However, when $p_2^{1,m} = p_2^{2,m}$ we have $p_2^b(q_1) = p_2^m$.

(b3) If $q_1 > N_1 + N_2 \bar{F}_2(p_2^{2,m})$, then $p_2^e(x|q_1) = \hat{p}_2(x, q_1)$ for all $x \in [0, 1]$ because $G(x) \leq 0$ for all x and hence $\hat{p}_2(x, q_1) \leq p_2^b(xN_1) = p_2^m$.

(b4) This result is immediate from the proof of parts (b1)–(b3). \square

Derivation of $p_1(q_1)$ When $p_2^{1,m} \leq p_2^{2,m}$. The derivation is similar to that in Section 3 and Proposition 1. We first show that mixed strategy REE are never Pareto-dominant REE. If $p_1^{1,m} <$

$p_2^{2,m}$, then $p_2^e(x|q_1)$ is increasing in x and Lemma 2 applies. If $p_1^{1,m} = p_2^{2,m}$, then the proof of Lemma 5 shows that we should distinguish three subcases (b1)–(b3). In cases (b2) and (b3), we have $p_2^e(0|q_1) < p_2^e(x|q_1)$ for any $x \in (0, 1)$ and so Lemma 2 applies again; in case (b1), $p_2^e(x|q_1)$ is constant. An $x \in (0, 1)$ is a Pareto-dominant REE if and only if $p_1 = p_2^m$. A proof similar to Proposition 1, (omitted here for the sake of brevity) now shows that $x = 1$ is the Pareto-dominant REE that is weakly preferred by the firm. \square

Claim 4 *The profit-maximizing price is $p_1(q_1) = p_1^w(q_1)$, and the associated Pareto-dominant REE is $x = 1$.*

Proof: If $p_1 > p_1^w(q_1)$ then $x = 0$ is the unique Pareto-dominant REE. If $p_1 = p_1^w(q_1)$ then $x = 1$ is the Pareto-dominant REE preferred by the firm. If $p_1 < p_1^w(q_1)$ then $x = 1$ is the unique Pareto-dominant REE. The profits are highest for $p_1 = p_1^w(q_1)$. \square

Claim 5 *A period-1 production level $q_1 > N_1$ is never optimal.*

Proof: $p_2^e(x|q_1)$ is weakly decreasing in q_1 . Therefore $p_1(q_1)$ is also weakly decreasing in q_1 . Increasing q_1 above N_1 does not increase sales $Q_1 = \min(xN_1, q_1)$ and but does reduce $p_1(q_1)$ and hence is suboptimal. \square

Proof of Lemma 6: The condition in the lemma is equivalent to $N_2\bar{F}_2(p_2^m) + N_1\bar{F}_1(p_2^m) \geq N_1$ or $Q_2^m \geq N_1$. To prove the result, it suffices to show that $q_1^* \geq Q_2^m$ because this inequality implies that $q_1^* \geq N_1$ —in other words, no excess demand in equilibrium and hence no buying frenzy.

Assume to the contrary that $q_1^* < Q_2^m$. Then $p_2^e(0|q_1) = p_2^m$ and $p_1(q_1) = \mu - \delta^c T_1(p_2^m)$; that is, $p_1(q_1)$ becomes independent of q_1 . The firm's profit function,

$$\pi(q_1) = q_1(\mu - \delta^c T_1(p_2^m)) + \delta^m p_2^e(1|q_1) [(N_1 - q_1)\bar{F}_1(p_2^e(1|q_1)) + N_2\bar{F}_2(p_2^e(1|q_1))],$$

is increasing in q_1 . To see this, observe that

$$\frac{d\pi(q_1)}{dq_1} = \mu - \delta^c T_1(p_2^m) - \delta^m p_2^e(1|q_1)\bar{F}_1(p_2^e(1|q_1)) > 0.$$

The inequality follows because

$$\begin{aligned} \mu - \delta^c T_1(p_2^m) &> p_2^m \bar{F}_1(p_2^m) \\ &\geq p_2^e(1|q_1)\bar{F}_1(p_2^e(1|q_1)) \\ &\geq \delta^m p_2^e(1|q_1)\bar{F}_1(p_2^e(1|q_1)). \end{aligned}$$

The second inequality follows because $p\bar{F}_1(p)$ is single-peaked at $p_2^{1,m}$ and because (i) $p_2^{1,m} < p_2^m < p_2^e(1|q_1)$ if $p_2^{2,m} > p_2^{1,m}$ and (ii) $p_2^e(1|q_1) \leq p_2^m \leq p_2^{1,m}$ if $p_2^{2,m} \leq p_2^{1,m}$. Therefore, $q_1 = Q_2^m$ —a contradiction. \square

Proof of Proposition 3: (i) $p_2^{1,m} < p_2^{2,m}$. Then $p_2^e(0|q_1) < p_2^e(1|q_1)$ by Lemma 5, so a buying frenzy occurs whenever $q_1 < N_1$. In other words, a sufficient condition is

$$\left. \frac{d\pi(q_1)}{dq_1} = \mu - \delta^c T_1(p_2^e(0|q_1)) + \delta^c q_1 \bar{F}_1(p_2^e(0|q_1)) \frac{dp_2^e(0|q_1)}{dq_1} - \delta^m p_2^e(1|q_1) \bar{F}_1(p_2^e(1|q_1)) \right|_{q_1=N_1} < 0 \quad (10)$$

or, equivalently, $\delta^c > \delta_2^c$. Observe that the coefficient of δ^c in (10) is negative because $p_2^e(0|q_1)$ is decreasing in q_1 . To see this, note that $p_2^e(0|q_1) = \min(p_2^b(xN_1), \hat{p}_2(x, q_1))$ by Lemma 4. Because $p_2^b(xN_1)$ is constant in q_1 and $\hat{p}_2(x, q_1)$ is decreasing in q_1 (see (5)), we conclude that $\frac{dp_2^e(0|q_1)}{dq_1} \leq 0$.

(ii) $p_2^{1,m} = p_2^{2,m}$. Since the optimal production level is at most N_1 (Claim 5), it follows from Lemma 5 that $p_2^e(1|q_1) = p_2^m = p_2^{1,m} = p_2^{2,m}$ and so is constant in q_1 . Unlike case (i), in this case a buying frenzy occurs only if $q_1^* \in (Q_2^m, N_1)$. That is, we must also eliminate the corner at Q_2^m in which no frenzy occurs because $p_2^e(1|Q_2^m) = p_2^e(0|Q_2^m)$ (see Lemma 5). Hence a set of sufficient conditions for a buying frenzy equilibrium to exist is $\frac{d\pi}{dq_1} |_{q_1=N_1} < 0$ and $\frac{d\pi}{dq_1} |_{q_1=Q_2^m} > 0$ or $\delta_2^c < \delta^c < \delta_1^c$. Similarly to case (i), these conditions are not necessary and do not guarantee the *uniqueness* of the frenzy equilibrium (unlike the model in Section 3 and Proposition 2).

Finally, consider the case $f_2(\cdot) = f_1(\cdot) = f(\cdot)$ discussed in the text. In this case, the firm's optimal policy is to induce a unique buying frenzy when $F(p_2^m) > \frac{N_2}{N_1+N_2}$ and $\delta_2^c < \delta^c < \delta_1^c$ where $\delta_1^c = \frac{\mu - \delta^m p_2^m \bar{F}(p_2^m)}{T_1(p_2^m) + \frac{Q_2^m \bar{F}_1(p_2^m)}{(N_1+N_2)f(p_2^m)}}$ and $\delta_2^c = \frac{\mu - \delta^m p_2^{2,m} \bar{F}(p_2^{2,m})}{T_1(p_2^w(N_1)) + \frac{N_1 \bar{F}(p_2^w(N_1))}{(N_1+N_2)f(p_2^w(N_1))}}$. Uniqueness of the equilibrium follows because the objective function over the relevant values of q_1 is strictly concave (cf. the proof of Proposition 2). \square

7. Frenzies with Second-Period Arrivals when $p_2^{1,m} > p_2^{2,m}$

This appendix extends the analysis in Section 5 to the case $p_2^{1,m} > p_2^{2,m}$. We first characterize the behavior of $p_2^e(x|q_1)$, which is significantly different in this case.

Lemma 7 *Assume that $p_2^{1,m} > p_2^{2,m}$. (i) If $q_1 \leq Q_2^m$, then $p_2^e(x|q_1) = p_2^b(xN_1)$ with $\frac{dp_2^e(x|q_1)}{dx} < 0$ for $x \in [0, \frac{q_1}{N_1}]$ and $p_2^e(x|q_1) = p_2^b(q_1)$ for $x \in [\frac{q_1}{N_1}, 1]$. (ii) If $Q_2^m \leq q_1 \leq N_1 + N_2 \bar{F}_2(p_2^{2,m})$ then: $p_2^e(x|q_1) = \hat{p}_2(x, q_1)$ with $\frac{dp_2^e(x|q_1)}{dx} > 0$ for $x \in [0, \hat{x}(q_1)]$; $p_2^e(x|q_1) = p_2^b(xN_1)$ with $\frac{dp_2^e(x|q_1)}{dx} < 0$ for $x \in [\hat{x}(q_1), \frac{q_1}{N_1}]$; and $p_2^e(x|q_1) = p_2^b(q_1)$ for $x \in [\frac{q_1}{N_1}, 1]$. (iii) If $q_1 \geq N_1 + N_2 \bar{F}_2(p_2^{2,m})$ then $p_2^e(x|q_1) = \hat{p}_2(x, q_1)$ with $\frac{dp_2^e(x|q_1)}{dx} > 0$ for $x \in [0, 1]$.*

In Lemma 7, $\hat{x}(q_1)$ —and the conditions under which it exists and is unique—are the same as in Section 5. Figure 7 schematically shows $p_2^e(x|q_1)$ in the five possible cases implied by Lemma 7. In effect, $p_2^e(x|q_1)$ can take three shapes. When case (i) in Lemma 7 applies, $p_2^e(x|q_1)$ is weakly decreasing as in panels (b) and (c). Panels (a) and (d) correspond to case (ii) where, $p_2^e(x|q_1)$

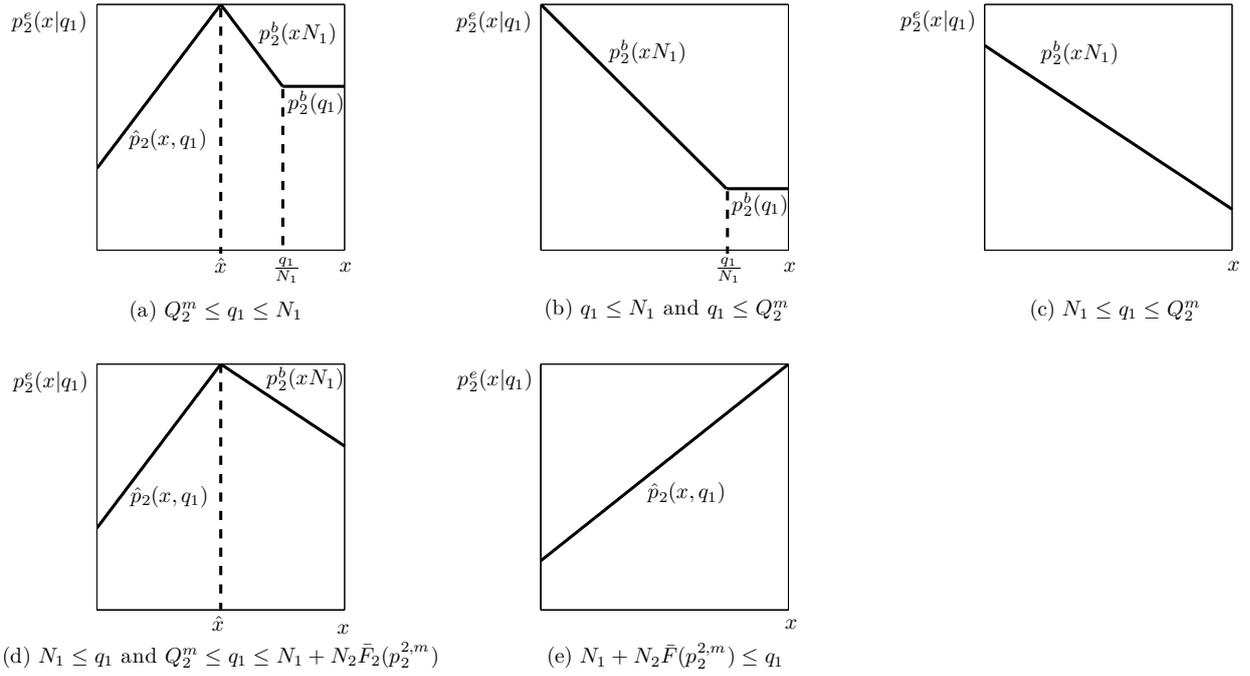


Figure 7 $p_2^e(x|q_1)$ when $p_2^{1,m} > p_2^{2,m}$.

is increasing up to a threshold and then is weakly decreasing. Finally, in panel (e), $p_2^e(x|q_1)$ is increasing in x . The important difference between the analysis here and that in Sections 3 and 5 is that the expected period-2 price, $p_2^e(x|q_1)$, is increasing in x only in panel (e). Otherwise, $p_2^e(x|q_1)$ may be decreasing in x , which implies that $p_2^e(0|q_1)$ is not always smaller than $p_2^e(x|q_1)$. Thus the waiting equilibrium need not Pareto dominate all mixed strategy REE. We rule out all mixed strategy REE such that $p_2^e(x|q_1) > p_2^e(0|q_1)$ by applying Lemma 2, and we show in Appendix B.1 that the remaining mixed strategy REE are equivalent to a pure strategy REE associated with a different firm announcement. Therefore, it suffices to look for Pareto-dominant REE among the pure strategy REE. These results are summarized in the following proposition.

Proposition 4 *Assume that the period-1 production is q_1 . Without loss of generality, we can restrict the analysis to the announcement $\tilde{p}_1(q_1) = \min(p_1^b(q_1), p_1^w(q_1))$ and the associated pure strategy, Pareto-dominant REE $x = 1$.*

The interpretation of $\tilde{p}_1(q_1)$ in Proposition 4 differs from that of $p_1(q_1)$ in Section 5: whereas $p_1(q_1)$ was the profit-maximizing price, here $\tilde{p}_1(q_1)$ is the profit-maximizing price conditional on selling q_1 . The difference is that, in the former, selling q_1 was always optimal. However, if $p_2^{1,m} > p_2^{2,m}$ then not selling at all (i.e., $x = 0$) may dominate selling at $\tilde{p}_1(q_1)$.¹² Yet the profits under $x = 0$ are weakly

¹² This will be the case when the profits from selling $\min(N_1, q_1)$ at $\tilde{p}_1(q_1)$ (i.e., $\tilde{p}_1(q_1)q_1 + \delta_m p_2^b(q_1)Q_2^b(p_2^b(q_1), q_1)$) are lower than the profits under $x = 0$ (i.e., $\delta^m Q_2^m p_2^m$ if $q_1 \leq Q_2^m$ and $\min(N_1, q_1)\hat{p}_2(q_1)$ otherwise).

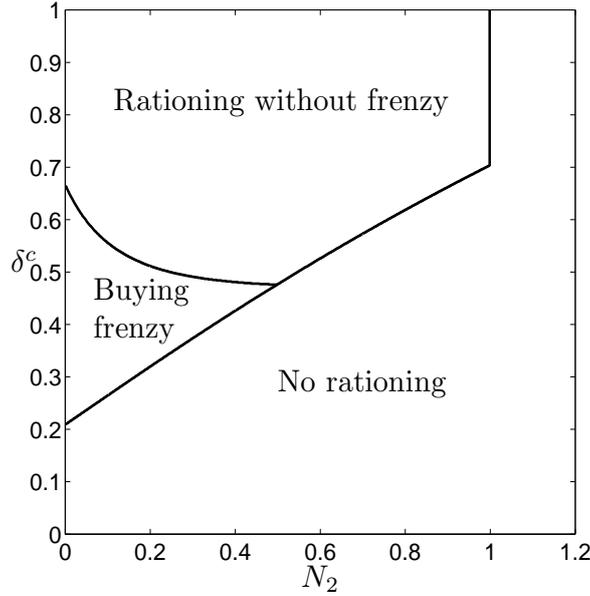


Figure 8 The firm's optimal policy when $p_2^{1,m} > p_2^{2,m}$. The valuation distribution of early arrivals is $U[0, b_1]$ and that of late arrivals is $U[0, b_2]$. In this figure, $\delta^m = 1$, $N_1 = 1$, and $(b_1, b_2) = (10, 5)$.

dominated by profits under alternative announcement $q_1 = 0$ and $p_1 > v_{\max}$, which implies that such early production volumes will never be the firm's choice in equilibrium. The firm optimization problem for determining the production level in period 1 is thus

$$\max_{q_1} \tilde{p}_1(q_1)q_1 + \delta^m p_2^e(1|q_1)Q_2^b(p_2^e(1|q_1), q_1). \quad (11)$$

We now focus on the conditions that induce a buying frenzy in the market. Figures 6(a) and 6(b) display the only equilibrium period-2 prices that can be part of a frenzy because a buying frenzy requires that $q_1^* < N_1$. However, such a frenzy cannot occur when $p_2^e(x|q_1)$ is as in Figure 7(b)—that is, when $q_1 \leq N_1$ and $q_1 < Q_2^m$. In this case $p_2^e(1|q_1) < p_2^e(0|q_1)$, which implies that $\tilde{p}_1(q_1) = p_1^b(q_1)$ by Proposition 4. Hence an individual customer is indifferent between buying and waiting. As a result, a buying frenzy is possible only under conditions of Figure 7(a). If $p_2^b(q_1) \leq p_2^e(0|q_1)$, then an individual customer is indifferent between buying and waiting because both strategies yield the surplus $\mu - p_1 = \delta^c T_1(p_2^b(q_1))$. Therefore, a frenzy can occur only if $\hat{p}_2(0, q_1) < p_2^b(q_1)$. For general distributions f_i , $i \in \{1, 2\}$, this inequality does not yield a threshold value for q_1 and we are therefore unable to establish sufficient conditions for a frenzy to occur. Instead, one must directly check that $q_1^* < N_1$ and $\hat{p}_2(0, q_1^*) < p_2^b(q_1^*)$.

Figure 8 shows the firm's optimal policy when customer valuations are uniformly distributed. This graph is derived by computing the firm's optimal policy and then checking the conditions $q_1^* < N_1$ and $\hat{p}_2(0, q_1^*) < p_2^b(q_1^*)$ directly.

7.1. Proofs of results in the extension

Proof of Lemma 7: A result similar to Claim 3 holds when $p_2^{1,m} > p_2^{2,m}$ (the same proof applies). We define the function $G(x)$ as in (9).

(i) From Lemma 4 it follows that if $x \in [0, \frac{q_1}{N_1}]$, then $p_2^e(x|q_1) = \min(p_2^b(xN_1), \hat{p}_2(x, q_1))$. Because $q_1 \leq Q_2^m$, we have $G(x) \geq 0$ for all $x \in [0, 1]$ (see (9)); hence $p_2^e(x|q_1) = p_2^b(xN_1)$, which is decreasing in x because $p_2^{1,m} > p_2^{2,m}$ by Lemma 3. Lemma 3 shows that if $x \in [\frac{q_1}{N_1}, 1]$ then $p_2^e(x|q_1) = p_2^b(q_1)$, which is constant in x .

(ii) If $Q_2^m \leq q_1 \leq N_1 + N_2 \bar{F}_2(p_2^{2,m})$, then $G(x)$ has a solution $\hat{x}(q_1) \in [0, 1]$ and consequently $p_2^e(x|q_1) = \hat{p}_2(x, q_1)$ for $x \in [0, \hat{x}(q_1)]$ and also $p_2^e(x|q_1) = p_2^b(xN_1)$ for $x \in [\hat{x}(q_1), \frac{q_1}{N_1}]$. By Lemma 3, $p_2^b(xN_1)$ is decreasing in x and $\hat{p}_2(x, q_1)$ is increasing in x (see the proof of Lemma 5). Finally, by Lemma 4, if $x \in [\frac{q_1}{N_1}, 1]$ then $p_2^e(x|q_1) = p_2^b(q_1)$, which is constant in x .

(iii) If $q_1 \geq N_1 + N_2 \bar{F}_2(p_2^{2,m})$ then $G(x) \leq 0$ for all $x \in [0, 1]$; this implies that $p_2^e(x|q_1) = \hat{p}_2(x, q_1)$, which is increasing in x (see the proof of Lemma 5). \square

Proof of Proposition 4: We first show that mixed strategy REE can be ignored without loss of generality. If $p_2^e(1|q_1) \leq p_2^e(0|q_1)$, define $\tilde{x}(q_1) = \min\{x \in (0, 1) \text{ s.t. } p_2^e(x|q_1) = p_2^e(0|q_1)\}$. Lemma 2 implies that we can ignore mixed strategy REE when $p_2^e(1|q_1) > p_2^e(0|q_1)$ and when $p_2^e(1|q_1) \leq p_2^e(0|q_1)$ and $x < \tilde{x}(q_1)$. The only case left to consider occurs when $p_2^e(1|q_1) \leq p_2^e(0|q_1)$ and the mixed strategy $x \geq \tilde{x}(q_1)$. We now focus on this case by dividing the interval into $x \in [\tilde{x}(q_1), \min(\frac{q_1}{N_1}, 1))$ and $x \geq \min(\frac{q_1}{N_1}, 1)$.

Claim 6 *Any mixed strategy REE x such that $x \geq \min(\frac{q_1}{N_1}, 1)$ is Pareto equivalent to the pure strategy REE $x = 1$.*

Proof: Assume that $x_0 \geq \min(\frac{q_1}{N_1}, 1)$ is an REE. Since $p_2^e(x|q_1)$ is constant for $x \in [\min(\frac{q_1}{N_1}, 1), 1]$, it follows that $x = 1$ is also an REE. The two strategies x_0 and $x = 1$ are Pareto equivalent and yield the same firm revenue. It is thus without loss of generality that we ignore x_0 . \square

The only mixed REE left are $x \in [\tilde{x}(q_1), \min(\frac{q_1}{N_1}, 1))$. We cannot eliminate these mixed strategy REE by showing that they are Pareto equivalent to the pure strategy REE $x = 1$, as we did in the proof of Proposition 1, because we can no longer compare the profits under the two REE. So instead we show that, for any REE $x_0 \in [\tilde{x}(q_1), \min(\frac{q_1}{N_1}, 1))$, there exists a *different* firm announcement that induces a Pareto-dominant REE yielding the same firm revenue. Hence x_0 can be ignored without loss of generality.

Claim 7 *Assume that $p_2^e(1|q_1) \leq p_2^e(0|q_1)$ and that $x_0 \in [\tilde{x}(q_1), \min(\frac{q_1}{N_1}, 1))$ is an REE for firm announcement (q_1, p_1) . Then, $x' = 1$ is a Pareto-dominant REE that is weakly preferred by the firm when announcing $(q'_1 = x_0 N_1, p'_1 = p_1)$, and it gives the same revenue as does the REE x_0 .*

Proof: We first derive the period-2 expected price for firm announcement (q'_1, p'_1) . Because $q'_1 = x_0 N_1 < N_1$, Lemma 7 implies that $p_2^e(x|q'_1) = \hat{p}_2(x|q'_1)$ for $x \leq \hat{x}(q'_1)$, $p_2^e(x|q'_1) = p_2^b(xN_1)$ for $x \in [\hat{x}(q'_1), x_0]$, and $p_2^e(x|q'_1) = p_2^b(x_0 N_1)$ for $x \geq x_0$. Note that $\hat{x}(q'_1)$ might not exist, in which case $p_2^e(x|q'_1) = p_2^b(xN_1)$ for $x \in [0, x_0]$ and $p_2^e(x|q'_1) = p_2^b(x_0 N_1)$ for $x \in [x_0, 1]$. Our proof will apply in both cases.

The next step is to show that $x \in [x_0, 1]$ are the only Pareto-dominant REE for the announcement $(q'_1 = x_0 N_1, p'_1 = p_1)$. Doing so requires that we rule out $x \in [0, x_0)$. For any $x \in [0, x_0)$, we have $\mu - p'_1 = \mu - p_1 = \delta_c T_1(p_2^e(x_0|q_1)) = \delta_c T_1(p_2^e(x_0|q'_1)) > \delta_c T_1(p_2^e(x|q'_1))$. The second equality holds because x_0 is a mixed strategy for the announcement (q_1, p_1) , the third equality holds because $p_2^e(x_0|q_1) = p_2^b(x_0 N_1) = p_2^e(x_0|q'_1)$, and the inequality follows because $p_2^e(x_0|q'_1) = p_2^b(x_0 N_1)$ and $p_2^e(x|q'_1) = \min(\hat{p}_2(x, q'_1), p_2^b(xN_1))$. Moreover, since $\hat{p}_2(x, q_1)$ is decreasing in q_1 , it follows that $\hat{p}_2(x, q'_1) \geq \hat{p}_2(x, q_1) > p_2^b(x_0 N_1)$. Finally, $p_2^b(xN_1)$ is decreasing in x by Lemma 3 and so $p_2^b(xN_1) > p_2^b(x_0 N_1)$. We conclude that $p_2^e(x_0|q'_1) < p_2^e(x|q'_1)$. Therefore, buying dominates waiting for $x \in [0, x_0)$ and x cannot be an REE.

Consider next $x \geq x_0$. We have $\mu - p'_1 = \mu - p_1 = \delta_c T_1(p_2^e(x_0|q_1)) = \delta_c T_1(p_2^e(x_0|q'_1)) = \delta_c T_1(p_2^e(x|q'_1))$ (because $x \geq x_0 = \frac{q'_1}{N_1}$). Any $x \in [x_0, 1]$ is then an REE and also a Pareto-dominant REE because the consumer surplus is constant and equal to $\mu - p_1$.

We conclude by comparing the profits under (q_1, p_1, x_0) and (q'_1, p'_1, x) for $x \in [x_0, 1]$. The prices are equal, $p_1 = p'_1$, and so are sales: $\min(xN_1, q'_1) = x_0 N_1 = \min(x_0 N_1, q_1)$. The firm's profit is the same in each case in period 1 and also period 2. Therefore, the firm is indifferent between REE x_0 for announcement (q_1, p_1) and Pareto-dominant REE $x' = 1$ for announcement (q'_1, p'_1) . \square

Now we have only to consider the pure strategy, Pareto-dominant REE $x = 0$ and $x = 1$. We distinguish two cases. If $p_1^w(q_1) \leq p_1^b(q_1)$, then $p_1 = p_1^w(q_1)$ is the highest period-1 price such that $x = 1$ is a Pareto-dominant REE; otherwise, $p_1 = p_1^b(q_1)$ is the highest price such that $x = 1$ is a Pareto-dominant REE. The function $\tilde{p}_1(q_1) = \min(p_1^w(q_1), p_1^b(q_1))$ matches the highest period-1 price in each case.

It is important to observe that, for a given q_1 , $x = 0$ is also a Pareto-dominant REE for $(q_1, \tilde{p}_1(q_1))$. For $x = 1$, firm profits are $\tilde{p}_1(q_1)q_1 + \delta_m p_2^b(q_1)Q_2^b(p_2^b(q_1), q_1)$; for $x = 0$, firm profits are $\delta^m Q_2^m p_2^m$ if $q_1 \leq Q_2^m$ and $q_1 \hat{p}_2(0, q_1)$ otherwise. The firm may prefer the Pareto-dominant REE $x = 0$, but the profits under $x = 0$ are weakly dominated by the profits under firm announcement $(q_1 = 0, p_1 = \infty)$. In other words, it is not optimal for the firm to choose such values for q_1 . We can therefore assume (without loss of generality) that the Pareto-dominant REE $x = 1$ is associated with announcement $\tilde{p}_1(q_1)$ even though it is not necessarily the preferred Pareto-dominant REE. This completes the proof of Claim 7. \square